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Quasi-cubic AT-Bézier Curve Based on Algebraic-trigonometric Function

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Abstract: In this paper, a new class of Bézierbasis, called AT-Bézier basis, is constructed based on algebraic-trigonometric function. Its properties including non negativity, normalization and the property of endpoints are discussed. Then quasi-cubic AT-Bézier curve is generated with new basis functions. The AT-Bézier curves not only have much properties resembling the traditional cubic Bézier curves, such as convex hull, geometry invariance and affine invariance, but also can come toG^1 continuity when two AT-Bézier curves stitch together.
Keywords: Algebraic-trigonometric, function, control, vertex, curve, stitching,G^1, continuity, quasi-cubic, AT-Bézier curve

1. INTRODUCTION

Bézier curves is an important method of free curve and surface designing. It has been widely used in computer aided geometric design (CAGD) and computer graphics (CG). Among them, the Bézier curve constructed on Bernstein basis is more common. Because Béziercurve can not describe conic curves except parabola, it has some limitations in practical application. Therefore, it is necessary to discuss curve and surface modeling based on non-polynomial functions.

Li [1] gave quasi-cubic trigonometric parametric curves based on the space $\{1, \sin u, \cos u, \sin^2 u\}$; Chen [2] defined C-Bézier curves in the space $\{1, t, t^2, \cdots, t^{n-2}, \sin t, \cos t\}$,which was similar to Bézier curves; Su [3,4] proposed a class of trigonometric polynomial curves and hyperbolic polynomial curves with characteristics of Bézier curves. Quasi-Bézier curves was constructed based on triangular polynomials [5-7]. Combining the concept of weights with singular blending technique, Zhang [8] generalized the Bézier curve and obtained α -Bézier curve which could reshape the curve by varying the blending parameters. By increasing the power of basis functions and adding control parameters to the basis functions, some Bézier curves and surfaces with shape parameters were presented[9,10]. Geng[11] extended the T- Bézier curve of trigonometric function with the idea of weighting.

Based on the characteristics of Bézier curve, this paper proposes a new class of cubicBézier basis in the function space $\{1, t, t\sin\frac{\pi}{2}t, t\cos\frac{\pi}{2}\}$, and then establishes quasi-cubic algebraic-trigonometric Bézier curve, which shares the same properties of cubic Béziercurve and can attain G^1 continuity.

2. QUASI-CUBIC AT-BEZIER BASIS

Definition 1.The following four functions

$$\begin{cases} X_{0,3}(t) = 1 - t\sin\frac{\pi}{2}t - \frac{2}{\pi}t\cos\frac{\pi}{2}t \\ X_{1,3}(t) = \frac{2}{\pi}t - \frac{2}{\pi}t\sin\frac{\pi}{2}t \\ X_{2,3}(t) = -\frac{2}{\pi}t + \frac{2}{\pi}t\sin\frac{\pi}{2}t + \frac{2}{\pi}t\cos\frac{\pi}{2}t \\ X_{3,3}(t) = t\sin\frac{\pi}{2}t \end{cases} \tag{1}$$

are called quasi-cubic Algebraic-trigonometric Bézier basis (AT-Bézier basis),where $t \in [0,1]$.

From Eq.(1), AT-Bézier basis functions have the following properties:

1) Non-negativity: $X_{i,3}(t) \ge 0, i = 0,1,2,3$;

2) Normalization: $\sum_{i=0}^{3} X_{i,3}(t) = 1$;

3) Properties of the endpoints:

$$\begin{cases} X_{0,3}(0) = X_{3,3}(1) = 1 \\ X_{0,3}(1) = X_{3,3}(0) = 0 \\ X_{1,3}(0) = X_{1,3}(1) = 0 \\ X_{2,3}(0) = X_{2,3}(1) = 0 \end{cases}$$

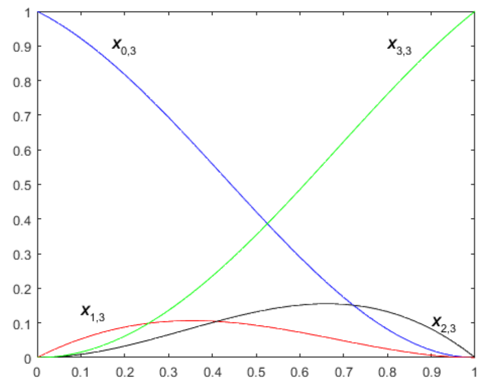

Figure 1 quasi-cubic AT-Bézier basis

3. QUASI-CUBIC AT-BEZIER CURVE

3.1 Definition of quasi-cubic AT-Bézier curve

Definition 2. For $0 \le t \le 1$, the quasi-cubic AT-Bézier curve is defined as

$$P(t) = \sum_{i=0}^{3} X_{i,3}(t) V_i \quad (2)$$

where V_0, V_1, V_2, V_3 are the control vertexes, $X_{i,3}(i = 0,1,2,3)$ are the AT-Bézier basis functions defined as Eq.(1).

From Eq.(1),the matrix form of AT-Bézier curve can be denoted as follows:

$$P(t) = \sum_{i=0}^{3} X_{i,3}(t) V_i = \begin{pmatrix} 1 & t & t\sin\frac{\pi}{2}t & t\cos\frac{\pi}{2}t \end{pmatrix} M \begin{pmatrix} V_0 & V_1 & V_2 & V_3 \end{pmatrix}^T \quad (3)$$

By derivation, we can get

$$P'(t) = \begin{pmatrix} 1 & \sin\frac{\pi}{2}t & \cos\frac{\pi}{2}t & t\sin\frac{\pi}{2}t & t\cos\frac{\pi}{2}t \end{pmatrix} A \begin{pmatrix} V_0 & V_1 & V_2 & V_3 \end{pmatrix}^T \quad (4)$$

where

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{2}{\pi} & -\frac{2}{\pi} & 0 \\ -1 & -\frac{2}{\pi} & \frac{2}{\pi} & 1 \\ -\frac{2}{\pi} & 0 & \frac{2}{\pi} & 0 \end{bmatrix} \quad A = \begin{bmatrix} 0 & \frac{2}{\pi} & -\frac{2}{\pi} & 0 \\ -1 & -\frac{2}{\pi} & \frac{2}{\pi} & 1 \\ -\frac{2}{\pi} & 0 & \frac{2}{\pi} & 0 \\ 1 & 0 & -1 & 0 \\ -\frac{\pi}{2} & -1 & 1 & \frac{\pi}{2} \end{bmatrix}$$

3.2 Properties of quasi-cubic AT-Bézier curve

From Eq.(2), we know that the algebraic-trigonometric Bézier curve has the same interpolating properties to the traditional cubic Bézier curve.

1) Properties of the endpoints: The AT-Bézier curve starts at the first control vertex and ends at the last control vertex, and is tangent to the first and last edges of the characteristic polygon, i.e:

$$P(0) = V_0,\ P(1) = V_3,\ P'(0) = \frac{2}{\pi}(V_1 - V_0),\ P'(1) = V_3 - V_2$$

2) Convex hull property: Because of the normalization and non-negativity of AT-Bézier Basis, the AT-Bézier curve is located in the convex hull generated by the control vertexes.

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3) Geometric Invariance and Affine Invariance: The shape of AT-Bézier curve depends only on the control vertexes, but has nothing to do with the selection of plane coordinate system. If the characteristic polygon is scaled or sheared, the new curve is obtained by the same affine transformation of AT-Bézier curve.

3.3 Application of quasi-cubic AT-Bézier Curve

Select (0,0), (0.25,0.5), (0.5,0.8), (0.75,0) as the control vertexes. The traditional cubic Bézier curve(dashed line) and quasi-cubic AT-Bézier (solid line) curve are plotted respectively, as shown in Fig.2.

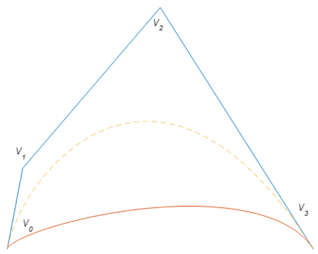

Figure 2 AT-Bézier curve and traditional cubic Bézier curve

4. CONTINUITY OF QUASI-CUBIC AT-BEZIER CURVE

Curve stitching is often used in the design of complex free curves. When stitching curves, the specified continuity requirement should be met at the joint. There are two quasi-cubic AT-Bézier curves as follows:

$$P(t)=\sum_{i=0}^{3}X_{i,3}(t)V_i\ ,\qquad Q(t)=\sum_{i=0}^{3}X_{i,3}(t)W_i$$

where $V_i\,(i=0,1,2,3)$ and $W_i\,(i=0,1,2,3)$ are the given control vertexes, $X_{i,3}(i=0,1,2,3)$ are the AT-Bézier basis.

1) G^0 continuity: Only if the last endpoint of $P(t)$ is the same as the first endpoint of $Q(t)$,i.e.: $Q(0)=P(1)$,we can get

$$V_3=W_0 \qquad (5)$$

2) G^1 continuity: The necessary and sufficient condition of G^1 continuity of curves $P(t)$ and $Q(t)$ is:

$$Q(0)=P(1)\,,\qquad Q'(0)=\lambda P'(1)$$

where $\lambda>0$. By the equations of $Q'(0)=\frac{2}{\pi}(W_1-W_0)$, $P'(1)=V_3-V_2$ and Eq.(5), there must also have one of the G^1 continuity conditions as following:

$$W_1=W_0+\frac{2\lambda}{\pi}(V_3-V_2) \qquad (6)$$

When the two quasi-cubic AT-Bézier curves satisfy Eq.(5) and Eq.(6),they attain G^1 continuity at the joint.

Its geometric significance is that when two quasi-cubic AT-Bézier curves are G^1 continuity, the control vertexes $V_2, V_3(=W_0), W_1$ must be collinear and arranged in order, that is, the two curves have common tangent at the common stitching vertex $V_3(=W_0)$. An example of G^1 continuity of two AT-Bézier curves is given in.Fig.3.

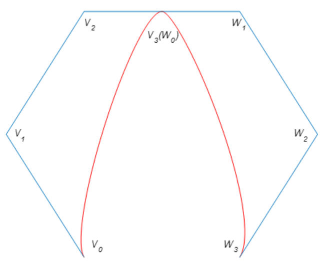

Figure 3 G^1 continuous stitching of two quasi-cubic AT-Bézier curves

3) C^1 continuity: If set $\lambda = 1$ in Eq.(6), the two curves not only have the same tangent line at the stitching point,and also have the same tangent vectors , therefore, they attain C^1 continuity.

5. CONCLUSION

In this paper, we construct an quasi-cubic algebraic-trigonometric Bézier basis function(AT-Bézier basis) in the space $\{1, t, t\sin\frac{\pi}{2}t, t\cos\frac{\pi}{2}\}$,and define the quasi-cubic AT-Bézier curve. It inherits the advantages of Bézier, and has similar properties as Bézier curve, such as convex hull, geometric invariance, continuity, etc. At the same time, because AT-Bézier basis is a mixed function of algebra and triangular function, quasi-cubic AT-Bézier curve can accurately represent algebraic curve and transcendental curve. The disadvantage is that the curve can not adjust its shape when the characteristic polygon remains unchanged. In the future, we will focus on adding shape parameters to make the curve more flexible and widely used.

6. ACKNOWLEDGEMENTS

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