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Alternative Formulae for Calculating Third and Fourth Power of a Number

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Abstract: The present paper provides alternative formulae for calculating a third and fourth power of a number. The number can be any number selected from a group consisting of a natural number, a whole number, an integer, a rational number, and an irrational number. The formulae enable easy and fast calculation of the third and fourth power of a given number.

Keywords: Formulae, fourth power, numbers, and Third power

I. INTRODUCTION

Numbers are an integral part of our life. Every day we deal with numbers. For example, we wake up at a particular time in a clock. The indicia which depict time in the clock are numbers. While purchasing daily utilities, again we encounter numbers, in form of cost of the utilities we purchase, or the amount of the utilities, etc. From childhood, we are taught counting things, manipulating numbers, by adding, subtracting, multiplying, and dividing numbers.

Various methods and formulae have been devised to manipulate numbers. More specifically, for easy and simple multiplication of numbers, numerous methods and formulae have been developed. Calculation of powers of numbers, which is basically multiplication of a number equal to the exponent of the number is easy for small numbers, but as the number increases, the calculation becomes difficult and at times impossible.

To simplify the calculation of numbers, attempts have been made, wherein numerous formulae, methods, and strategies have been developed which allow speedy calculation of the power of a given number.

A known method is multiplying numbers successively. For example, to calculate third power of a four-digit number 1032, the number 1032 is multiplied successively as shown below:

1032 x 1032 = 1065024, which is further multiplied by 1032 again, that is, 1065024 x 1032 = 1099104768.

Another method is by using the algebraic expansion formula:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

In order to use the above–mentioned formula for calculation of cube of 1032, 1032 can be written as sum of 1000 and 32.

$$(1000 + 32)^3 = (1000)^3 + 3(1000)^2(32) + 3(1000)(32)^2 + (32)^3$$
$$= 10^9 + 3(10^6)(32) + 3(10^3)(1024) + 32768$$
$$= 10^9 + 96000000 + 3072000 + 32768 = 1099104768.$$

As is evident from the above, the conventionally known methods are cumbersome and lengthy. Further, for larger numbers, it is difficult to keep track of the calculations.

Therefore, there exists a need for overcoming the above–mentioned drawbacks of the known methods or formulae and provide a suitable alternative for calculating the third and fourth power of a given number, wherein the formula or method is simple, and provides an easy way to calculate the respective powers.

II. FORMULA TO CALCULATE THIRD POWER OF A GIVEN NUMBER

In order to calculate a cube of a number, the author of the present paper has discovered the following formula:

$$x^3 = 6(x – 1)^2 + (x – 1)(x – 2)(x – 3) + x \ldots(I.A)$$

Or

$$x^3 = (x – 1)[6(x – 1) + (x – 2)(x – 3)] + x \ldots (I.B)$$

Or alternatively, the formula:

$$x^3 = x[(x – 1)(x + 1) +1] \ldots(II)$$

Calculating cube of a number employing the formulae (I.A), (I.B) and (II):

EXAMPLE 1

x^3 = 1099104768, wherein x = 1032

Using formula, (I.A):

$x = 1032; (x - 1) = 1031; (x - 2) = 1030; (x - 3) = 1029;$

Therefore,

$$x^3 = 6(1031)^2 + (1031)(1030)(1029) + 1032$$
$$x^3 = 6377766 + 1092725970 + 1032$$
$$x^3 = 1099104768$$

Using formula, (I.B):

$$x^3 = (1031)[6(1031) + (1030)(1029)] + 1032$$
$$x^3 = (1031)[6186 + 1059870] + 1032$$
$$x^3 = (1031)[1066056] + 1032$$
$$x^3 = 1099103736 + 1032$$
$$x^3 = 1099104768$$

Finally using formula, (II):

$$x^3 = (1032)[(1031)(1033) +1]$$
$$x^3 = (1032)(1065023+1)$$
$$x^3 = (1032)(1065024)$$
$$x^3 = 1099104768$$

EXAMPLE 2

$$x = \sqrt{5}$$
$$x^3 = x((x - 1)(x + 1) +1) \ldots(II)$$
$$(\sqrt{5})^3 = (\sqrt{5})\left((\sqrt{5} - 1)(\sqrt{5} + 1) + 1\right)$$
$$5\sqrt{5} = 5\sqrt{5}$$

EXAMPLE 3

$$x = 28$$
$$x^3 = (x - 1)[6(x - 1) + (x - 2)(x - 3)] + x \ldots (I.B)$$
$$(28)^3 = (28 - 1)[6(28 - 1) + (28 - 2)(28 - 3)] + 28$$
$$21952 = (27)[6(27) + (26)(25)] + 28$$
$$21952 = (27)[162 + 650] + 28$$
$$21952 = (27)[812] + 28$$
$$21952 = 21952$$

III. FORMULA TO CALCULATE FOURTH POWER OF A GIVEN NUMBER

For calculating the fourth power of a given number, the author has devised the following formula:

$$x^4 = x(x - 1)(x + 1)x + x^2 \ldots (III.A)$$

or

$$x^4 = x(x^2 - 1)x + x^2 \ldots (III.B)$$

EXAMPLE 4

For $x = 25$,

$$x^4 = 390625;$$

Using formula, (III.A)

$$x^4 = (25)(25 - 1)(25 +1)(25) + (25)^2$$
$$x^4 = (25)(24)(26)(25) + (25)^2$$
$$x^4 = (625)(624) + (625)$$
$$x^4 = 390000 + 625$$
$$x^4 = 390625$$

Using formula, (III.B):

$$x^4 = (25)(25^2 - 1)(25) + (25)^2$$
$$x^4 = (625)(624) + (625)$$
$$x^4 = 390000 + 625$$
$$x^4 = 390625$$

Thus, employing the above–mentioned formulae for calculating third power or fourth power of a given number, the calculation becomes relatively simple, and easy.

The author of the present paper has verified the above–mentioned formulae by using excel spreadsheet for x varying in the range of 1 to 30,000, for numbers which are square root, cube roots. The above–mentioned formulae have been observed to be applicable at least to the above–mentioned ranges.

IV. PROOF FOR CALCULATING CUBE OF A NUMBER BY INDUCTION METHOD

Theorem: For any number x, $x^3 = 6(x-1)^2 + (x-1)(x-2)(x-3) + x$, wherein x is one of natural number, a whole number, an integer, a rational number, and an irrational number.

Proof: Let P(x) be the statement "$x^3 = 6(x-1)^2 + (x-1)(x-2)(x-3) + x$". The above–mentioned formula is proved in two steps herein below:

For x = 1,

$$P(1) = (1)^3 = 6(1-1)^2 + (1-1)(1-2)(1-3) + 1$$
$$(1)^3 = 6(0)^2 + (0)(-1)(-2) + 1$$
$$1 = 1$$

Therefore, for x = 1, the statement P(1) holds true

We have to prove that the statement P(x) holds true for x = k, that is, P(k) holds true.

Thus, we have

$$k^3 = 6(k-1)^2 + (k-1)(k-2)(k-3) + k$$
$$\text{Right hand side (RHS)} = 6(k-1)^2 + (k-1)(k-2)(k-3) + k$$
$$= 6(k^2 - 2k + 1) + (k2 - 3k + 2)(k-3) + k$$
$$= 6k^2 - 12k + 6 + k^3 - 3k^2 + 2k - 3k^2 + 9k - 6 + k$$
$$= k^3 = \text{Left hand side (LHS)}$$

Hence, $k^3 = 6(k-1)^2 + (k-1)(k-2)(k-3) + k$.

Since, P(k) is true, P(k + 1) is true by induction.

Therefore, the statement $x^3 = 6(x-1)^2 + (x-1)(x-2)(x-3) + x$ is true for all x.

Similarly, the statement $x^4 = x(x-1)(x+1)x + x^2$ can be proved using induction method.

V. CONCLUSION

I have provided formulae for calculating a third and fourth power of a given number, the number being selected from a natural number, a whole number, an integer, a rational number, and an irrational number. The formulae facilitate speedy and easy calculation of third and fourth power of a given number with reduced human efforts. The above–mentioned formulae may also be employed to develop and implement a computer code, which may reduce computation time.

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