



On unsteady two-dimensional convective heat and mass transfer flow of incompressible viscous fluid through a vertical porous medium

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Abstract: In this paper we consider unsteady two-dimensional convective heat and mass transfer flow of a viscous, incompressible, electrically conducting optically thin fluid which is bounded by a vertical infinite plane surface. A uniform applied homogeneous magnetic field is considered in the transverse direction with first order chemical reaction. An analytical solution for two-dimensional oscillatory flow on unsteady mixed convection of an incompressible viscous fluid, through a porous medium bounded by an infinite vertical plate in presence of chemical reaction and thermal radiation are presented. The surface absorbs the fluid with a constant suction and the free stream velocity oscillates about a constant mean value. The resulting nonlinear partial differential equations were transformed into a set of ordinary differential equations using two-term series. The closed form solutions for velocity, temperature, concentration, skin friction, Nusselt number, and Sherwood number have been obtained, using the regular perturbation technique. Numerical evaluation of the analytical solutions were performed and the results are presented in graphical and tabular form. This was done to illustrate the influence of the various parameters involved in the problem on the solutions.

Keywords: MHD flow, Oscillatory flow, Heat and mass transfer, Thermal radiation -Chemical reaction.
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1. Introduction

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. The application of MHD flow is found in metrology, solar physics and in motion of earth's core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics [1,2].

Oscillatory flows has known to result in higher rates of heat and mass transfer, many studies have been done to understand its characteristics in different systems such as reciprocating engines, pulse combustors and chemical reactors. The detailed study on fluid mechanics of oscillatory and modulated flows have been made by Cooper et al. [3]. The numerically studied the influence of convective heat transfer from periodic open cavities in a channel with oscillatory flow have been studied by Fusegi [4], Gomaa and Taweel [5] and Abdelkader and Lounes [6] respectively. The influence of MHD oscillatory flow on free convection radiation through a porous medium with constant suction velocity and the effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel have been studied by El-Hakim [7], Makinde and Mhone [8] and Mehmood and Ali [9] respectively.

The MHD oscillatory flow past a vertical porous plate through porous medium in the presence of thermal and mass diffusion with constant heat source has been studied by Gholizadeh [10]. The work of Makinde [11] is of particular interest since it demonstrated the possibility of achieving significant unsteady incompressible flow in a porous channel. The MHD mixed convection from a vertical plate embedded in porous medium with convective boundary condition has been analyzed by Makinde and Aziz [12]. Rabi et al [2] have investigated the influence of chemical reaction effect on MHD oscillatory flow through a porous medium bounded by two vertical porous plates with heat source Soret effect.

Chemical reactions are classified as either heterogeneous or homogeneous processes. These processes take place in numerous industrial applications, e.g., polymer production, manufacturing of ceramics and food processing Cussler, [13]. The diffusion of chemically reactive species in a laminar boundary layer flow are analyzed by Chambre and Young [14]. The exact solution for hydrodynamic boundary layer flow and heat transfer is studied by Vajravelu [15]. Das et al. [16], Muthucumaraswamy [17, 18] and Anjali Devi and Kandasamy [19] studied the effect of a first-order chemical reaction on the flow in different cases. An analytical



solution for heat and mass transfer by laminar flow of a Newtonian, viscous, electrically conducting fluid is presented by Chamkha [20]. Kandasamy et al. [21] studied the nonlinear MHD flow, with heat and mass transfer characteristics, of an incompressible, viscous, electrically conducting, Boussinesq fluid with chemical reaction and thermal stratification effects.

The role of thermal radiation is of major importance in engineering areas occurring at high temperatures and knowledge of radiative heat transfer becomes very important in nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles and space vehicles. Hakeem and Sathiyathan [22], Srinivas and Muthuraj [23], Pal and Talukdar [24], Chamkha [25], Bakr [26] and Prakash et al. [27] have examined the radiation effect of an oscillatory flow under different conditions. The influence of chemical reaction on unsteady MHD slip flow in a Planer channel with varying concentration has been examined by Sivaraj et al. [28].

The main objective of this study is to investigate the effect of periodic heat and mass transfer on unsteady mixed convection MHD flow past an infinite vertical porous flat plate with constant suction in presence of chemical reaction and thermal radiation when the free stream velocity is oscillating with time. The boundary layer equations governing the problem under consideration are solved by multi-parameter perturbation technique and giving more importance on analytic solution. For this study, air ($Pr=0.71$) and water ($Pr=6.2$) are the only fluids under consideration. The closed form solutions for velocity, temperature, skin friction, concentration, Nusselt number, and Sherwood number are presented. The effects of pertinent parameters on fluid flow of heat and mass transfer characteristics are studied in detail. This work is presented as follows. First, the problem is formulated, and then the solution of the problem is presented. Following are results and discussion, and finally, conclusions are summarized.

2. Formulation of the problem

Consider unsteady two-dimensional convective heat and mass transfer flow of a viscous, incompressible, electrically conducting optically thin fluid which is bounded by a vertical infinite plane surface. A uniform applied homogeneous magnetic field is considered from the transverse direction with first order chemical reaction. We assume that the surface absorbs the fluid with a constant velocity and the fluid far away the surface oscillates with time and assumed to be in the form $\bar{U}(t) = U_0(1 + \varepsilon e^{i\alpha t})$, where U_0 is the mean stream velocity and $\varepsilon (\ll 1)$ the amplitude of the free stream variation. A Cartesian coordinate system (x', y') is assumed, where x' -axis lies along the plate and y' -axis in the normal direction. Then, under the usual Boussinesq's approximation, the equation governing flow field under consideration are

Continuity equation

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0$$

Momentum equation

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' + g\beta(T' - T'_\infty) + g\beta_c(C' - C'_\infty) \quad (1)$$

Energy equation

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y'} \quad (2)$$

Concentration equation

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - \gamma(C' - C'_\infty) \quad (3)$$

where (u', v') are velocity components in x' and y' directions, g is acceleration due to gravity, T' is the temperature of the fluid, C' is the species concentration, β is the coefficient of thermal expansion, β_c is the volumetric expansion coefficient, ν is the kinematic viscosity of the fluid, k is effective thermal conductivity, ρ is the density of the fluid, c_p is the specific heat at constant pressure, D is the diffusion coefficient, B_0 is



the electromagnetic induction, σ is the conductivity of the fluid, v_0 constant suction/ injection and p is non dimensional pressure.

The corresponding boundary conditions of the problem are:

$$\begin{aligned} u' = 0, \quad T' = T'_w + (T'_\infty - T'_w), \quad C' = C'_w + \varepsilon(C'_\infty - C'_w)e^{i\omega t'} \text{ at } y' = 0 \\ u' = U', \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{at } y' \rightarrow \infty \end{aligned} \quad (5)$$

And the radiative heat flux is given by

$$\frac{\partial q_r}{\partial y'} = 4\alpha^2(T'_0 - T') \quad (6)$$

Where α – is the radiation absorption coefficient .

Eliminating the modified pressure gradient under the usual boundary layer approximation (2) reduced to

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \frac{\partial U'}{\partial t'} + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} (u' - U') + g\beta(T' - T'_\infty) + g\beta_c(C' - C'_\infty) \quad (7)$$

introducing the following dimensionless quantities:

$$\begin{aligned} y = \frac{v_0}{\nu} y', \quad t = \frac{v_0^2}{\nu} t', \quad \omega = \frac{\nu \omega'}{v_0^2}, \quad u = \frac{u'}{U_0}, \quad U = \frac{U'}{U_0}, \\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad k = \frac{v_0^2}{\nu^2} k' \end{aligned} \quad (8)$$

With the help of the non-dimensional variables above, Eqs (1), (3), (4) and (7) reduced to

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_c \phi + M(U - u) \quad (9)$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{p_r} \frac{\partial^2 \theta}{\partial y^2} + N\theta \quad (10)$$

$$\frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - K_r \phi \quad (11)$$

The reduced boundary conditions are;

$$\begin{aligned} u = 0, \quad \theta = 1 + \varepsilon e^{i\omega t} \quad \phi = 1 + \varepsilon e^{i\omega t} \quad \text{at } y = 0 \\ u = 1 + \varepsilon e^{i\omega t}, \quad \theta = 0, \quad \phi = 0 \quad \text{at } y \rightarrow \infty \end{aligned} \quad (12)$$

Where $G_r = \frac{\nu g \beta}{U_0 v_0^2} (T'_w - T'_\infty)$ is Grashof number, $G_c = \frac{\nu g \beta_c}{U_0 v_0^2} (C'_w - C'_\infty)$ is the modified Grashof number, $M = \frac{\sigma \beta_0^2 \nu}{\rho v_0^2}$ is magnetic number, $Sc = \frac{\nu}{D}$ is the Schmidt number, $p_r = \frac{\rho \nu c_p}{k}$ is Prandtl number, $K_r = \frac{\nu \gamma}{v_0^2}$ is the chemical reaction parameter and $N = \frac{4\alpha^2 \nu}{\rho c_p v_0^2}$ is radiation parameter

3. Method of Solution



In order to solve the differential Eqs. (9), (10) and (11) we assume that

$$\begin{aligned} u(y,t) &= u_0(y) + \varepsilon e^{i\omega t} u_1(y), \\ \theta(y,t) &= \theta_0(y) + \varepsilon e^{i\omega t} \theta_1(y), \\ \phi(y,t) &= \phi_0(y) + \varepsilon e^{i\omega t} \phi_1(y) \end{aligned} \tag{13}$$

Where u_0, θ_0 and ϕ_0 are respectively the mean velocity, mean temperature and mean concentration. On using (13) into (9), (10) and (11) and neglecting the higher order of ε , and simplifying we get the following set of equations

$$\left[\frac{d^2}{dy^2} + \frac{d}{dy} - M \right] u_0 = -G_r \theta_0 - G_c \phi_0 - M \tag{14}$$

$$\left[\frac{d^2}{dy^2} + \frac{d}{dy} - M \right] \theta_0 = 0 \tag{15}$$

$$\left[\frac{d^2}{dy^2} + \frac{d}{dy} - M \right] \phi_0 = 0 \tag{16}$$

$$\left[\frac{d^2}{dy^2} + \frac{d}{dy} - M \right] u_1 = -G_r \theta_1 - G_c \phi_1 - (M + i\omega) \tag{17}$$

$$\left[\frac{d^2}{dy^2} + p_r \frac{d}{dy} + p_r (N - i\omega) \right] \theta_1 = 0 \tag{18}$$

$$\left[\frac{d^2}{dy^2} + Sc \frac{d}{dy} + Sc(K_r + i\omega) \right] \phi_1 = 0 \tag{19}$$

The corresponding reduced boundary conditions are

$$\begin{aligned} u_0 = 0, u_1 = 0, \quad \theta_0 = 1, \theta_1 = 1, \quad \phi_0 = 1, \phi_1 = 1 \quad \text{at } y = 0 \\ u_0 = 1, u_1 = 1, \quad \theta_0 = 0, \theta_1 = 0, \quad \phi_0 = 0, \phi_1 = 0 \quad \text{at } y \rightarrow \infty \end{aligned} \tag{20}$$

Solving equations (14) to (19) with the conditions in Eq. (20), we get

$$\begin{aligned} u = -A_1 e^{m_1 y} - A_2 e^{m_2 y} + (A_1 + A_2 - 1)e^{m_3 y} + 1 + \\ \varepsilon e^{i\omega t} (-A_3 e^{m_4 y} - A_4 e^{m_5 y} + (A_3 + A_4 - 1)e^{m_6 y} + 1) \end{aligned} \tag{21}$$

$$\theta = e^{m_1 y} + \varepsilon e^{i\omega t} e^{m_4 y}, \tag{22}$$

$$\phi = e^{m_2 y} + \varepsilon e^{i\omega t} e^{m_5 y}, \tag{23}$$

Where

$$m_1 = \frac{-pr - \sqrt{pr^2 - 4prN^2}}{2},$$

$$m_2 = \frac{-Sc - \sqrt{Sc^2 + 4ScKr}}{2},$$



$$m_3 = \frac{-1 - \sqrt{1 + 4M}}{2},$$

$$m_4 = \frac{-pr - \sqrt{pr^2 - 4pr(N^2 - i\omega)}}{2},$$

$$m_5 = \frac{-Sc - \sqrt{Sc^2 + 4Sc(kr + i\omega)}}{2},$$

$$m_6 = \frac{-1 - \sqrt{1 + 4(M + i\omega)}}{2},$$

$$A_1 = \frac{G_r}{m_1^2 + m_1 - M},$$

$$A_2 = \frac{G_c}{m_2^2 + m_2 - M},$$

$$A_3 = \frac{G_r}{m_4^2 + m_4 - (M + i\omega)},$$

$$A_2 = \frac{G_c}{m_5^2 + m_5 - (M + i\omega)},$$

The local skin friction coefficient, local Nusselt number and local Sherwood number are important physical quantities for this type of heat and mass transfer problem. These are defined as follows:
 The dimensionless shearing stress on the surface of a body is written as:

$$\tau'_\omega = \mu \left. \frac{\partial u'}{\partial y'} \right|_{y'=0} \quad (24)$$

There, the local skin-friction factor is given by:

$$C_f = \frac{2\tau'_\omega}{\rho U_0 V_0} = 2 \left. \frac{\partial u}{\partial y} \right|_{y=0} \quad (25)$$

The rate of heat transfer at the surface in terms of the local Nusselt number can be written as:

$$Nu = \frac{x}{T_\infty - T_\omega} \left. \frac{\partial T'}{\partial y'} \right|_{y=0} \quad (26)$$

$$Nu \text{Re}_x^{-1} = - \left. \frac{\partial \theta}{\partial y} \right|_{y=0} \quad (27)$$

Where $\text{Re}_x^{-1} = -\frac{xV_0}{\nu}$ is the local Reynolds number.

The rate of heat transfer at the surface in terms of the local Sherwood number can be written as:

$$Sh = \frac{x}{C_\infty - C_\omega} \left. \frac{\partial C'}{\partial y'} \right|_{y=0}$$



$$Sh Re_x^{-1} = -\left. \frac{\partial \phi}{\partial y} \right|_{y=0} \quad (28)$$

Results and Discussion

Numerical evaluation of the analytical solutions reported in the previous section was performed and the results are presented in graphical and tabular form. This was done to illustrate the influence of the various parameters involved in the problem on the solutions. In plotting the results.

The values of Grashof number G_r have been chosen as they are interesting from physical point of view. The free convection currents are due to temperature difference $T_m - T_\infty$ and hence $G_r > 0$ when $T_m - T_\infty > 0$ which physically corresponding to cooling of the plate by free-convection current, $G_r < 0$ corresponds to heating of the boundary surface by free-convection currents as $T_m - T_\infty < 0$ and $G_r = 0$ corresponds to the absence of free convection currents.

Figure 1. demonstrates the variation of the velocity distribution with the chemical reaction in the cases of cooling ($G_r > 0$) and heating ($G_r < 0$) by free convection currents. It observed that the velocity in the cases of cooling and heating decreases near the boundary layer with an increases in the chemical reaction parameter K_r . Also, figure 1 shows the effects of heating of the surface leads to a fall in the velocity inside the boundary layer while the effect of cooling by free convection current, ($G_r > 0$) is to increase the velocity more than in the case of heating of the surface.

Figures 2 and 3 depict the effects of chemical reaction on the concentration profiles with two values of Grashof number $G_r = 2$ (cooling of the surface) and $G_r = -2$ (heating of the surface), respectively. An increase in the values of K_r from 1, 1.5, 2, 2.5, and 3 causes a significant decrease in concentration profiles throughout the concentration boundary layer in the case of cooling and heating of the surface by free convection current.

Figure 4. illustrates the influence of the magnetic parameter M on the velocity profiles with two values of $G_r = -2$ (heating of the surface) and $G_r = 2$ (cooling of the surface). Application of a transfer magnetic field to an electrically conducting fluid gives rise to a resistive-type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer. This is obvious from decreases in the velocity profiles. Moreover, the magnetic field is found to cause an over shoot in the velocity profiles (i.e, the velocity profiles exceed the values at the edge of the momentum boundary layer) in the case of cooling of the surface. However, no overshoot is observed in the case of heating of the surface whatever the values of magnetic field parameter are. Also, the effect of the magnetic field is found to be more pronounced for the case of cooling of the surface than for the case of heating of the surface.

Figure 5 and 6 show the effects of the Grashof number G_r on the velocity profiles in the case of air ($p_r = 0.71$) and water ($p_r = 6.2$). From these figures, we observe that the effects of cooling by free convection currents occur when $G_r > 0$. The velocity increases inside the boundary layer as G_r increases. In addition the curves show that the peak value of the velocity increases rapidly near the surface as the Grashof number increases and then decays to the free-stream velocity in the two cases air and water. Also figures 5 and 6 show the effect of heating by free convection currents when $G_r < 0$. An increase in greater heating of the surface leads to a fall in the velocity in the two cases of air and water. Also, we notice that the effects of Grashof number are found to be more pronounced for the fluid with small Prandtl number.

Figures 7 -9, respectively. It was observed that the velocity and concentration decrease as the Schmidt number Sc increases, that noted in the two cases of Prandtl number $Pr = 0.71, 6.2$ (air and water).

Figure 10 depicts the velocity profile for different values of radiation parameter N in two cases of Prandtl number $Pr = 0.71, 6.2$. It was observed that the velocity decreases as the radiation parameter N decreases, velocity distribution for various values of radiation parameter N in case of the Prandtl number $Pr = 0.71$ has a



clear impact but its effect hasn't a clear impact in case the Prandtl number $Pr = 6.2$ so we zoomed in water case to clarify influence.

The temporal development of temperature with different values of time t is elucidated in fig 11. It is clear that the velocity of the fluid decreases as time increases. that noted in the two cases of Prandtl number $Pr = 0.71, 6.2$, this is more evident in the case of air than in water case. It illustrated that the influence of the various parameters involved in the problem on the velocity of the air is greater than the velocity of the water under the same conditions.

Table 1 and 2 show effects of variations of Grashof number, modified Grashof number, chemical reaction, Schmidt number, magnetic parameter, frequency of the fluid and radiation parameter on the coefficients of skin friction, heat transfer and mass transfer respectively for $\omega t = \pi / 2, \varepsilon = 0.5$ in the two cases air and water respectively. It is clear that as Grashof number, modified Grashof number, Schmidt number, magnetic parameter and radiation parameter increase, skin friction coefficient increases, while decreasing by increasing chemical reaction and frequency of the fluid. Nusselt number decreases as frequency of the fluid and radiation parameter increase and Sherwood number increases as the chemical reaction and Schmidt number increase, while decrease by increasing frequency of the fluid. On the other hand, the Nusselt number increases as Pr increases.

Finally, the effects of various values of t on the coefficients of skin friction, heat transfer and mass transfer across the boundary layer are presented in Table 3. It can be concluded that skin friction decreases as time increases. Heat transfer and mass transfer increasing with time at values 0,10, 20, while decreasing at values 20, 30, 40, 50.

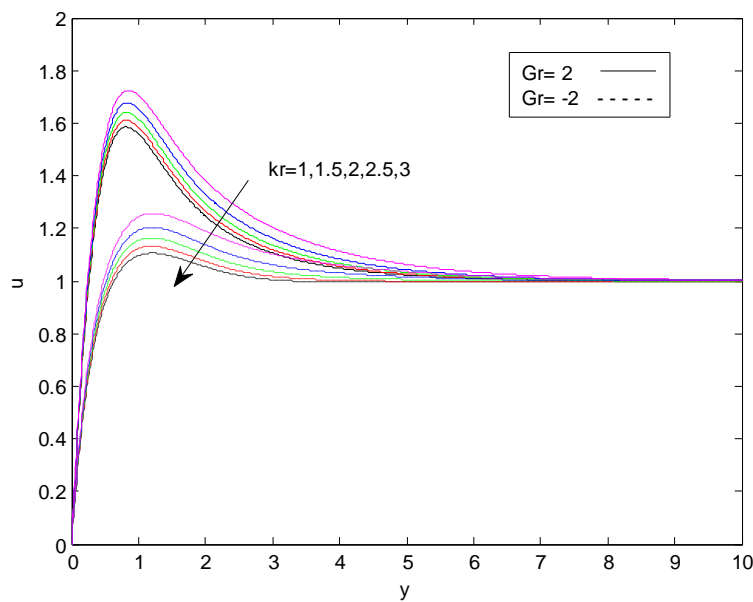


Figure 1. Velocity distribution for various values of Grashof number Gr and chemical reaction parameter k_r for $\omega = 10, \omega t = \pi / 2, \varepsilon = 0.5, N = 0.2, M = 5, G_c = 5, Pr = 0.71, Sc = 0.22$.

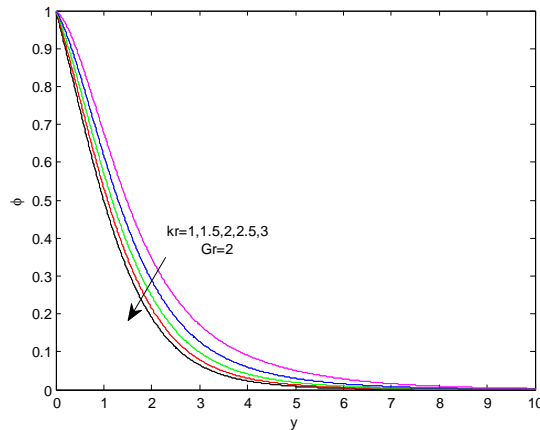


Figure 2. Concentration distribution for various values and chemical reaction parameter k_r for $\omega = 10, \omega t = \pi / 2, \varepsilon = 0.5, N = 0.2, M = 5, G_c = 5, Pr = 0.71, Sc = 0.22$.

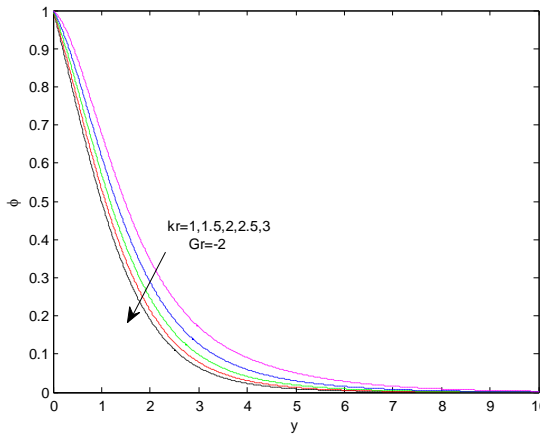


Figure 3. Concentration distribution for various values chemical reaction parameter k_r for $\omega = 10, \omega t = \pi / 2, \varepsilon = 0.5, N = 0.2, M = 5, G_c = 5, Pr = 0.71, Sc = 0.22$.

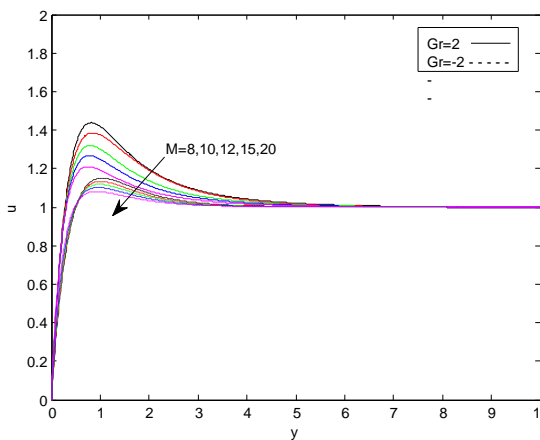


Figure 4. Velocity distribution for various values of Grashof number Gr and magnetic field parameter M for $\omega = 10, \omega t = \pi / 2, \varepsilon = 0.5, N = 0.2, kr = 2, G_c = 5, pr = 0.71, Sc = 0.22$

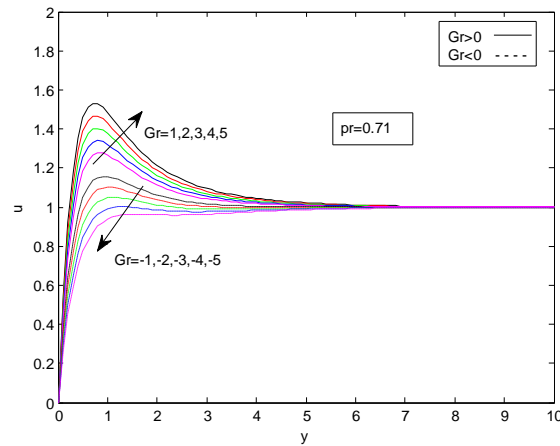


Figure 5. Velocity distribution for various values of Grashof number Gr for $\omega = 10, \omega t = \pi / 2, \varepsilon = 0.5, N = 0.2, M = 10, G_c = 5, kr = 3, Sc = 0.22$.

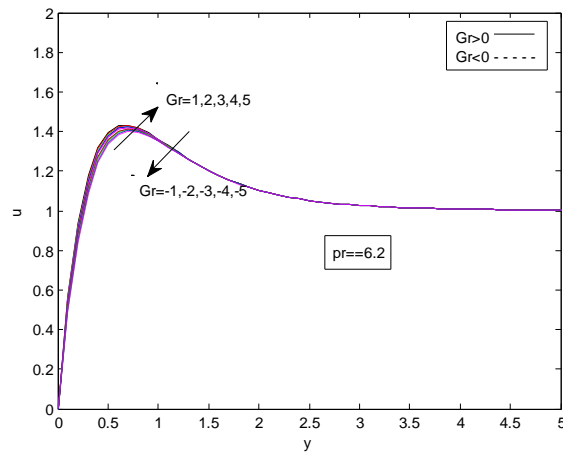


Figure 6. Velocity distribution for various values of Grashof number Gr for $\omega = 10, \omega t = \pi / 2, \varepsilon = 0.5, N = 0.2, M = 10, G_c = 5, kr = 3, pr = 6.2, Sc = 0.22$.

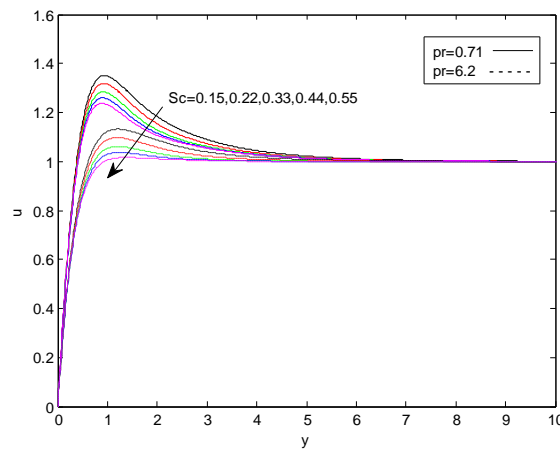


Figure 7. Velocity distribution for various values of Prandtl number Pr and the Schmidt number Sc for $\omega = 10, \omega t = \pi / 2, \varepsilon = 0.5, N = 0.2, M = 5, kr = 3, G_r = G_c = 2$.

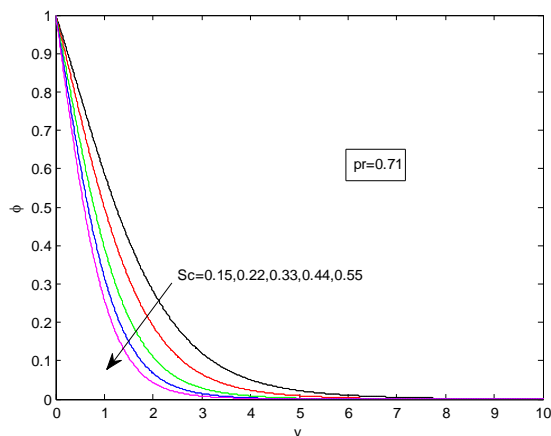


Figure 8. Concentration distribution for various values of the Schmidt number Sc for $\omega=10, \omega t = \pi/2, \varepsilon=0.5, N=0.2, M=5, kr=3, G_r = G_c=2$

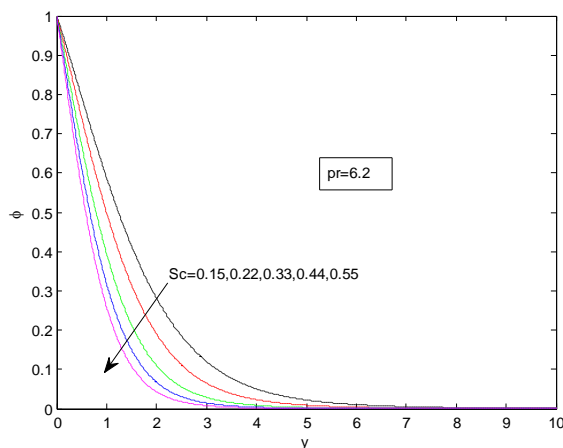


Figure 9. Concentration distribution for various values of the Schmidt number Sc for $\omega=10, \omega t = \pi/2, \varepsilon=0.5, N=0.2, M=5, kr=3, G_r = G_c=3, pr=6.2$

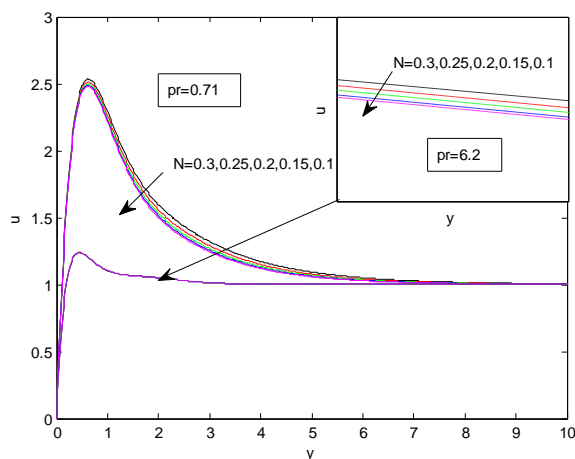


Figure 10. Velocity distribution for various values of radiation parameter N for $\omega=10, \omega t = \pi/2, \varepsilon=0.5, Sc=0.66, M=5, kr=3, G_r = G_c=5$

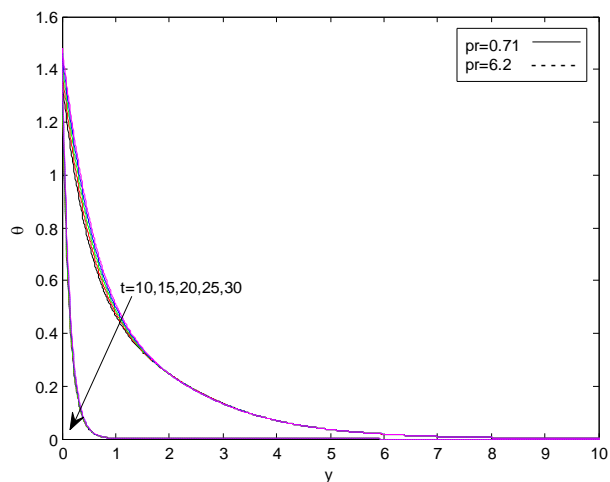


Figure 11. temperature distribution for various values of Prandtl number Pr and the time t for $\omega = 5$, $\omega t = \pi / 2$, $\varepsilon = 0.5$, $N = 0.2$, $M = 5$, $kr = 6$, $G_r = G_c = 5$, $Sc = 0.22$

Table 1. Effects of variations of Grashof number, modified Grashof number, chemical reaction, Schmidt number, magnetic parameter, frequency of the fluid and radiation parameter on the coefficients of skin friction, heat transfer and mass transfer respectively for $p_r = 0.71$, $\omega t = \pi / 2$ and $\varepsilon = 0.5$.

Pr	Gr	Gc	kr	Sc	M	ω	N	C_f	$Nu Re_x^{-1}$	$Sh Re_x^{-1}$
0.71	-2	-2	1	0.22	5	5	0.2	0.9681985	0.0104001	0.2278289
	0							2.4504063		
	2							3.9326142		
		0						6.0647835		
		2						8.1969528		
			2					8.0915512		0.4788251
			3					8.0292530		0.6521084
				0.44				8.1965396		0.9988825
				0.66				8.6646388		1.2987692
					10			8.9164825		
					15			10.088027		
						10		9.5646458	-0.268168	0.9979852
					15		9.0859922	-0.481072	0.7734869	
						0.3	9.1014427	-0.546189		
						0.4	9.1369066	-0.686658		

Table 2. Effects of variations of Grashof number, modified Grashof number, chemical reaction, Schmidt number, magnetic parameter, frequency of the fluid and radiation parameter on the coefficients of skin friction, heat transfer and mass transfer respectively for $p_r = 6.2$, $\omega t = \pi / 2$ and $\varepsilon = 0.5$.

Pr	Gr	Gc	kr	Sc	M	w	N	C_f	$Nu Re_x^{-1}$	$Sh Re_x^{-1}$
6.2	-2	-2	1	0.22	5	5	0.2	1.9013410	4.4633888	0.2578289
	0							2.4504063		
	2							2.9994716		
		0						5.1316409		
		2						7.2638104		
			2					7.1584088		0.4788251
			3					7.0961104		0.6521084
				0.44				7.2633971		0.9988814
				0.66				7.7314962		1.2987690
					10			8.1253424		



					15			9.4666094		
						10		8.9347129	3.5775681	0.9979852
						15		8.4445792	2.9172851	0.7734869
							0.3	8.4469053	2.8608185	
							0.4	8.4502701	2.7802543	

Table 3. Effects of various values of time on the coefficients of skin friction, heat transfer and mass transfer respectively for $\omega = 5, \varepsilon = 0.5, N = 0.2, M = 10, G_r = G_c = 2, k_r = 3, Sc = 0.22$.

Pr	T	C_f	$Nu Re_x^{-1}$	$Sh Re_x^{-1}$
0.71	0	14.6751855	1.5203172	1.4799403
	10	14.6269275	1.6628313	1.5335311
	20	14.2328840	1.7356010	1.5448208
	30	13.5206649	1.7335276	1.5130185
	40	12.5401740	1.6567563	1.4403525
	50	11.3601122	1.5106664	1.3319143
6.2	0	13.4775922	9.9940552	1.4799403
	10	13.4600055	10.304803	1.5335310
	20	13.1268349	10.325115	1.5448208
	30	12.5014249	10.053568	1.5130185
	40	11.6275967	9.5091880	1.4403525
	50	10.5665778	8.7301189	1.3319143

Conclusion

An analytical solution for two-dimensional oscillatory flow on unsteady mixed convection of an incompressible viscous fluid, through a porous medium bounded by an infinite vertical plate in presence of chemical reaction and thermal radiation are presented. The governing boundary layer equations for the velocity, temperature, and concentration fields were solved using the method of small perturbation approximation. Numerical evaluation of the analytical solutions were performed and the results were presented in graphical and tabular form. This was done to illustrate the influence of the various parameters involved in the problem on the solutions. Two cases be considered air ($P_r = 0.71$) and water ($P_r = 6.2$). It can concluded that the velocity in the case of cooling and heating decreases near the boundary with an increases in the chemical reaction. Also, increasing the chemical reaction is to decreases the concentration profile throughout the boundary layer. We found also that for two values of prendtl number, under study, the concentration distribution decreases as the Schmidt number increases. Moreover, the Nusselt number decreases as the Prandtl number increases. It is interesting to note that the rate of heat mass transfer increase with increasing time for both air and water when $t \leq 20$, while the reverse behavior is observed when $t \geq 20$.

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