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Transmuted Inverse Exponential Software Reliability Model

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Abstract: In this paper we study the Hausdorff approximation of the Heaviside step function $h_r(t)$ by sigmoidal curve model based on the transmuted inverse exponential software reliability model and find an expression for the error of the best approximation. Some comparisons are made.

Keywords: transmuted inverse exponential software reliability model, Hausdorff approximation, Heaviside step function, sigmoidal curve model

I. INTRODUCTION

The transmuted inverse exponential distribution – (TIED) is popular for modeling lifetime data in engineering, reliability, biomedical sciences and life testing [1]. The (TIED) is based on the contents of [2].

For the generalized inverted exponential distribution, see [3]–[5].

Some software reliability models, can be found in [6]–[17].

A new class of Gompertz–type software reliability models and some deterministic reliability growth curves for software error detection, also approximation and modeling aspects, can be found in [19]–[22].

In this note we study the Hausdorff approximation of the Heaviside step function $h_r(t)$ by sigmoidal curve model based on the transmuted inverse exponential software reliability model and find an expression for the error of the best approximation.

II. TRANSMUTED INVERSE EXPONENTIAL SOFTWARE RELIABILITY MODEL

We consider the transmuted inverse exponential cumulative distribution function – (TIECDF):

$$M(t;\theta,\lambda) = \omega e^{-\frac{\theta}{t}}\left(1+\lambda-\lambda e^{-\frac{\theta}{t}}\right), \tag{1}$$

where θ is a scale parameter and λ is the transmuted parameter.

We examine the special case $\omega = 1$,

$$t_0 = -\frac{\theta}{\ln\left(\frac{1+\lambda-\sqrt{1+\lambda^2}}{2\lambda}\right)},$$

i.e. $M(t_0;\theta,\lambda) = \frac{1}{2}$.

The one–sided Hausdorff distance d between the Heaviside step function

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0, \end{cases} \tag{2}$$

and the sigmoid (1) satisfies the relation

$$M(t_0+d;\theta,\lambda) = 1-d. \tag{3}$$

The following theorem gives upper and lower bounds for d:

Theorem. Let

$$a = -e^{-\frac{2\theta}{t_0}}\left(e^{\frac{\theta}{t0}} - 1\right)\left(e^{\frac{\theta}{t_0}} - \lambda\right),$$

$$b = 1 + \frac{e^{-\frac{\theta}{t_0}}\theta}{t_0^2} - \frac{2e^{-\frac{2\theta}{t_0}}\theta\lambda}{t_0^2} + \frac{e^{-\frac{\theta}{t_0}}\theta\lambda}{t_0^2}.$$

For the one–sided Hausdorff distance d between h_{t_0} and the curve (1) the following inequalities hold for:

$$\frac{0.99b}{-a} > e^{0.99},$$

$$d_l = \frac{1}{0.99\frac{b}{-a}} < d < \frac{\ln(0.99\frac{b}{-a})}{0.99\frac{b}{-a}} = d_r. \quad (4)$$

Proof. Let us examine the functions:

$$F(d) = M(t_0 + d; \theta, \phi) - 1 + d, \quad (5)$$

$$G(d) = a + bd. \quad (6)$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.
Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 1).
In addition $G'(d) > 0$.
Further, for $\frac{0.99b}{-a} > e^{0.99}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

This completes the proof of the theorem.

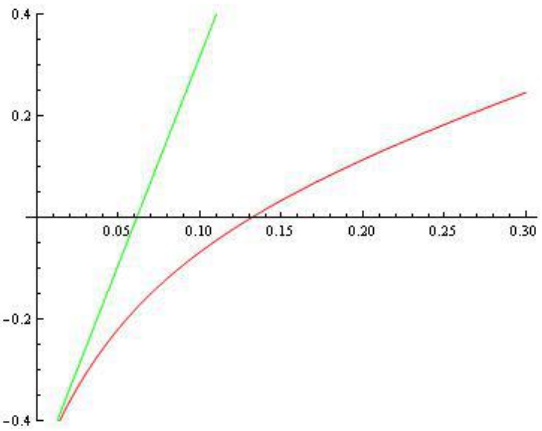

Figure 1: The functions $F(d)$ and $G(d)$ for $\theta = 0.08$, $\lambda = 0.9$.

The model (1) for $\theta = 0.08$, $\lambda = 0.9$, $t_0 = 0.0679563$ is visualized on Fig. 2.

The model (1) for $\theta = 0.04$, $\lambda = 0.96$, $t_0 = 0.0331172$ is visualized on Fig. 3.

The model (1) for $\theta = 0.01$, $\lambda = 0.99$, $t_0 = 0.00817702$ is visualized on Fig. 4.

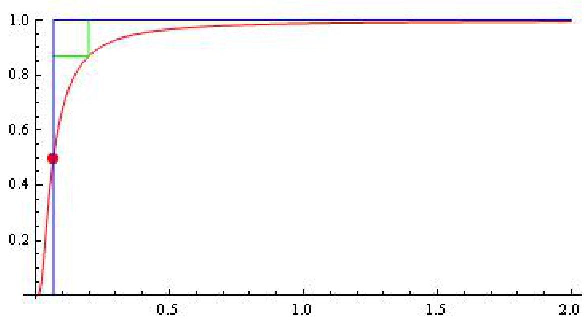

Figure 2: The model (1) with $\theta = 0.08$, $\lambda = 0.9$, $t_0 = 0.0679563$; H–distance $d = 0.131394$; $d_l = 0.0617319$; $d_r = 0.17192$.

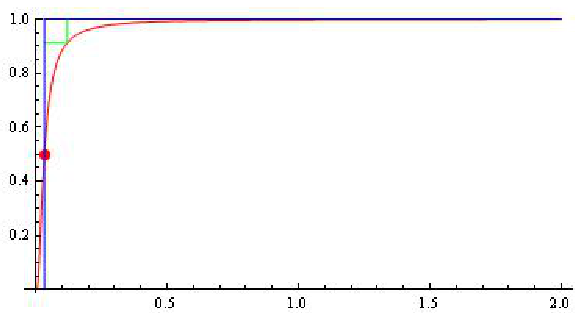

Figure 3: The model (1) with $\theta = 0.04$, $\lambda = 0.96$, $t_0 = 0.0331172$; H–distance $d = 0.0876238$; $d_l = 0.0313529$; $d_r = 0.108557$.

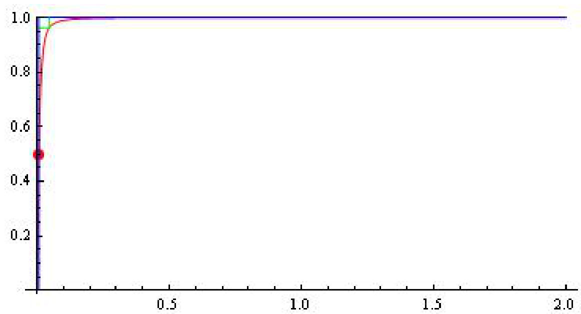

Figure 4: The model (1) with $\theta = 0.01$, $\lambda = 0.99$, $t_0 = 0.00817702$; H–distance $d = 0.0386002$; $d_l = 0.00802311$; $d_r = 0.0387149$.

III. REMARKS

The estimation of remaining errors in the software is the deciding factor for the release of the software or the amount of more testing which is required software growth reliability models are using for the correct estimation of the remaining errors.

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IV. NUMERICAL EXAMPLES

We examine the following data (see Table 1). (The data were reported by Musa [24] and represent the failures observed during system testing for 25 hours of CPU time.)

Hour	Number of failures	Cumulative failures
1	27	27
2	16	43
3	11	54
4	10	64
5	11	75
6	7	82
7	2	84
8	5	89
9	3	92
10	1	93
11	4	97
12	7	104
13	2	106
14	5	111
15	5	116
16	6	122
17	0	122
18	5	127
19	1	128
20	1	129
21	2	131
22	1	132
23	2	134
24	1	135
25	1	136

Table 1: Failures in 1 Hour (execution time) intervals and cumulative failures [24], [23].

$$f[t\_] := 136\, e^{-1.7646399225342981/t}\left(0.4406925851762932 + 0.5593074148237068\, e^{-1.7646399225342981/t}\right)$$

$$g[t\_] := 136\left(1 - \left(1 - e^{-3.1446927524130714`/t}\right)^{1.1268337951971146`}\right)$$

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data1 = {{1, 27}, {2, 43}, {3, 54}, {4, 64}, {5, 75}, {6, 82}, {7, 84}, {8, 89}, {9, 92},
   {10, 93}, {11, 97}, {12, 104}, {13, 106}, {14, 111}, {15, 116}, {16, 122}, {17, 122},
   {18, 127}, {19, 128}, {20, 129}, {21, 131}, {22, 132}, {23, 134}, {24, 135}, {25, 136}};
d2 = Plot[f[t], {t, 0, 25}, PlotStyle → {Thick}, AspectRatio → 0.6, PlotRange → {0, 136}];
d3 = Plot[g[t], {t, 0, 25}, PlotStyle → {Dashed}, AspectRatio → 0.6, PlotRange → {0, 136}];
Show[d2, d3, ListPlot[data1, Joined → True, Mesh → Full,
  MeshStyle → Directive[PointSize[Large], Thick]]]
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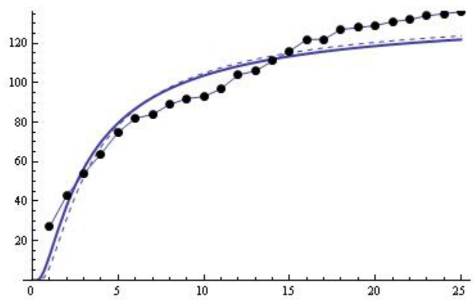

Figure 5: Comparison between $g(t)$ – (dashed) and $f(t)$ – (thick).

In [22] we consider the generalized inverted exponential cumulative distribution function – (GIECDF) for modeling lifetime data in software error detection:

$$g(t) = \omega\left(1-\left(1-e^{-\frac{\theta}{t}}\right)^{\phi}\right). \qquad (7)$$

The fitted model (7) based on the data of Table 1 for the estimated parameters:

$$\omega = 136; \ \theta = 3.1446927524; \ \phi = 1.1268337951$$

is plotted on Fig. 5.

We consider the transmuted inverse exponential cumulative distribution function – (TIECDF):

$$f(t) = \omega e^{-\frac{\theta}{t}}\left(1+\lambda-\lambda e^{-\frac{\theta}{t}}\right). \qquad (8)$$

The fitted model (8) based on the data of Table 1 for the estimated parameters:

$$\omega = 136; \ \theta = 1.7646399225; \ \lambda = -0.55930741482$$

is also plotted on Fig. 5.

From the presented comparisons, cf. Fig. 5, it can be seen that in some cases the model reliability proposed in this article is "flexible" compared to many other, seemingly refined models.

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