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Study of the Natural Frequencies and Mode Shapes of the Torsional Vibrations of Woodworking Shapers

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Abstract: An investigation of the natural frequencies and mode shapes of the woodworking shaper's saw unit is presented in this study. This study is done on the base of a mechanic-mathematical model for investigation of free torsional vibrations of a woodworking shaper's saw developed by the author. The main features in the construction of a kind of wood shapers are rendered an account of this model. The two most commonly used driving mechanisms are modeled – with a wedge belt and with a ribbed belt. The studies allow a comparison of the vibration behavior of the mechanism for both drives. The results of this study allow the determination of the resonant work regimes. The exactly determination of these regimes is important for introduction of adequate measures which can guarantee their using. The investigation`s results can be used as a base for making some recommendations concerning the increase of reliability of the woodworking shapers as well as the accuracy and quality of their production.

Keywords: woodworking shapers, modeling, torsional vibrations.

I. Introduction

Wood processing through milling is one of the most frequently used methods for wood cutting in the woodworking and furniture industries. The quality of the machined surfaces depends on the selection of the optimal cutting mode [3]. The cutting mode influences the dynamic behavior of the working saw unit of the woodworking shapers. The enhanced requirements for reducing the level of vibration and noise accompanying the operation of modern woodworking machines significantly improve the study of dynamic processes in them [4, 9]. It mainly concerns woodworking shapers which are machines with a high level of vibration and noise. Discovering the causes for originate and increase of the vibration and noise level requires understanding the essence of the phenomena characteristic of the machine and its individual elements [10, 15]. It is necessary to conduct concrete studies in which the machine can be considered as a mechanical vibrating system with known characteristics [1].

The main task of the research – reducing the level of the vibration and noise – needs of formulation concrete measures and ways to influence the vibrating system. This in turn demands the introduction of specific requirements for the construction and operation of its elements. For this pufirstly it is necessary to have mechanical-mathematical modeling and composing of equations describing the vibration of the elements of the woodworking shaper [12, 14]. Well-targeted research can be done by solving these equations in different conditions [11, 13]. Some recommendations for the construction's design and the work regimes of the machine are formed on their base.

The vibration characteristics of the woodworking shaper's saw units depend on their natural vibration frequencies as well as any mechanical vibration system. It is known that when the frequencies of the external influences, which cause vibrations, are equal to a frequency of their natural frequencies, the phenomenon "resonance" appears. Resonance regimes can lead to significant increase of vibration amplitudes. Significant vibration amplitudes change the normal work regimes of the woodworking shaper's saw unit and damage the accuracy and quality of the production [7]. Extra stress, which is caused by increase of vibration amplitudes, sometimes can reach such values that can damage or even destroy some machine's elements. This causes reduction of the reliability of the woodworking shaper.

A cardinal rule in the engineer's practice is that resonant effects are unwilling and they must be avoided by a suitable selection of parameters of the wood shaper's saw units and other details and units of the machine, as well as the work regimes [5]. Therefore, it is necessary to make in advance an evaluation of the resonant danger when the wood shaper is designed and dimensioned. It does not allow danger work regimes during the operation exploitation. To solve this problem it is a must to study the natural frequencies. In cases that the resonant danger is available, some changes in the construction of the wood shaper or in the work regimes are advisable.

The aim of this study is to make an investigation of the natural frequencies and mode shapes of the torsional vibrations of the woodworking shaper's saw unit. The investigation is done on the base of an adequate mechanic-mathematical model for investigation of free torsional vibrations of the woodworking shapers developed by the author. The model presents features in the construction of a kind of woodworking shapers. Two most commonly used driving mechanisms are modelled – with a wedge belt and with a ribbed belt. The studies allow a comparison of the vibration behaviour of the mechanism for both drives. Some recommendations concerning the prevention from the resonant work regimes can be made on the base of this study. It is connected with the increase of reliability of the woodworking shapers as well as with the accuracy and quality of the production.

II. Mechanic-Mathematical Model

The kind of woodworking shapers that are commonly used in the practice of forestry industry [6] are examined in the proposed study. Fig. 1 shows the general view, and Fig. 2 – a scheme of this type of woodworking shapers [8]. The machine body is marked with *1*, *2* is the electric motor, *3* – the belt drive, which can be a wedge belt or a ribbed belt, *4* – the vibration isolators between the machine and the floor, *5* – the spindle with its bearing units, *6* – the abror with morse cone, *7* – the work table, *8* – the cutter.

Fig. 1. General view of wood shaper

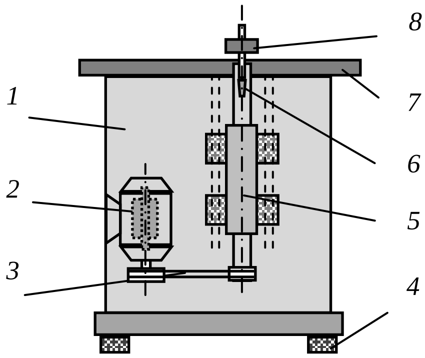

Fig. 2. Scheme of wood shapers

The next figures show the components of the woodworking shaper's saw unit that is the subject of this study. Fig. 3 shows the electric motor, and Fig. 4 – the spindle with its bearing units. Fig. 5 shows the spindle with the fitted cutter.

Fig. 3. Drive electric motor

Fig. 4. Spindle with its bearing units

Fig. 5. Spindle with fitted cutter

The mechanic-mathematical model for investigation of the dynamical processes and vibrations in the woodworking shaper's saw unit is built by the author. The model is shown on the fig. 6. This model includes four discrete mass connected with three massless elastic elements. φ_i, $i = 1, 2, 3, 4$ are the angles of the rotation of the corresponding rotor. The elasticity coefficients of the electric motor's shaft, the belt and the spindle are taken into account. The elasticity angular coefficient of the electric motor's shaft is marked with c_{12}, and this one of the spindle – with c_{34} (*N.m/rad*). The elasticity linear coefficients of the two parts of the belt between the belt puller are c_{23} and c_{32} (*N/m*).

The necessary reduced mass inertia moments ($kg{\cdot}m^2$) render in account (fig. 6):
J_1 – the mass inertia moment of the electric motor's rotor;
J_2 – the mass inertia moment of the belt puller on the electric motor's shaft;
J_3 – the mass inertia moment of the belt puller on the spindle;
J_4 – the mass inertia moment of the wood shaper's saw with cutter arbor.

The other some symbols on fig. 6 are:
d_{12}, d_{34} – diameters of the electric motor's shaft and spindle (m);
c_{12}, c_{34} – elasticity coefficients of the electric motor's shaft and spindle (m);
h_{12}, h_{14}, h_{34}, l_{14} – computing length (m);
r_1, r_2, r_3, r_4 – radius of the rotors (m);
G – modulus of shearing.

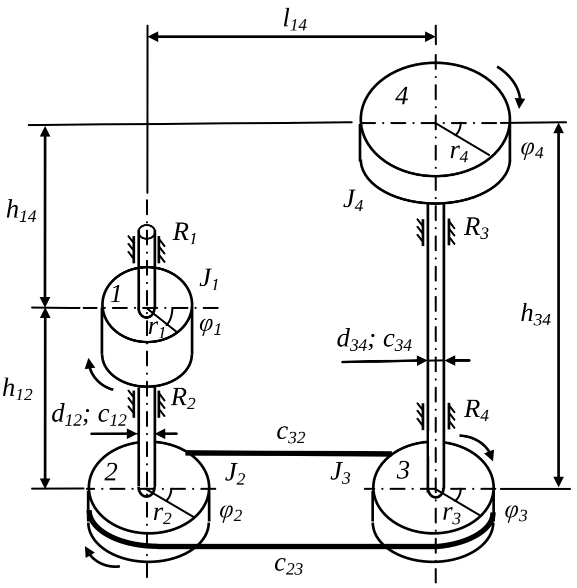

Fig. 6. Mechanic-mathematical model of woodworking shaper's saw unit

The investigation of the torsional vibrations of the woodworking shaper's saw unit requires formulation and solution of the differential equations which describe these processes. Therefore, the priority of the matrix mechanics is used [2].

The mechanic-mathematical model is done by using the applied engineer program (Mathematica). An algorithm for formulation of the matrixes which describe the mass-inertial properties and the elastic properties of the mechanical system is developed. The differential equations which describe the free vibrations are deduced by using the Lagrange's method.

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{q}}\right)-\frac{\partial E_K}{\partial q}+\frac{\partial E_P}{\partial q}=0\,, \qquad (1)$$

where q_i are the generalized coordinates, E_K and E_P are respectively the kinetic and the potential energy of the multibody systems.

The vector of the generalized coordinates is

$$\mathrm{q}=[\varphi_1 \quad \varphi_2 \quad \varphi_3 \quad \varphi_4]^{\mathrm{T}}\,. \qquad (2)$$

The kinetic energy of the mechanical system is obtained as a sum of the kinetic energy of the four basic bodies (the electric motor's rotor, the belt puller on the electric motor's shaft, the belt puller on the spindle, wood shaper's saw). By analogy the potential energy of the mechanical system is obtained as a sum of the potential energies received from the deformations of the electric motor's shaft, the belt and the spindle

$$E_K=\frac{1}{2}J_1.\dot{\varphi}_1^{\,2}+\frac{1}{2}J_2.\dot{\varphi}_2^{\,2}+\frac{1}{2}J_3.\dot{\varphi}_3^{\,2}+\frac{1}{2}J_4.\dot{\varphi}_4^{\,2}\,,$$

$$E_P = \frac{1}{2}c_{12}.(\varphi_1 - \varphi_2)^2 + \frac{1}{2}c_{23}.(r_2.\varphi_2 - r_3.\varphi_3)^2 + \\ + \frac{1}{2}c_{32}.(r_3.\varphi_3 - r_2.\varphi_2)^2 + \frac{1}{2}c_{34}.(\varphi_3 - \varphi_4)^2 . \tag{3}$$

The obtained system of parametric differential equations which describes the free torsional vibrations of the mechanical system is

$$\mathrm{M}.\ddot{\mathrm{q}} + \mathrm{C}.\mathrm{q} = 0 \ . \tag{4}$$

The matrix, which characterizes the mass-inertial properties □ and the elastic properties □ of the mechanical system, are

$$\mathrm{M} = [a_{ij}], \qquad a_{ij} = \frac{\partial^2 E_K}{\partial \dot{q}_i . \partial \dot{q}_j}, \qquad \mathrm{M} = \begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 \\ 0 & 0 & J_3 & 0 \\ 0 & 0 & 0 & J_4 \end{bmatrix}, \tag{5}$$

$$\mathrm{C} = [c_{ij}], c_{ij} = \frac{\partial^2 E_P}{\partial q_i . \partial q_j}, \mathrm{C} = \begin{bmatrix} c_{12} & -c_{12} & 0 & 0 \\ -c_{12} & c_{12} + c_{23}.r_2^{\ 2} + c_{32}.r_2^{\ 2} & -c_{23}.r_2.r_3 - c_{32}.r_2.r_3 & 0 \\ 0 & -c_{23}.r_2.r_3 - c_{32}.r_2.r_3 & c_{34} + c_{23}.r_3^{\ 2} + c_{32}.r_3^{\ 2} & -c_{34} \\ 0 & 0 & -c_{34} & c_{34} \end{bmatrix}$$

Particular solutions to the system of the differential equations (4) are searched as

$$q_r = h_r.sin(\omega_r.t + \varphi), \tag{6}$$

where h_r is the amplitude of the small vibration on the generalized coordinate q_r with natural frequency ω_r, and φ is the initial phase.

After differentiation of (6) and substituting in (4) it obtains a system of linear algebraic equations. In the matrix description they are:

$$\left|\mathrm{C} - \omega^2.\mathrm{M}\right|.\mathrm{V} = 0 \ . \tag{7}$$

To determine the natural frequencies and the mode shapes, it is necessary to solve the task about finding the natural values and the natural vectors of the equations (7).

The satisfaction of the equations (7) requires the following

$$det\left|\mathrm{C} - \omega^2.\mathrm{M}\right| = 0 \ . \tag{8}$$

The roots of the characteristics equation determine the natural frequencies. The natural frequencies form the matrix of the natural values. They are

$$\omega = diag[\omega_{r,r}], \qquad i = 1, 2, \ldots, 4 \ . \tag{9}$$

The natural frequencies [*Hz*] are determinated by (9)

$$\mathrm{f}_{\mathrm{r}} = \frac{\omega_{\mathrm{r,r}}}{2\pi} \quad Hz \ . \tag{10}$$

The natural values of the system (8) determine the natural vectors of the mechanical system.

A natural vector $\mathbf{v_r}$, which gives correlation between amplitudes of the vibrations, corresponds to every natural frequency ω_r. The vector's components define the matrix of the natural vectors (modal matrix) of the system (7) that is

$$V = \left[v_{r,j} \right]_{4\times 4}\ , \tag{11}$$

where $v_r = \left[v_{r,1} \quad v_{r,2} \quad v_{r,3} \quad v_{r,4} \right]$ is the natural mode vector on the generalized coordinate for r^{-th} natural frequency.

III. Results

Some investigations of the wood shaper's saw unit with both drives: first – with a wedge belt and second – with a ribbed belt are carried out. The mass, elastic and geometrical characteristics of the elements of the wedge-belt mechanism are shown in the Table 1, and characteristics of the ribbed-belt mechanism – in Table 2.

Tabl. 1 – Data 1

J_1 – inertia moment of the electric motor's rotor (kg·m^2)	0, 0102
J_2 – inertia moment of the belt puller *2* (kg·m^2)	0, 0740
J_3 – inertia moment of the belt puller *3* (kg·m^2)	0, 0060
J_4 – inertia moment of the shaper saw (kg·m^2)	0, 0141
c_{12} – stiffness of the electric motor's shaft (Nm/rad)	14016
c_{34} – stiffness of the spindle (Nm/rad)	10324
c_{23} – stiffness of the belt (N/m)	$4,5.\cdot10^5$
c_{32} – stiffness of the belt (N/m)	$4,5.\cdot10^5$
d_1 – diameter of the electric motor's shaft (mm)	30
d_3 – diameter of the spindle (mm)	44
r_2 – radius of the belt puller *2* (mm)	190
r_3 – radius of the belt puller *3* (mm)	88
l_1 – distance between the belt puller *2* and the electric motor (mm)	240
l_3 – distance between the shaper saw and the belt puller *3* (mm)	460

Tabl. 2 – Data 2

J_1 – inertia moment of the electric motor's rotor (kg·m^2)	0, 0102
J_2 – inertia moment of the belt puller *2* (kg·m^2)	0, 0729
J_3 – inertia moment of the belt puller *3* (kg·m^2)	0, 0048
J_4 – inertia moment of the shaper saw (kg·m^2)	0, 0141
c_{12} – stiffness of the electric motor's shaft (Nm/rad)	14016
c_{34} – stiffness of the spindle (Nm/rad)	10324
c_{23} – stiffness of the belt (N/m)	$5.\cdot10^5$
c_{32} – stiffness of the belt (N/m)	$5.\cdot10^5$
d_1 – diameter of the electric motor's shaft (mm)	30
d_3 – diameter of the spindle (mm)	44
r_2 – radius of the belt puller *2* (mm)	190
r_3 – radius of the belt puller *3* (mm)	88
l_1 – distance between the belt puller *2* and the electric motor (mm)	240
l_3 – distance between the shaper saw and the belt puller *3* (mm)	460

The calculations are done with help of the applied engineer program Mathematica (www.mathematica.com). The natural frequencies [s^{-1}] (and in [min^{-1}], [Hz]) for two investigations are respectively

1875.8; 1264.54; 706.65; 0; (17912.5; 12075.5; 6748.5; 0), (298.54; 201.26; 112.47; 0).
2100.43; 1268.53; 732.78; 0; (20057.6; 12113.6; 6997.54; 0) (334.29; 201.89; 116.63; 0).

The calculated natural frequencies [*Hz*] and mode shapes of the torsional vibrations of the two types of mechanism research are illustrated graphically on Fig. 7 and Fig. 8.

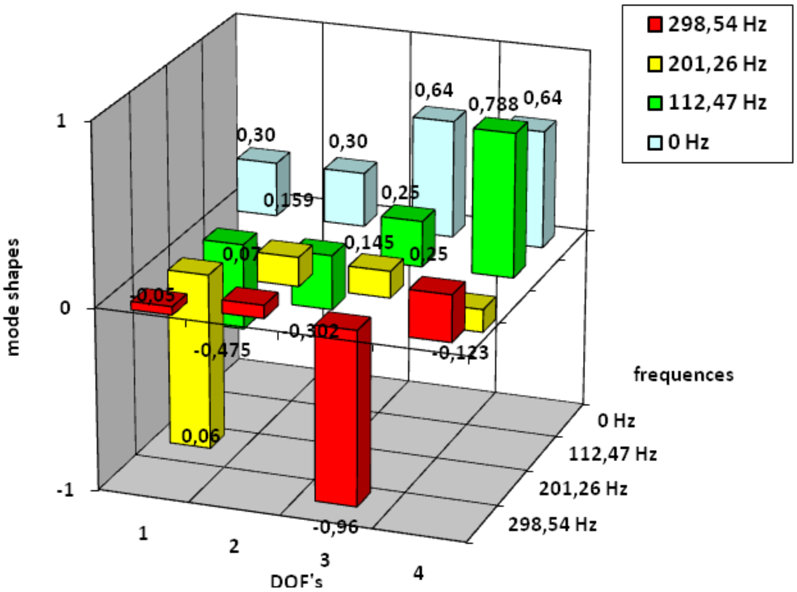

Fig.7 Mode shapes 1

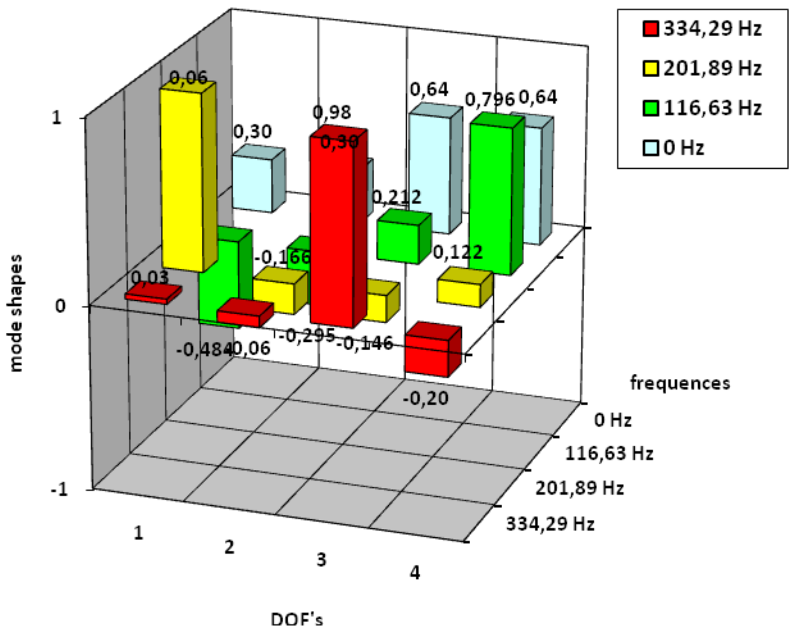

Fig.8 Mode shapes 2

The diagrams, which are obtained as a result of the research, illustrate the natural frequencies and the mode shapes of torsional vibrations of the investigated mechanism. The comparison of the two diagrams shows the significant influence of the type of belt drive (wedge belt or ribbed belt) used on the studied factors. The obtained results are a prerequisite for analyzing the overall vibration behavior of the machine considering its various configurations.

IV. Conclusion

The natural frequencies and mode shapes of the torsional vibrations of the woodworking shaper's saw unit are obtained and illustrated in the presented study. The subject of the research is one mechanism during the usage of two different types of belt drive - wedge belt or ribbed belt. Investigations are made on the base of the specific mechanic – mathematical model developed by the author for analyzing the torsional vibrations of a kind of woodworking shapers. The calculations are done numerically by using modern software. The advantages of the model are: it renders an account of the characteristics of the construction of this class woodworking shapers; it provides an ability to analyze and compare the vibration behaviour of the system using a different type of belt drives. The results of this study allow formation of some recommendations for the avoidance of resonance regimes. The obtained results can be used as a base concerning the increasing of the reliability of the woodworking shaper, accuracy and quality of processing products.

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