International Journal of Latest Research in Engineering and Technology (IJLRET)
ISSN: 2454-5031
www.ijlret.com || Volume 06 - Issue 08 || August 2020 || PP. 01-12

Optimal Planning of Radial Distribution Networks Through Mixed Integer Quadratic Programming

Mahmoud Ali Farrag[1], Maged Gamal Zahra[1], ShaimaaOmran[2]
[1]*(Electrical Power & Machines department, Faculty of Engineering, Cairo University, GamaaStreet,12613, Cairo, Egypt)*
[2]*(National Research Centre, Engineering Research Division, Systems and Information Department, El-Buhouth street, 12622, Dokki, Cairo, Egypt)*

Abstract: A model for optimal planning of radial distribution networks using the mixed integer quadratic programming technique is presented in this paper. The paper proposes a simplified load flow model for radial networks, and then the model is modified to be used in the planning process. An objective function composed of the capital cost and cost of energy losses is formed. The energy loss cost is developed as a quadratic function in both real and reactive power flow. The verification of the presented model has been made through its application to a 24-bus radial distribution network.
Keywords: Capital cost, Distribution system, Energy losses cost, Mathematical programing, Optimal routing.

Nomenclature

N:	Number of load buses
M:	Total number of branches
V_i:	Voltage magnitude of bus i
δ_i:	Voltage angle of bus i
p_k^s:	Real power flow on branch k at its start bus
p_k^e:	Real power flow on branch k at its end bus
q_k^s:	Reactive power flow on branch k at its start bus
q_k^e:	Reactive power flow on branch k at its end bus
P_i^D:	Real power demand at bus i
Q_i^D:	Reactive power demand at bus i
r_k:	Resistance of branch k
x_k:	Reactance of branch k
z_k:	Magnitude of impedance of branch k
$W1(i)$:	Set of lines connected to bus i which is the start bus
$W2(i)$:	Set of lines connected to bus i which is the end bus
r_a :	Resistance of cable size a per unit length
x_a :	Reactance of cable size a per unit length
r_b :	Resistance of cable size b per unit length
x_b :	Reactance of cable size b per unit length
$\bar{k}$:	Large positive number
z_k^a:	Zero-one variable associated with branch k and size a
z_k^b:	Zero-one variable associated with branch k and size b
F:	Total network cost
Fc:	Annualized capital cost of the network
Fl:	Annual loss cost
l_k:	Length of branch k
g:	Annual recovery factor
Cl:	Cost of unit power loss in $ per unit power loss
T:	Time per year (8760 hrs)
l_s :	Loss factor
C_e:	Cost of unit energy in $/kWh
p_{ma} :	Max real power flow permitted on cable size a
p_{mb} :	Max real power flow permitted on cable size b
q_{ma} :	Max reactive power flow permitted on cable size a
q_{mb} :	Max reactive power flow permitted on cable size b
V_{min} :	Min voltage permitted at load bus

Pl_k: Real power loss of branch k
Ql_k: Reactive power loss of branch k
I_k: Current magnitude flowing on branch k
p_k: Real power flow on branch k when losses are neglected
q_k: Reactive power flow on branch k when losses are neglected

1. Introduction

Distribution system is a crucial element in the modern power system as it manages and delivers the electrical energy generated to end-users[1]. Distribution system planning (DSP) is a challenging task as its objective is to design an optimum reliable network that meets the load growth demand, and the technical and operational constraints at minimum total cost[2, 3]. The distribution system configurations depend on the available paths, substations, and load point locations. The radial configuration and the weakly meshed configuration are the most commonly used ones[4, 5]. This paper primarily adopts the radial feeders' configuration. The optimization of the DSP was solved using mathematical[6, 7]as well as heuristic techniques[1-3, 5, 8-13]. Mathematical optimization was implemented in[6] where a multi-criteria analysis to evaluate several options for a rural DSP was used. The multi-objective formulations of the problem considered the optimal sizing and siting of energy storage system. Furthermore, [7] proposed a multi-energy system expansion planning model with the aim to minimize the total cost of the system. A mixed integer second order cone programming model is deployed to optimize the size, the placement, and the type of all the infrastructure components of the multi-energy system. Reference [13]is one of the references that adopted the heuristic techniques to solve the DSP, it used an improved harmony search algorithm to solve the complex DSP optimization problem with the objective of minimizing the total annual system cost. Also, [11]that described a method to solve the DSP using the GA heuristic technique to solve this multi-objective MINLP problem. Moreover, [5] proposed using the steepest descent and the simulated annealing algorithm to solve the radial DSP with the goal to find the routes that minimize the total annual system cost. Reference[8] addressed the DSP problem and solved it using the discrete particle swarm optimization (PSO) technique. Reference[2] formulated the distribution system expansion planning problem as a MINLP problem and solved it using GA. Reference[3]introduced a multi-stage planning method for DSP. The problem is formulated as a MINLP model and solved using GA with the aim to minimize the total network investment cost. Many researchers considered integration and penetration of DG [1, 3, 8, 9, 11, 12, 14-18] in their studies. Some of these studies are [1]that presented a two stage MINLP model for DSP considering Demand Response (DR) techniques and DG integration. The model optimized the DG allocation, the price offerings, and smart metering so as to minimize the total cost and the CO_2 emissions. Reference [14] solved the distribution expansion planning as a long term multi-year problem. The present value approach was used to solve this MINLP problem, co-optimizing the allocation of the DG in the network with the objective to minimize the total system cost. Reference[15]presented a DSP model that uses non-linear convex AC power flow equations. The model considered the uncertainty in the load and the DG wind generation. Moreover, energy storage systems were taken into account when designing the DSP problem in some studies[9, 10, 12,19].Also, contingency analysis was of concern when solving DSP problem in[20] and[4].In addition, the smart grid (SG) technologies were implemented in some studies addressing the DSP[16, 21,22].In this paper, the DSP problem is formulated as a mixed integer quadratic programming problem, where an objective function that minimizesthe capital cost and cost of energy losses is formed. The contribution of the paper is deriving a simplified form of the power flow equations and modifying this form to be used to solve the DSP problem. This proposed model uses the active and reactive power flow values in the energy loss cost formulations instead of the current. Thus, the power factor is taken into consideration in the model rather than considering it constant on all buses as an approximation commonly suggested in other studies. Section 2 introduces the formulations of the simplified form for the load flow equations. Section 3 presents the proposed load flow equations used in the planning phase. Section 4 describes the complete mathematical planning model. Section 5 presents a final form for the objective function and the constraints used for the distribution system planning. Section6 provides a verification of the proposed model using a 24-bus distribution system and presents the results and analysis of the case studies. Section 7 concludes the results of the paper and envisions direction for future research.

2. A Simple Form for the Load Flow Equations

The load flow equations can be derived in a simple form as follows:

a) The active and reactive power flow at the start and the end of each branch; as that shown in Fig.1(a) are constructed as non-linear function in the voltages of the two end buses as follows:

$r_k p_k^s + x_k q_k^s = |V_i|^2 - |V_i||V_j| \cos \delta_{ij}$.(1)

$x_k p_k^s - r_k q_k^s = |V_i||V_j| \sin \delta_{ij}$.(2)

$r_k p_k^e + x_k q_k^e = |V_j|^2 - |V_i||V_j| \cos \delta_{ji}$.(3)

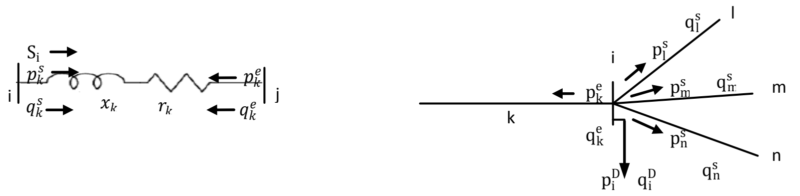
$x_k p_k^e - r_k q_k^e = |V_i||V_j| \sin \delta_{ji}$.(4)

Where $\delta_{ji} = \delta_j - \delta_i$.

b) The active and reactive power balance linear equations for bus i shown in Fig. 1(b) can be presented as follows

$\sum_{j \in W1(i)} p_j^s + \sum_{j \in W2(i)} p_j^e + P_i^D = 0$..(5)

$\sum_{j \in W1(i)} q_j^s + \sum_{j \in W2(i)} q_j^e + Q_i^D = 0$. (6)



(a) (b)

Figure 1. (a) Branch k between bus i and bus j (b) Bus i with lines branching from it

The power flow equations for branch k can be developed through simple mathematical operations on Eqs.1 - 4 yielding the following equations.

By adding Eqs.1 and 2 after squaring them, then dividing by $|V_i|^2$, Eq.7 is obtained as follows:

$|V_i|^2 = |V_j|^2 + 2r_k p_k^s + 2x_k q_k^s - \left(\frac{Z_k^2 {S_k^s}^2}{|V_i|^2}\right)$.(7)

Eq. 8 is simply Eq. 2

$x_k p_k^s - r_k q_k^s = |V_i||V_j| \sin \delta_{ij}$.(8)

Adding Eqs. 2 and 4 and noting that $\sin \delta_{ij} = -\sin \delta_{ji}$, Eq. 9 is obtained:

$x_k(p_k^s + p_k^e) = r_k(q_k^s + q_k^e)$. (9)

Eq. 10 is obtained by subtracting Eq. 2 from 1:

$|V_i|^2 = |V_j|^2 + r_k(p_k^s - p_k^e) + x_k(q_k^s - q_k^e)$.(10)

The total number of equations is (4m + 2n) for any network. But for radial networks where the number of feeder sections, m, is equal to number of load buses, n, the total number of equations is 6n. As the voltage angle, δ, is only existing in one equation, Eq. 8, then this equation can be deleted and variable, δ, can be ignored. In existing radial feeders both feeder configuration and section impedances are known while in planning phase the radial configuration is still unknown, as well as, the feeder section impedances(selected feeder size) are unknown.

3. Load Flow Equations in the Planning Phase

In the planning phase, the above Eqs. 7,9, and 10 suffer from two drawbacks as follows:

a) Each feeder section may be deleted or added. If section k, for example, is deleted, this means that the four powers p_k^s, q_k^s, p_k^e, and q_k^e will be zero and Eq. 7 with its present form will mean that V_i will be equal to V_j which is a fatal error.

b) If section k is selected, its impedances are still unknown as the best size to be selected is not yet known.

To overcome the above two serious problems, the above three equations are rewritten as follows[23, 24]assuming that there are two available cable sizes to select from.

$\bar{k}(z_k^a + z_k^b - 1) \leq |V_i|^2 - |V_j|^2 - 2r_k p_k^s - 2x_k q_k^s + \left(\frac{Z_k^2({p_k^s}^2 + {q_k^s}^2)}{|V_i|^2}\right) \leq \bar{k}(1 - z_k^a + z_k^b)$.(11)

$x_k(p_k^s + p_k^e) = r_k(q_k^s + q_k^e)$. (12)
$\bar{k}(z_k^a + z_k^b - 1) \leq |V_i|^2 - |V_j|^2 - r_k(p_k^s - p_k^e) - x_k(q_k^s - q_k^e) \leq \bar{k}(1 - z_k^a + z_k^b)$.(13)

$r_k = l_k\,(r_a\, z_k^a + r_b\, z_k^b)$.(14)

$x_k = l_k\,(x_a\, z_k^a + x_b\, z_k^b)$.(15)

$(z_k^a + z_k^b) \leq 1$.(16)

Now if line k is deleted, z_k^a and z_k^b equals zero, then

a- Both r_a and x_a will be zero from Eqs. 14 and 15.
b- The four powers p_k^s, q_k^s, p_k^e, and q_k^e will be constrained to be zero(this will be elaborated in Eqs. 31-34 later on), and Eqs 11 and 13 will be converted to $-\bar{k} \leq |V_i|^2 - |V_j|^2 \leq \bar{k}$.

as $\bar{k}$ is large positive value, so there is no limit on the buses voltages.

If line k is selected and any cable size is selected, then

a) z_k^a or z_k^b will be equal to one from Eq. 16
b) The impedance of section k will be known from Eqs. 14 and 15
c) Eqs. 11 and 13 will be reduced to Eqs.7 and 10.

By the above modifications, Eqs.11 and 13 will be correct for selecting or deleting section k.

4. Mathematical Planning Model

Cost Function. The cost function which contains both the capital cost of the feeder sections and the cost of annual loss can be given as follows:

$F = Fc + Fl$.(17)

Where $Fc = g\, \sum_i^M l_i\,(c_a z_i^a + c_b z_i^b)$.(18)

and $Fl = Cl\, \sum_{i=1}^M (p_i^s + p_i^e)$.(19)

where $Cl = l_s\;\; T\;\; C_e$.(20)

The losses cost is a linear function as the power loss on section i is the summation of both the power at its start bus and end bus.

Hence, the cost function now is a linear one as a direct result of the simple representation of load flow equations.

Constraints Equations. The constraints equations consist of the following forms of equations:

Relations related to each line. For line k of two end buses i and j:

$\bar{k}(z_k^a + z_k^b - 1) \leq V_i^2 - V_j^2 - 2r_k p_k^s - 2x_k q_k^s + NT \leq \bar{k}(1 - z_k^a + z_k^b)$.(21)

$x_k(p_k^s + p_k^e) - r_k(q_k^s + q_k^e) = 0$.(22)

$\bar{k}(z_k^a + z_k^b - 1) \leq -r_k(p_k^s - p_k^e) - x_k(q_k^s - q_k^e) \leq \bar{k}(1 - z_k^a + z_k^b)$.(23)

Where $NT = \frac{(r_k^2 + x_k^2)\,({p_k^s}^2 + {q_k^s}^2)}{|V_i|^2}$.(24)

$r_k = l_k\,(r_a\, z_k^a + r_b\, z_k^b)$.(25)

$x_k = l_k\,(x_a\, z_k^a + x_b\, z_k^b)$.(26)

$\left(z_k^a + z_k^b\right) \leq 1.$(27)

Power balance equations. For bus i:

$\sum_{j \in W1(i)} p_j^s + \sum_{j \in W2(i)} p_j^e + P_i^D = 0.$(28)
$\sum_{j \in W1(i)} q_j^s + \sum_{j \in W2(i)} q_j^e + Q_i^D = 0.$ (29)

Radiality constraints. To guarantee radiality constraint, the number of selected sections should be equal to the number of buses

$\sum_{i=1}^{m}\left(z_i^a + z_i^b\right) = N.$(30)

Limit constraints. This constraint is used to force the real and reactive power flow on each selected feeder to be less than its thermal limit.

For line i:

$-\left(p_{ma} \ z_i^a + p_{mb} z_k^b\right) \leq \mathrm{p}_\mathrm{i}^\mathrm{s} \leq \left(p_{ma} z_i^a + p_{mb} \ z_k^b\right).$(31)

$- \ \left(q_{ma} \ z_i^a + q_{mb} z_k^b\right) \leq \mathrm{q}_\mathrm{i}^\mathrm{s} \leq \left(q_{ma} z_i^a + q_{mb} \ z_k^b\right).$(32)

$-\left(p_{ma} \ z_i^a + p_{mb} z_k^b\right) \leq \mathrm{p}_\mathrm{i}^\mathrm{e} \leq \left(p_{ma} z_i^a + p_{mb} \ z_k^b\right).$(33)

$-\left(q_{ma} \ z_i^a + q_{mb} z_k^b\right) \leq \mathrm{q}_\mathrm{i}^\mathrm{e} \leq \left(q_{ma} z_i^a + q_{mb} \ z_k^b\right).$(34)

These constraints equations guarantee that if line i is deleted its four power flows over it will be equal to zero. If the line i is selected, its four power flows over it will be within the acceptable limits.

Voltage limit. It is given for all buses as follows

For bus i:

$V_i^2 \geq V_{min}^2.$(35)

Linearization of the constraints equations. The nonlinear term only exists in Eq. 24 which can be simply reduced to

$NT = (r_k^2 + x_k^2)|I_k|^2.$(36)

Where $|I_k|^2 = \left(\frac{\left(\mathrm{p}_\mathrm{k}^{\mathrm{s}\,2} + \mathrm{q}_\mathrm{k}^{\mathrm{s}\,2}\right)}{|\mathrm{V}_\mathrm{i}|^2}\right).$(37)

Thus$NT = r_k Pl_k + x_k Ql_k.$(38)

As the power loss Pl_k and Ql_k are small percentage of the power flow 2 $p_k^s$and $2q_k^s$, these losses can be neglected. This will lead to the following results:

a- The cost of losses will be zero as Pl is neglected on all branches.
b- The power flow on each branch will be fixed from start to end line.

$p_k = p_k^s = p_k^e.$(39)
$q_k = q_k^s = q_k^e.$(40)

c- The three constraint equations related to each branch will be reduced to one equation only as follows: For branch k of two end buses i and j

$V_i^2 = V_j^2 - 2\mathrm{r}_\mathrm{k} p_k - 2\mathrm{x}_\mathrm{k} q_k.$(41)

Where the power is flowing from bus i to j

d- Neglecting branch k losses will have small effect on bus voltages.

Losses Cost. In order to regain the losses cost, note that for line k

$z_k^2 \frac{S_k^2}{V_i^2} = \frac{(z_k^2 p_k^2 + z_k^2 q_k^2)}{V_i^2} = z_k^2 I_k^2$.(42)

$\frac{(z_k^2 p_k^2 + z_k^2 q_k^2)}{V_i^2} = {r_k}^2 I_k^2 + {x_k}^2 I_k^2$.(43)

$\frac{(z_k^2 p_k^2 + z_k^2 q_k^2)}{V_i^2} = r_k Pl_k + x_k Ql_k$.(44)

If $V_i = 1$ then $r_k Pl_k = z_k^2 p_k^2 + z_k^2 q_k^2 - x_k Ql_k$. (45)

As $\frac{Pl_k}{Ql_k} = \frac{r_k}{x_k}$.(46)

Eq. 47 is obtained by substituting for Ql_k from 46 in 45

$(r_k + x_k^2/r_k)Pl_k \cong z_k^2 p_k^2 + z_k^2 q_k^2$.(47)

$\left(\frac{r_k^2 + x_k^2}{r_k}\right) Pl_k = \frac{z_k^2}{r_k} Pl_k \cong z_k^2 p_k^2 + z_k^2 q_k^2$.(48)

$Pl_k \cong r_k p_k^2 + r_k q_k^2$.(49)

Then the losses cost on branch k can be represented as follows to reach a good degree of accuracy

$Cl_k = C' (r_k \; p_k^2 + r_k q_k^2)$.(50)

5. Final Mathematical Planning Model

The final planning model adopted in this research can be represented as follows

Min $F = Fc + Fl$.(51)

Where $Fc = g \sum_i^M l_i \,(c_a z_i^a + c_b z_i^b)$.(52)

And $Fl = \sum_{i=1}^M C' (r_i \; p_i^2 + r_i q_i^2)$.(53)

Subject to

a. Voltage drop relation for each branch k
$V_i^2 = V_j^2 - 2r_k p_k - 2x_k q_k$.(54)

b. Bus balance equation for bus i
$\sum_{j \in W1(i)} p_j + P_i^D = 0$.(55)
$\sum_{j \in W1(i)} q_j + Q_i^D = 0$. (56)

c. Limit equations for branch k
$-\left(p_{ma} \; z_i^a + p_{mb} z_k^b\right) \le p_k \le \left(p_{ma} z_i^a + p_{mb} \; z_k^b\right)$.(57)

$-\left(q_{ma} \; z_i^a + q_{mb} z_k^b\right) \le q_k \le \left(q_{ma} z_i^a + q_{mb} \; z_k^b\right)$.(58)

d. Bus voltage limit for bus i
$V_i^2 \ge V_{min}^2$.(59)

A further simplification can be obtained as the number of real variables can be reduced if all the loads have the same power factor. Also, assuming that all bus voltages have zero angles, then:

$q_k = w \, p_k$, where $w = 0.4843$, $w^2 = 0.234568$.(60)

So the function of losses cost can be reduced to

$Fl = \sum_{i=1}^{M} C^{'}(r_i\ p_i^2\ +\ r_i w^2 p_i^2)$.(61)

$Fl = \sum_{i=1}^{M} C^{'}(r_i +\ r_i w^2)\ p_i^2$.(62)

Also, the reactive power balance equation can be neglected.

Moreover, the voltage drop equation will be

$V_i^2 =\ V_j^2 -\ p_k(2\,r_k -\ 2x_k\ w)$.(63)

6. Model Verification

The verification of the presented distribution system planning model is performed by its implementation to a 24 bus system described and depicted in this section. The LINGO® 17.0 optimization software tool[25] is used to simulate the proposed model on an Intel® Core™ i7 @ 1.73 processor with a 4 GB installed memory on a 64-bit MS Windows® operating system. The planning model developed has been applied to the initial distribution network shown in Fig.2. It is a 10kV distribution system that has 42 routes and 24 buses. The lines and loads data associated with this network is found in[5]. The model development has been made in the light of the following data:

1. Overhead lines (aluminium steel reinforced cables) are used where the two sizes considered have the data shown in Table 1. As the cost per unit length of the two sizes is the same, only larger size conductor, b, will be considered in the implementation as its losses cost will be smaller.

Table 1. The 24 bus distribution network branches conductors data

Feeder code	Size of conductor [mm^2/mm^2]	Capacity [Ampere]	Conductor impedance [Ω/km]	Cost [kUS$/km]	Failure rate per [km yr]	Time for repair [hr]
a	16/2.5	95	2.1 + j 0.4	15	0.2	3
b	25/4	125	1.2 + j 0.4	15	0.2	3

2. The input data to get the cost equation is

The annual recovery factor g = 0.1,
The loss factor l_s = 0.6,
Load power factor (p.f.) = 0.9,
Total network load = 2550 kVA .
Total network real power of the load = 2295 kW.
Four case studies are investigated for the verification of the proposed model. Fig. 3 depicts these four case studies.

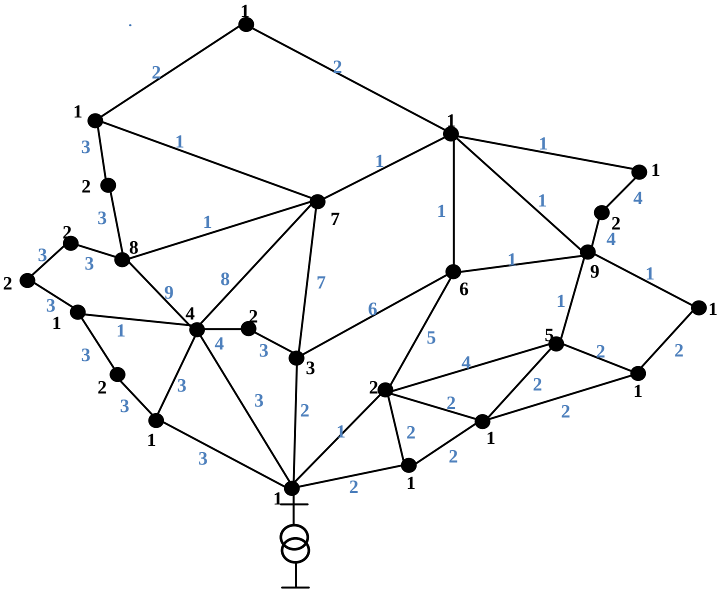


Figure 2. The initial 24 bus distribution network

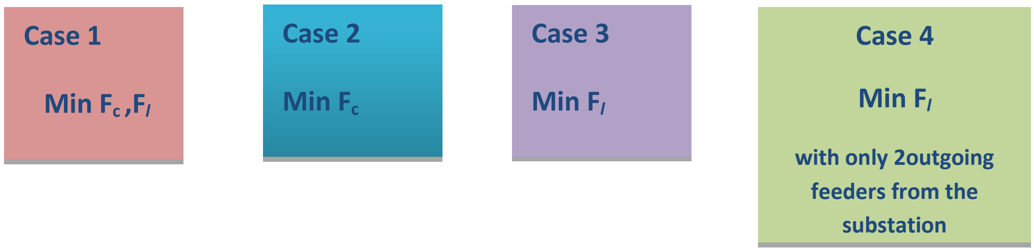


Figure 3. The 4 case studies tested on the 24 bus distribution network

Case 1. In this case both capital and losses cost are considered where the optimum planned network obtained is shown in Fig.4. The costs associated with this planned network are: $Fc = 36{,}000$ \$,= 15,288.2 \$, $F =$ 51,288.2 \$, power losses = 0.089 p.u. = 89 kW. This yields a power loss percent of 3.87% of the total system load.
Case 2. In this case, it is required to minimize the capital cost only under the assumption that the available budget to this project is small. The output of this case is shown in Fig.5. The different costs associated with this planned distribution system are: $Fc = 35{,}475$ \$, = 19,161.22 \$, $F = 54{,}636.27$ \$, power losses = 0.1116 p.u. = 111.6 kW. This yields a power loss percent of 4.858% of the total system load.
Case 3. In this case, it is required to minimize only the losses cost to the least possible value. This case may be adopted if the cost of unit energy is expected to largely increase with time. In this case the program selected all the outgoing lines of the substation as depicted in Fig.6. The different costs associated with this planned distribution system are: $Fc = 46{,}500$ \$, = 9,669.5 \$, $F = 56{,}169.5$\$, power losses = 0.05629 p.u. = 56.29 kW. This yields a power loss percent of 2.45% of the total system load.
Case 4. In this case, it is required to minimize the cost of losses only while maintaining two outgoing feeders from the substation. The planned network obtained is presented in Fig.7. The output results are given as follows: $Fc = 40{,}725$ \$, = 14,973.2 \$, $F = 55{,}698.2$ \$, power losses = 0.06872 p.u. = 68.72 kW. This yields a power loss percent of 2.99% of the total system load.
The costs and power losses obtained for the four case studies are tabulated in Table 2 and illustrated in Fig. 8 and Fig. 9.

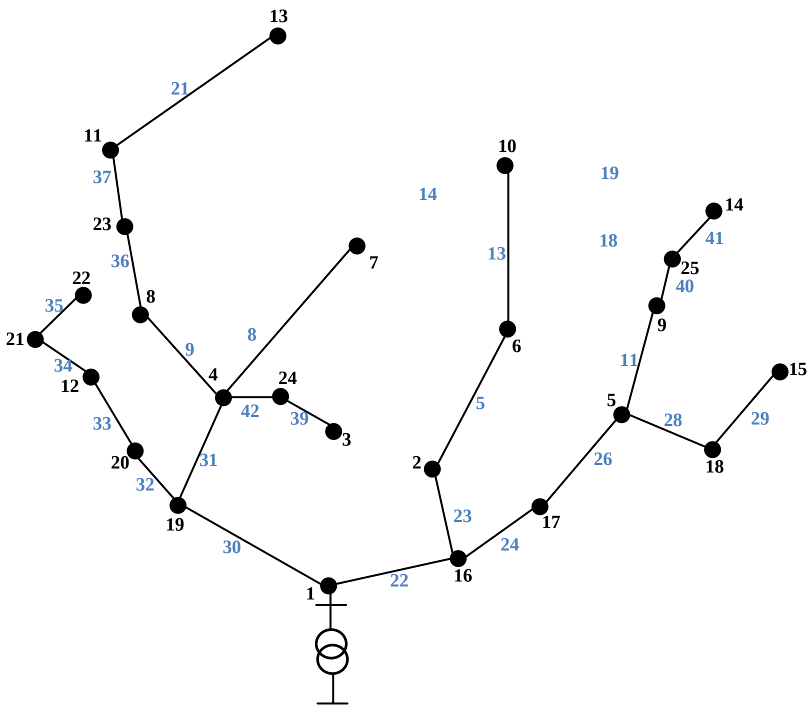


Figure 4. Case 1 results for considering the capital and losses cost

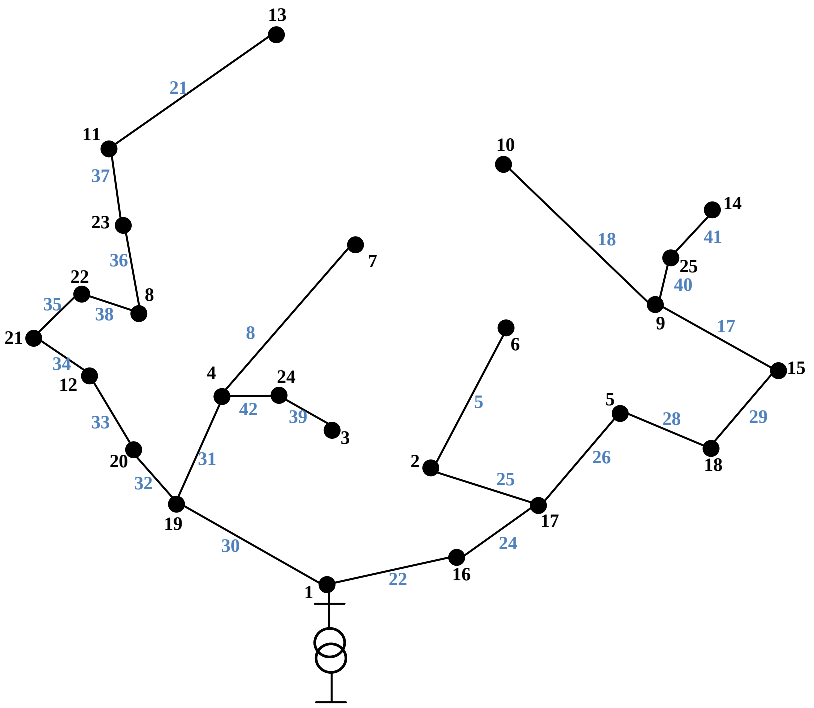


Figure 5. Case 2 results for considering the capital cost only

Table 2. The costs and power losses for the four case studies

Case study	Total cost [$]	Capital cost [$]	Losses cost [$]	Power losses [kW]
1. Min total cost	51,288.2	36,000	15,288.2	89
2. Min capital cost	54,636.3	35,475	19,161.2	111.6
3. Min losses cost	56,169.5	46,500	9,669.5	56.29
4. Min losses cost with outgoing lines constraints	55,698.2	40,725	14,973.2	68.7

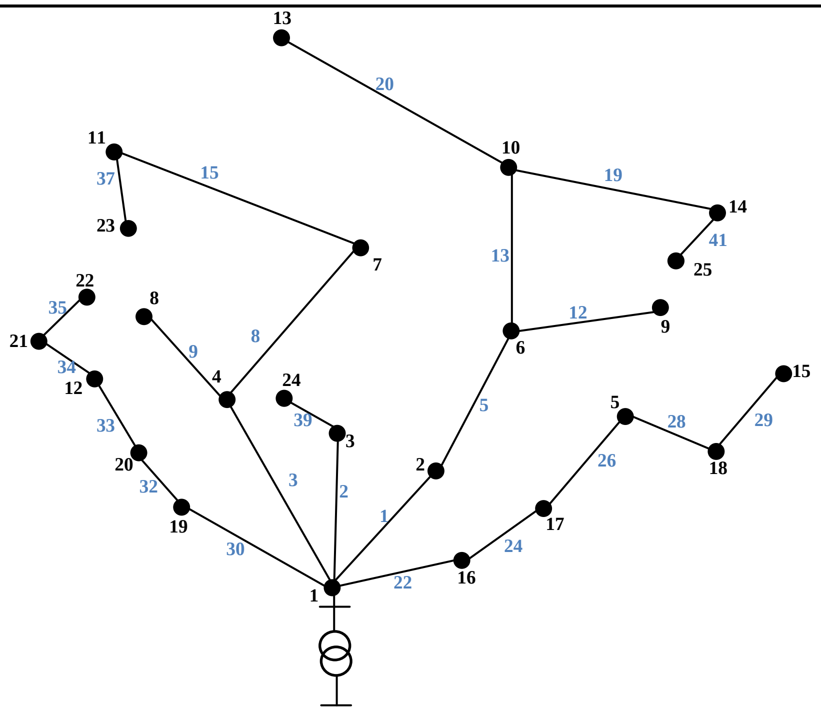


Figure 6. Case 3 results for considering the losses cost only

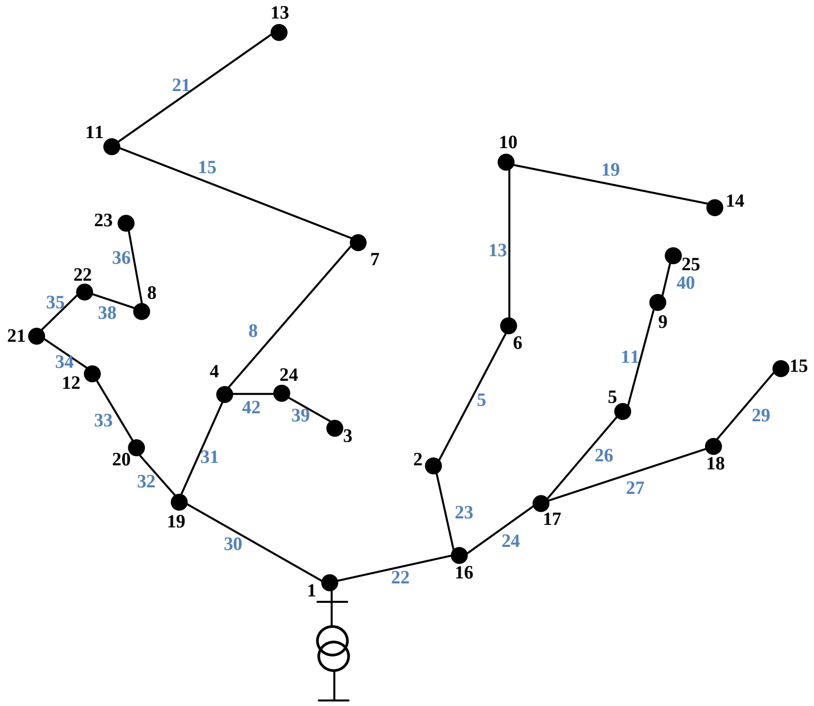


Figure 7. Case 4 results for considering the losses cost and maintaining two outgoing feeders from the substation

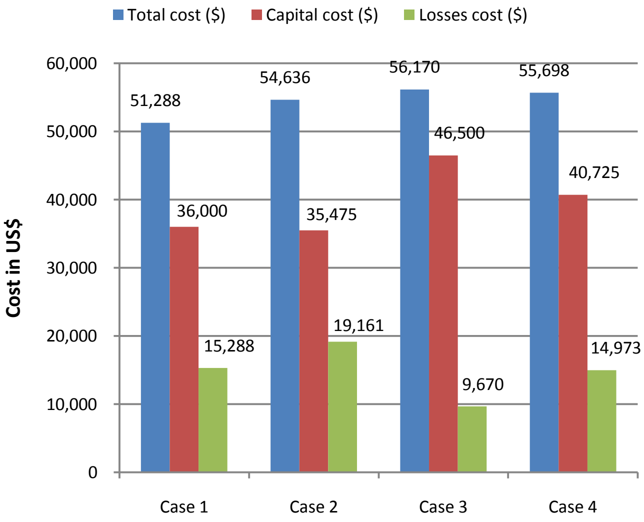


Figure 8. Different costs obtained for the 4 case studies of the 24 bus radial distribution network

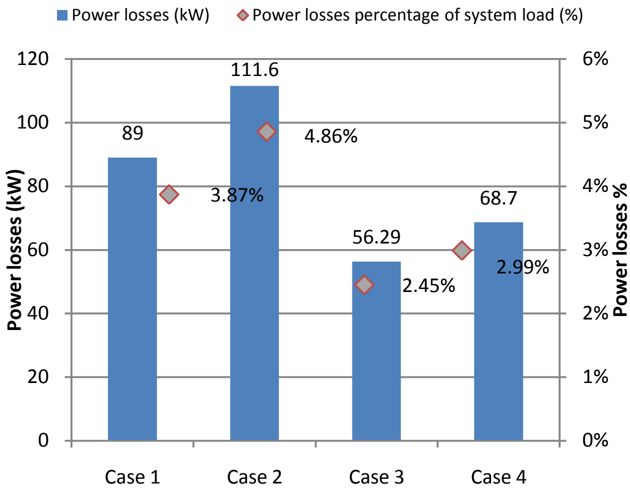


Figure 9. Power losses for the 4 case studies of the 24 bus radial distribution network

The analysis of the four case studies and observations from Table 2, Fig.8 and Fig.9 show that:

1- The minimum network cost is that which minimizes both the capital as well as the losses cost. If one cost type only is minimized, the other cost type will be larger such that the total cost is larger.
2- Lines number 20 and 30 are the best lines outgoing from the substation as they are selected in all the case studies investigated.
3- The most important lines that are selected in all four case studies are 5, 8, 22, 24, 26, 29, 30, 32, 33, 34, 35, 39.
4- The lines that are found in most networks (3 out of 4 case studies) are 13, 21, 28, 31, 36, 37, 40, 41, 42.
5- The lines that are excluded from the four case studies are 4, 6, 7, 10, 14, 16.

7. Conclusion

The paper presents a model that can be used for optimal planning of distribution networks using mixed integer quadratic programming technique. The model can be easily applied for planning of new networks as well as expansion of existing old ones. The results obtained for the case studies adopted prove the accuracy and robustness of the presented model. Analysis of the 4 case studies performed revealed that the optimum case is the one that considered minimization of both the capital cost and energy losses cost yielding a minimum total cost of 51,288.2 $ and a power loss percentage of 3.87% of the total system load. It was also observed that there is a specific set of lines that are preferable for addition; as they have been selected in the 4 case studies performed. On the other hand, a certain set of lines were constantly excluded in the 4 case studies. For the future work, the impact of placement of shunt capacitors in radial distribution feeders will be investigated. The use of capacitor banks is intended for reactive power compensation and voltage improvement. Moreover, the proposed planning model is to be extended to enable considering the challenges facing the distribution grid as the penetration of distributed generation.

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