



Vendor-Buyer Model with Imperfect Supply Process and Environmental Impacts

W. Ritha, S. Infancy Vimal Priya

Department of Mathematics, Holy Cross College (Autonomous), Tiruchirapalli -2 (Tamilnadu)

Abstract: One of the most important problems the world is facing today is the environmental pollution, which is continuously increasing and causing severe and irreparable damage to the earth. In an effort the new actions could be taken to reduce the negative impact of human species on the environment. This paper presents an integrated vendor-buyer inventory model for a single-vendor and a single-buyer considering a single item. The purpose of this paper is to investigate the impact of emission costs on the replenishment order sizes and the total profit of a Vendor-Buyer in an imperfect supply process, where the buyer receives the batches containing a percentage of imperfect quality items. In this context, an integrated inventory model with imperfect supply process and emission costs, which are the result of warehousing, waste disposal, packaging and transportation is derived with numerical example.

Keywords: Inventory, Coordinated, imperfect supply, Environmental impacts

1. Introduction

In the past few decades, firms have realized that an efficient management across the different parties in a supply chain is critical to reducing inventory costs. This efficient management can be achieved through greater cooperation and better coordination among the different parties. This means that the vendor and the buyers should work cooperatively towards minimizing their costs and maximizing their profits. Integrated inventory management has received a great deal of attention. The classical EOQ model assumes that items produced are of perfect quality. However, the quality of a product is not always perfect but directly affected by the reliability of the production process used to produce the products. Therefore, the process may produce defectives or poor quality items. While considering the production process with imperfect items, it may cause various emissions to the environment. Environmental pollution is one of the greatest challenges that the world is facing today. It began since industrial revolution, increasing day by day and causing irreparable damage to Mother Earth. Environmental pollution has its own causes, effects and solutions. Pollution is the contamination of the environment by introduction of contaminants that can cause damage to environment and harm or discomfort to humans or other living species. In this context, we considered the environmental impacts, which are the results of warehousing, production, waste disposal transportation and packaging. The purpose of this paper is to develop an integrated vendor-buyer inventory model for items with imperfect quality and emission costs. The rest of this paper is organized as follows. In Section 2, Literature review. In Section 3, we develop a mathematical model that integrates the vendor's and the buyer's total profit cost and takes into consideration imperfect-quality items and emissions. Section 4 provides numerical example to illustrate important aspects of the model. Finally, the paper concludes in Section 5.

2. Literature Review

Inventory models with imperfect quality items have received significant attention in the literature. Many researchers have developed the models with integrated inventory model consisting of a vendor and a buyer under various cases. Benjaafar, Li, and Daskin[4] investigated single and multiple lot-sizing models and showed how carbon emission constraints could be incorporated into both models. Bouchery et al.[8] included sustainable criteria into the classical EOQ model and formulated a new multi-objective type model, which considered carbon emission in inventory holding and transportation. Arıkan and Jammernegg[2] studied a new vendor problem with two supply options and carbon emission constraints, where emissions from manufacturing, warehousing and transportation are considered. A model that linked carbon emission and price to the demand offered by Hovelaque and Bironneau[22], who made a trade-off between retailers' total profit and carbon emission reduction, with some policies as results. Helmrich et al.[21] developed a Non-deterministic Polynomial-time (NP) hard model that includes a constraint on the set-up, holding and production emission. Dye and Yang[11] combined carbon emission in an inventory model with deteriorating items and trade credit under different emission policies to obtain joint decisions on emission reduction, trade credit and replenishment policies. Jaber et al.[2013] developed a JELS model inspired by The European Union Emissions



Trading System, which includes emission tax and emission penalty. In their model, the emission released from the supply chain is linked to the production rate of the manufacturer.

Zanoni, Mazzoldi, and Jaber[44] investigated the model of Jaber et al.[26] with a vendor-managed inventory with consignment stock (VMI-CS) agreement in lower total costs and lower emissions levels. Finally, Hammami, Noura, and Frein[18] optimized a multi-echelon supply chain model with environmental measures and lead time constraint with the objective to minimize the total cost of the supply chain. Salameh and Jaber[37] proposed a type of EOQ model where buyer receives a lot mixed with imperfect and good quality items. They maximized the total profit of the buyer with the assumption that after the screening process, imperfect quality items could be sold in a single batch at a lower price. All the extensions of this model until 2011 are reviewed in Khan, Jaber, Guiffrida, and Zolfaghari[28] and Khan, Jaber, and Bonney[31]. Salameh and Jaber[37], Goyal and Cardenas-Barron[17] presented an easy to implement version of this model. Rezaei[35] extended this model by relaxing the assumption of the basic model and assumed that shortage can be incurred, where the shortage is completely backordered. The extension of this model with backorders is also offered by Eroglu and Ozdemir[13] and Wee, Jonas, and Wang[42]. Furthermore, other extensions that worth mentioning here are included, supply chain (Khan, Jaber, & Ahmad[30]; Huang[24], learning in inspection (Khan, Jaber, & Wahab, 2010), inspection errors (Khan, Jaber, & Bonney, 2011), supplier-buyer relationship in inspection (Rezaei & Salimi[36]), repairing or buying from local sources (Jaber, Zanoni, & Zavanella[27]) and inspection speed (Hauck & Vörös[22]). Nima Kazemi[33] extended an EOQ models for items with imperfect quality and emission considerations using different cases.

3. Model Formulation

In this section, we extended the model for a buyer with imperfect quality items and emissions in the Nima Kazemi[33] as the integrated Vendor-Buyer model considering imperfect quality and environmental impacts including transportation, packaging and its emission costs.

The following notations and assumptions are used to develop the model.

Notations

P	Vendor's production rate
D	buyer's demand rate (units/yr)
x	buyer's screening rate (unit/time)
S_v	vendor's setup cost for a batch
S_b	buyer's ordering cost (\$/order)
n	number of shipments in a batch for vendor to buyer
F	fixed transportation cost
E	initial cost of packaging equipment
m	number of parcels
k	cost of material used for packaging (\$/parcel)
h_b	buyer's unit holding cost for both good and imperfect items (\$/unit/yr)
b	space occupied by an item (m^3 /unit)
c_{eh}	average of carbon emission cost of the warehouse (\$/ m^3)
a	weight of an item stored in the warehouse (ton/unit)
t_{eh}	rate of the emission tax for the warehouse (\$/ton)
μ	average annual rate of inventory obsolescence for good quality items
c_p	buyer's purchasing price (\$/unit)
p_{sc}	scrap price (\$/unit)
β	social cost of emission from packaging
c_{eo}	average carbon emission cost coefficient of obsolete items (\$/ton)
α	social cost of emission from transportation
h_v	vendor's holding cost (\$/unit/yr)
t	screening time
d	buyer's screening cost (\$/unit)
c	unit cost for the vendor
v	selling price of imperfect items for buyer (\$/unit)
s	selling price of good items for buyer (\$/unit)
γ	fraction of imperfect items which is a random variable with support in [0,1]
f(γ)	probability density function of fraction of imperfect items
T	cycle length



H_e	emission cost of inventory holding(\$/unit/yr)
T_e	emission tax for inventory holding(\$/unit/yr)
W_e	annual emission cost of obsolescence disposal(\$/yr)
W_c	annual cost of obsolescence(\$/yr)
V_e	cost of emission from transportation
P_e	cost of emission from packaging(\$/yr)
Q_e	lot size with emission considerations(unit/cycle)

Assumptions

1. A single item is considered.
2. Demand rate is constant
3. Production rate is greater than the demand rate.
4. The defective units are sold with discounted price by the buyer as a single batch at the end of the screening period.
5. The production rate is uniform and finite.
6. Shortages and backorder are not allowed.
7. There is a single vendor and a single buyer.

Vendor's cost formation

The vendor's inventory cost per cycle consists of the setup and holding cost of an inventory,

$$TC_V(n, Q_e) = S_v + ch_v \left\{ \frac{Q_e}{2n} + \frac{(n-2)Q_e}{2n} \left(1 - \frac{D}{P} \right) \right\} \quad -- (1)$$

Buyer's cost formation

The buyer's inventory cost per cycle includes the ordering cost, unit purchasing cost, unit screening, holding, transportation, packaging and emission costs,

Where,

Emission costs = Emission cost of inventory holding + Emission tax for inventory holding
 + Holding cost of the obsoleted items + Emission cost of obsolescence disposal + Emission from packaging + Emission from transportation
 = $H_e + T_e + W_c + W_e + P_e + V_e$

$$\text{Emission costs} = \left\{ \frac{Q_e^2(1-\gamma)^2}{2D} + \frac{\gamma Q_e^2}{x} \right\} (bc_{eh} + at_{eh} + \mu(c_p - p_{sc}) + \mu ac_{eo} + m\beta) + \alpha Q_e$$

Thus, from the above considerations the buyer's inventory cost per cycle can be written as,

$$TC_B(n, Q_e) = S_b + c_p Q_e + dQ_e + nF + (E + mk) + \alpha Q_e + \left\{ \frac{Q_e^2(1-\gamma)^2}{2D} + \frac{\gamma Q_e^2}{x} \right\} (h_b + bc_{eh} + at_{eh} + \mu(c_p - p_{sc}) + \mu ac_{eo} + m\beta) \quad -- (2)$$

Coordinated Vendor-Buyer model

The total vendor-buyer coordinated cost per cycle is expressed as,

$$TC_e(n, Q_e) = TC_V(n, Q_e) + TC_B(n, Q_e) \\ = S_v + S_b + c_p Q_e + dQ_e + nF + (E + mk) + \alpha Q_e + ch_v \left\{ \frac{Q_e}{2n} + \frac{(n-2)Q_e}{2n} \left(1 - \frac{D}{P} \right) \right\} \\ + \left\{ \frac{Q_e^2(1-\gamma)^2}{2D} + \frac{\gamma Q_e^2}{x} \right\} (h_b + bc_{eh} + at_{eh} + \mu(c_p - p_{sc}) + \mu ac_{eo} + m\beta) \quad -- (3)$$

The total profit per cycle for the supply chain is the sum of the selling price of good items and the discounted selling price of imperfect items less total cost. Thus, it is written as

$$TPU_e(n, Q_e)\gamma = sQ_e(1-\gamma) + vQ_e\gamma \\ - \left[S_v + S_b + c_p Q_e + dQ_e + nF + (E + mk) + \alpha Q_e + ch_v \left\{ \frac{Q_e}{2n} + \frac{(n-2)Q_e}{2n} \left(1 - \frac{D}{P} \right) \right\} + \left\{ \frac{Q_e^2(1-\gamma)^2}{2D} + \frac{\gamma Q_e^2}{x} \right\} \right. \\ \left. + (h_b + bc_{eh} + at_{eh} + \mu(c_p - p_{sc}) + \mu ac_{eo} + m\beta) \right] \quad -- (4)$$

Since the cycle length in the emission case is $T = \frac{(1-\gamma)Q_e}{D}$, we have the expected value of cycle length as, $E[T] = \frac{(1-E[\gamma])Q_e}{D}$.

Using the Renewal-Reward theorem, the expected total profit per unit time of vendor-buyer is,

$$ETPU_e(n, Q_e) = \frac{E[TC_e(n, Q_e)]}{E[T]}$$



$$= sD + \frac{D}{(1-E[\gamma])} \left[\begin{array}{l} vE[\gamma] - c_p - d - n\alpha - \frac{1}{Q_e}(S_v + S_b + nF + E + mk) \\ -ch_v \left\{ \frac{1}{2n} + \frac{(n-2)}{2n} \left(1 - \frac{D}{P} \right) \right\} - Q_e \left\{ \frac{E[(1-\gamma)^2]}{2D} + \frac{E[\gamma]}{x} \right\} \\ (h_b + bc_{eh} + at_{eh} + \mu(c_p - p_{sc}) + \mu ac_{eo} + m\beta) \end{array} \right] \quad (5)$$

Taking the first derivative of $E[TPU_e(n, Q_e)]$ with respect to Q_e , we have

$$\frac{\partial(ETPU_e(n, Q_e))}{\partial Q_e} = \frac{D}{(1-E[\gamma])} \left[\frac{1}{Q_e^2}(S_v + S_b + nF + E + mk) - \left\{ \frac{E[(1-\gamma)^2]}{2D} + \frac{E[\gamma]}{x} \right\} \right] \quad (6)$$

Taking the second derivative, we have

$$\frac{\partial^2(ETPU_e(n, Q_e))}{\partial Q_e^2} = -\frac{2D}{(1-E[\gamma])Q_e^3}(S_v + S_b + nF + E + mk) \quad (7)$$

Clearly it can be seen in (7) that $\frac{\partial^2(ETPU_e(n, Q_e))}{\partial Q_e^2} < 0$ for all values of n, Q_e . Hence n and Q_e become optimum.

Then, $ETPU_e(n, Q_e)$ is strictly said to be concave.

By setting $\frac{\partial(ETPU_e(n, Q_e))}{\partial Q_e} = 0$, we get

$$\text{i.e. } \frac{1}{Q_e^2}(S_v + S_b + nF + E + mk) = \left\{ \frac{E[(1-\gamma)^2]}{2D} + \frac{E[\gamma]}{x} \right\} (h_b + bc_{eh} + at_{eh} + \mu(c_p - p_{sc}) + \mu ac_{eo} + m\beta)$$

$$\text{i.e. } Q_e^* = \sqrt{\frac{2Dx(S_v + S_b + nF + E + mk)}{\{xE[(1-\gamma)^2] + 2DE[\gamma]\}(h_b + bc_{eh} + at_{eh} + \mu(c_p - p_{sc}) + \mu ac_{eo} + m\beta)}} \quad (8)$$

4. Numerical Example

In this paper, we consider an example with the following parameters,

$$\begin{aligned} D &= 50,000, x = 175,200, S_v = 300, S_b = 100, n = 5, F = 25, E = 3000, m = 2500, k = 3, \\ h_b &= 5, b = 0.017, c_{eh} = 0.55, a = 0.002, t_{eh} = 13, \mu = 0.06, c_p = 25, p_{sc} = 6, \beta = 0.2, \\ c_{eo} &= 13, \alpha = 0.5, h_v = 2, d = 0.5, c = 10, v = 20, s = 50, P = 160,000. \end{aligned}$$

If the fraction of imperfect items follows a uniform distribution with $f(\gamma) = \begin{cases} 25, & \text{for } 0 \leq \gamma \leq 0.04 \\ 0, & \text{otherwise.} \end{cases}$, then we have $E[\gamma] = 0.02$, $E[(1-\gamma)^2] = 0.96$.

❖ From (8), Optimal lot size $Q_e^* = 1497.4$ units

❖ From (5), Expected total profit for Vendor-Buyer $ETPU_e(n, Q_e) = \$28056.95/\text{yr}$.

5. Conclusion

Environmental factors are the main concerns of the decision makers. Therefore, in this context, an Economic order quantity model with Vendor-Buyer coordination for items with imperfect quality and emission costs, which are the result of warehousing, transportation, packaging and waste disposal activities, is formulated. The equations to calculate the expected average total profit per unit time for vendor-buyer and optimal lot size quantities are presented. A numerical example is provided to demonstrate its practical usage.

References

- [1]. Andriolo, A., Battini, D., Grubbström, R.W., Persona, A., & Sgarbossa, F. (2014). A century of evolution from Harris's basic lot size model: Survey and research agenda. *International Journal of Production Economics*, 155, 16–38.
- [2]. Arıkan, E., & Jammernegg, W. (2014). The single period inventory model under dual sourcing and product carbon footprint constraint. *International Journal of Production Economics*, 157, 15–23.
- [3]. Battini, D., Persona, A., & Sgarbossa, F. (2014). A sustainable EOQ model: Theoretical formulation and applications. *International Journal of Production Economics*, 149, 145–153.
- [4]. Benjaafar, S., Li, Y., & Daskin, M. (2010). Carbon footprint and the management of supply chains: Insights from simple models (Working Paper). Minneapolis, MN: University of Minnesota.
- [5]. Benjaafar, S., Li, Y., & Daskin, M. (2013). Carbon footprint and the management of supply chains: Insights from simple models. *IEEE Transactions on Automation Science and Engineering*, 10, 99–116.
- [6]. Bonney, M., & Jaber, M.Y. (2011). Environmentally responsible inventory models: Non-classical models for a non-classical era. *International Journal of Production Economics*, 133, 43–53.
- [7]. Bonney, M., & Jaber, M.Y. (2013). Developing an input-output activity matrix (IOAM) for environmental and economic analysis of manufacturing systems and logistics chains. *International Journal of Production Economics*, 143, 589–597.



- [8]. Bouchery, Y., Ghaffari, A., Jemai, Z., & Dallery, Y. (2012). Including sustainability criteria into inventory models. *European Journal of Operational Research*, 222, 229–240.
- [9]. Bushuev, M.A., Guiffrida, A., Jaber, M., & Khan, M. (2015). A review of inventory lot sizing review papers. *Management Research Review*, 38, 283–298.
- [10]. Chen, X., Benjaafar, S., & Elomri, A. (2013). The carbon constrained EOQ. *Operations Research Letters*, 41, 172–179.
- [11]. Dye, C.-Y., & Yang, C.-T. (2015). Sustainable trade credit and replenishment decisions with credit-linked demand under carbon emission constraints. *European Journal of Operational Research*, 244(1), 187–200.
- [12]. El Saadany, A., Jaber, M., & Bonney, M. (2011). Environmental performance measures for supply chains. *Management Research Review*, 34, 1202–1221.
- [13]. Eroglu, A., & Ozdemir, G. (2007). An economic order quantity model with defective items and shortages. *International Journal of Production Economics*, 106, 544–549.
- [14]. Glock, C.H. (2012). Lead time reduction strategies in a single-vendor–single-buyer integrated inventory model with lot size-dependent lead times and stochastic demand. *International Journal of Production Economics*, 136, 37–44.
- [15]. Glock, C.H., Grosse, E.H., & Ries, J.M. (2014). The lot sizing problem: A tertiary study. *International Journal of Production Economics*, 155, 39–51.
- [16]. Glock, C.H., & Kim, T. (2014). Shipment consolidation in a multiple-vendor–single-buyer integrated inventory model. *Computers & Industrial Engineering*, 70, 31–42.
- [17]. Goyal, S.K., & Cardenas-Barron, L.E. (2002). Note on: Economic production quantity model for items with imperfect quality—a practical approach. *International Journal of Production Economics*, 77, 85–87.
- [18]. Hammami, R., Nourira, I., & Frein, Y. (2015). Carbon emissions in a multi-echelon production–inventory model with lead time constraints. *International Journal of Production Economics*, 164, 292–307.
- [19]. Hauck, Z., & Vörös, J. (2015). Lot sizing in case of defective items with investments to increase the speed of quality control. *Omega*, 52, 180–189.
- [20]. He, P., Zhang, W., Xu, X., & Bian, Y. (2014). Production lot sizing and carbon emissions under cap-and-trade and carbon tax regulations. *Journal of Cleaner Production*, 103, 241–248.
- [21]. Helmrich, M.J.R., Jans, R., van den Heuvel, W., & Wagelmans, A.P. (2015). The economic lot-sizing problem with an emission capacity constraint. *European Journal of Operational Research*, 241, 50–62.
- [22]. Hovelaque, V., & Bironneau, L. (2015). The carbon-constrained EOQ model with carbon emission dependent demand. *International Journal of Production Economics*, 164, 285–291.
- [23]. Hua, G., Cheng, T., & Wang, S. (2011). Managing carbon footprints in inventory management. *International Journal of Production Economics*, 132, 178–185.
- [24]. Huang, C.-K. (2002). An integrated vendor-buyer cooperative inventory model for items with imperfect quality. *Production Planning & Control*, 13, 355–361.
- [25]. Jaber, M., Goyal, S., & Imran, M. (2008). Economic production quantity model for items with imperfect quality subject to learning effects. *International Journal of Production Economics*, 115, 143–150.
- [26]. Jaber, M.Y., Glock, C.H., & El Saadany, A.M. (2013). Supply chain coordination with emissions reduction incentives. *International Journal of Production Research*, 51, 69–82.
- [27]. Jaber, M.Y., Zanoni, S., & Zavanella, L.E. (2014). Economic order quantity models for imperfect items with buy and repair options. *International Journal of Production Economics*, 155, 126–131.
- [28]. Khan, M., Jaber, M., Guiffrida, A., & Zolfaghari, S. (2011). A review of the extensions of a modified EOQ model for imperfect quality items. *International Journal of Production Economics*, 132, 1–12.
- [29]. Khan, M., Jaber, M., & Wahab, M. (2010). Economic order quantity model for items with imperfect quality with learning in inspection. *International Journal of Production Economics*, 124, 87–96.
- [30]. Khan, M., Jaber, M.Y., & Ahmad, A.-R. (2014). An integrated supply chain model with errors in quality inspection and learning in production. *Omega*, 42, 16–24.
- [31]. Khan, M., Jaber, M.Y., & Bonney, M. (2011). An economic order quantity (EOQ) for items with imperfect quality and inspection errors. *International Journal of Production Economics*, 133, 113–118.
- [32]. Maddah, B., & Jaber, M.Y. (2008). Economic order quantity for items with imperfect quality: Revisited. *International Journal of Production Economics*, 112, 808–815.
- [33]. Nima Kazemi, Salwa Hanim Abdul-Rashid, Raja Ariffin Raja Ghazilla, Ehsan Shekarian & Simone Zanoni (2016). Economic order quantity models for items with imperfect quality and emission considerations. *International Journal of Systems Science: Operations & Logistics*, DOI: 10.1080/23302674.2016.1240254



- [34]. Papachristos, S., & Konstantaras, I. (2006). Economic ordering quantity models for items with imperfect quality. *International Journal of Production Economics*, 100, 148–154.
- [35]. Rezaei, J. (2005). Economic order quantity model with backorder for imperfect quality items. In *Proceedings of IEEE International Engineering Management Conference* (pp.466–470). St. John's, Newfoundland.
- [36]. Rezaei, J., & Salimi, N. (2012). Economic order quantity and purchasing price for items with imperfect quality when inspection shifts from buyer to supplier. *International Journal of Production Economics*, 137, 11–18.
- [37]. Salameh, M., & Jaber, M. (2000). Economic production quantity model for items with imperfect quality. *International journal of production economics*, 64, 59–64.
- [38]. Soni, H.N., & Patel, K.A. (2012). Optimal strategy for an integrated inventory system involving variable production and defective items under retailer partial trade credit policy. *Decision Support Systems*, 54, 235–247.
- [39]. Vastag, G., & Montabon, F. (2001). Linkages among manufacturing concepts, inventories, delivery service and competitiveness. *International Journal of Production Economics*, 71, 195–204.
- [40]. Wahab, M., & Jaber, M. (2010). Economic order quantity model for items with imperfect quality, different holding costs, and learning effects: A note. *Computers & Industrial Engineering*, 58, 186–190.
- [41]. Wahab, M., Mamun, S., & Ongkunaruk, P. (2011). EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact. *International Journal of Production Economics*, 134, 151–158.
- [42]. Wee, H.-M., Jonas, C., & Wang, K.-J. (2006). An integrated production-inventory model for deteriorating items with imperfect quality and shortage backordering considerations. *Lecture Notes in Computer Science*, 3982(LNCS), pp. 885–897.
- [43]. Zaroni, S., Jaber, M.Y., & Zavanella, L.E. (2012). Vendor managed inventory (VMI) with consignment considering learning and forgetting effects. *International Journal of Production Economics*, 140, 721–730.
- [44]. Zaroni, S., Mazzoldi L., & Jaber M.Y. (2014). Vendor-managed inventory with consignment stock agreement for single vendor-single buyer under the emission-trading scheme. *International Journal of Production Research*, 52(1), 20–31.