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Particle Swarm Optimization for Tracking Control of a Self-Balancing Electric Unicycle

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Abstract: This paper proposes an LQR controller for the balance and stability of an electric unicycle. Aparticle swarm optimization (PSO) algorithm is utilizedto automatically track the best solution of the weighting matrices Q and R of` the controller, rather than randomly choosing their weights. It also examines dynamics of the electric unicycle's schematic design, which has a backing seat for the passenger and is analogous to the Segway device equipped with a handle shaft for direction. Command tracking or regulation of some initial state condition were examined and demonstrated by time based simulations of the modelled system. The results evidently indicate that the LQR controller can effectively and efficiently attain attitude equilibrium for the two-wheeled unicycle.
Index Terms: Unicycle, LQR, PSO.

I. Introduction

This paper describes the controller design and dynamics analysis for the system model of an electric two-wheeled unicycle. The final aim is to make possible the use of the electric unicycle conveniently as a mean of personal transportation medium. At the present, the motorcycle and electric bicycles with longitudinal two-wheels are amongst the utmost common in use. Electric unicycle has the advantage of having a simpler build;consequently, it is beneficial in having most usage flexibility, lower fabrication cost, and minimum space occupation.

Proficient skill is needed by the rider to maintain the longitudinal attitude balance of the unicycle. This poses a barrier for the adoption of the unicycle as a mean of daily transportation. The Segway having lateral two-wheeled has the ability of self-stance balancing besides to requiring human force to drive it. Initial production of the Segway was in 2002 which sold over 50,000 units in 2009 [1], [2].Those longitudinal adjusting work and the parallel two-wheeled structure empower Segway's ability to perform same time pivoting or standing still. Though, Segway is not capable of crossing through limited passages because of the parallel two-wheel configuration. Furthermore, the Segway cannot be operated via human force and can only be driven by electricity.

Empowered by the late develop advancement of advances in state of mind detecting control and center engines and propelled by the spearheading Segway outline, an assortment of individual vehicles with the inclination of programmed disposition adjusting control have been industrially advancedand created. The Reabo [3] for instance beginning in Taiwan, has the same longitudinal adjusting capacity and horizontal two-wheeled design like the Segway. Besides, the Reabo is improved toward its maneuvering procedure, achieved by controlling relative pace inter alia its two wheels by twisting or moving the same control pole.

An unstable inverted pendulum is quite comparable to the underlying forces of the electric unicycle. In the literature [4]-[8], nonlinear global control approaches have been established, accounting every attitude position ranges for the design of the controller. However, a control system that can properly specify a reasonable attitude operating domain is important for a practical and harmless use of the unicycle [4]-[8].

The works in [9] and [10] taking into account the linear control approaches presented the linear quadratic regulator (LQR) and the proportionalderivative (PD) controller, respectively, to automatically balance the attitude of the electric unicycle. Though, the actuator constraints concern isn't stated in [9]. Moreover, regarding the riders' various uncertainties are not taken into account in [10]. Furthermore, as regards the modern advances of control methods relating to the electric unicycle, the workings in [11]-[13] studied the trajectory tracking of mobile robots of unicycle-type, fitted with one additional passive wheel and two actuated wheels. Thus, the topic of longitudinal attitude-poising isn't considered in the modern studies [11]-[13].

II. Electric Unicycle Dynamics Anaysis

Figure 1 illustrates the dynamicsmodel of the unicycle attitude control. In terms of mechanical maneuvering, there are two aspects: 1) the movement incident in contact with the ground and tire (translational); and 2) the motion surrounding the axis of the tire (rotational). The translational displacement and rotational motion are coupled together and affect one another. Electrical energy supplied by a battery powers the mode of operation, according to the attitude angle θ sensing signal and control of pivot motor torque τ_m. Maintaining the rotating axis in an upwards vertical position is the ultimate control objective, which can be translated to (attitude angle $\theta \approx 0$).

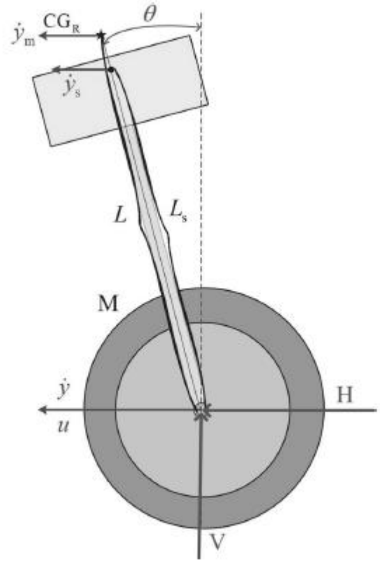

Figure 1. Schematic diagram of the electric unicycle.

First, the unicycle's moment of inertia of control/power module and rider is denoted as I and consider the rotation movement about the axis of the tire. The dynamics of mechanical rotation can be defined as:

$$I\ddot{\theta} = VL\sin\,\theta - HL\cos\,\theta \quad (1)$$

Where V and H symbolize the vertical and horizontal forces applied on the axis of the tire, respectively and *L* denote the length between the rider's center of gravity CG*R* and the tire rotation. When referring to Newton's second law of motion, it can be definedas:

$$H = m\frac{d^2}{dt^2}(y + L\sin\theta), \qquad V = m\frac{d^2}{dt^2}(L\cos\,\theta) + mg \quad (2)$$

where, $y, \dot{y}, \ddot{y}$ are the distance, velocity, and acceleration of the translational movement, respectively. Moreover, $\theta, \dot{\theta}, \ddot{\theta}$ are the attitude angle, angular velocity, and angular acceleration, respectively.By replacing the terms for the vertical force, horizontal force, and moment of inertia,

$$H = m\left(\ddot{y} + L\ddot{\theta}\cos\,\theta - L\dot{\theta}^2\sin\,\theta\right), \quad (3)$$
$$V = m\left(-L\ddot{\theta}\sin\,\theta - L\dot{\theta}^2\cos\,\theta\right) + g,$$
$$I = \frac{mL^2}{3}$$

The dynamics of rotation in (1) can be rewritten as: $\frac{mL^2}{3}\ddot{\theta} = mgL\sin\theta - mL\ddot{y}\cos\theta - mL^2\ddot{\theta}$. And further simplified as:

$$\frac{4L\ddot{\theta}}{3} = g\sin\theta - \ddot{y}\cos\theta \quad (4)$$

According to the analysis of force balance of the backwards and forwards translational movement of the tires that are in contact with the floor, the dynamics of mechanical translation can be defined as:

$$M\ddot{y} = u - H - K_d\dot{y} - K_s\mathrm{sgn}\,(\dot{y}) \quad (5)$$

Everywhere M refers to the unicycle's mass; and K_s and K_d are the static and dynamic friction coefficients, respectively, $u$is the applied force on the tire externally; between the ground and the tire. The translational dynamics (5) can be rewritten by replacing the equation for lateral force in (3):

$$M\ddot{y} = u - m\left(\ddot{y} + L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta\right) - K_d\dot{y} - K_s\,\mathrm{sgn}(\dot{y})$$

and further reorganized as:

$$(M+m)\ddot{y} + mL\ddot{\theta}\cos\theta - mL\dot{\theta}^2\sin\theta = u - K_D\dot{y} - K_S\,\text{sgn}(\dot{y}) \tag{6}$$

The design of the controller was implemented built on the unicycle's mechanical translational motion and rotation designated in (6) and (4), in that order. The goal is to maintain attitude angle in a relatively small range under normal operating conditions. To assist in controller design and further dynamics analysis, the following linearization approximation is performed in the nonlinear unicycle dynamics:

$$\sin\,\theta \cong \theta, \cos\,\theta \cong 1, \dot{\theta} \cong 0 \tag{7}$$

As regards $\ddot{y}$ and θ being unknown variables, the equations of the dynamic system (4) and (6) are organized as:

$$\begin{bmatrix} \frac{4L}{3} & 1 \\ mL & M+m \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} g\theta \\ u - K_d\dot{y} - K_s\,\text{sgn}(\dot{y}) \end{bmatrix} \tag{8}$$

Thereafter, the subsequent differential dynamical equations can be obtained for the unidentified $\ddot{\theta}$ and $\ddot{y}$ by:

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} \frac{3g(M+m)}{L(4M+mm)}\theta + \frac{3K_d}{L(4M+m)}\dot{y} - \frac{3}{L(4M+m)}(u - K_s\,\text{sgn}(\dot{y})) \\ \frac{-3mg}{(4M+m)}\theta - \frac{4K_d}{(4M+m)}\dot{y} + \frac{4}{(4M+m)}(u - K_s\,\text{sgn}(\dot{y})) \end{bmatrix} \tag{9}$$

As a result of setting the variables of states $x_1 = \theta, x_2 = \dot{\theta},\ x_3 = y, x_4 = \dot{y}$, the dynamics of electric unicycle can be obtained in the form of the state-space model:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{3g(M+m)}{L(4M+m)} & 0 & 0 & \frac{3K_d}{L(4M+m)} \\ 0 & 0 & 0 & 1 \\ \frac{-3mg}{(4M+m)} & 0 & 0 & \frac{-4K_d}{(4M+m)} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-3}{L(4M+m)} \\ 0 \\ \frac{0}{(4M+m)} \end{bmatrix} (u - K_s\,\text{sgn}(\dot{y})) \tag{10}$$

The work in [7] defined the electric unicycle's dynamic properties referring to equation (10). The only operating condition resulting in steady-state is a zero attitude angle $\theta = 0$ when a force is applied externally to the unicycle, in addition to fixed translational velocity $\dot{y} = (u - K_s)/K_d$. If the attitude angle is maintained and controlled at a non-zero value, referring to (10), the electric unicycle will either make an accelerating or reduced speed motion $g\theta$, subject to the shift angle's direction. As a result, the electric unicycle's control function of the movement can be accomplished by managing the angle position deviation, according to the illustration of the electric unicycle's dynamic properties. In the present literature [7], for stabilizing nonlinear control law design the segment of the dynamics of rotation in (10), i.e., (4), is exclusively presumed for maintaining the position angle around $\theta \cong 0$.

Table I: SYSTEM PARAMETERS

Symbol	Description	Value
m	Mass of rider	65 (kg)
L	Distance between tire's rotating	0.75 (m)
M	axis and the rider's center of	15 (kg)
K_d	Mass of the Unicycle	0.05
g	Dynamic friction coefficient	9.8 (m/s^2)

III. Controller Design

The friction coefficients kd and ks as made known in (10), dependent on path of movement's direction of the electric unicycles, while the coefficient of dynamic friction K_d is fed to the matrix of the system, the coefficient of static friction K_s introduces an additional nonlinear entry to the force of control u. If one of the variables of the state $x_4 = \dot{y}$ is measurable online, the input term $(-K_s\text{sgn}(\dot{y}))$ which is nonlinear is simply excluded. In this paper, during control design the dynamic coefficient K_d effect is considered, while the static friction coefficient K_s effect is not taken into account for the design of control laws established on the state-space model in (10) and can only be evaluated in the response time simulation.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{3g(M+m)}{L(4M+m)} & 0 & 0 & \frac{3K_d}{L(4M+m)} \\ 0 & 0 & 0 & 1 \\ \frac{-3mg}{(4M+m)} & 0 & 0 & \frac{-4K_d}{(4M+m)} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-3}{L(4M+m)} \\ 0 \\ \frac{4}{(4M+m)} \end{bmatrix} (u) \tag{11}$$

Equation (11) will be adopted for the formulation of the state feedback LOR controller which will yield the PD controller gains.

A. LQR Controller

The LQR technique is a frequently implemented method of optimum control. It provides a systematic technique to calculate the gain matrix of state feedback which is one of its advantages and an additional benefit is the stability of the designed system is ensured. Referring to LQR controller concept, $\dot{x} = Ax + BU$ is the state equation, and $J = \int_0^{\inf} (x^T Q x + u^T R u) dt$ is the control performance indicator. This performance indicator J implies a measure of the system energies (potential and kinetic), which needs to be minimal through finding the K matrix in the feedback control $u(t) = -kx(t)$, where R is either a positive definite or a real matrix which is also symmetric, and Q is a positive definite or positive semi-definite matrix or a real matrix which is also symmetric. Thus possible candidates of Q and R exist in a wide range, and since their choice affects the control cost J, it is critical to achieve minimum control cost while satisfying stability and performance quality. Q and R are the weight matrices for x as well as u.
The equation of Riccati is:

$$A^T P + PA + Q - PBR^{-1} B^T P = 0 \tag{12}$$

The matrix P can be obtained by solving the above equality. If a positive definite matrix P exists, then a stable system could be constructed. Then taking the P matrix into the subsequent equation to will acquire the matrix K.

$$K = R^{-1} B^T P \tag{13}$$

The open-loop system controllability is firstly examined. Following classical control stability calculations, the system was found to be controllable and the matrices of the state feedback were acquired through the LQR technique. Afterwards numerous experimental simulations, the following Q and R matrices were selected based on a PSO (Particle swarm Optimization) algorithm that was implemented on the system's state space model to select the best Q and R matrices that best minimize the objective function.

Figure (2) of the results was obtained by $Q = \text{eye}(4)$ and $R = 1$. Then, the MATLAB function lqr() was utilized to solve the equation of optimality. The state feedback matrix was acquired by executing $K = \text{lqr}(A, B, Q, R)$ in MATLAB which yielded $K = [-1620.7 - 331.3 - 1 - 13.1]$

B. Particle Swarm Optimization Algorithm

Particle Swarm Optimization (PSO) is a computational search and optimization technique; it was developed by Eberhart and Kennedy in 1995. PSO is inspired by the group behavior of bird flocking or fish schooling.It has been successfully applied in many areas: function optimization, artificial neural network training, fuzzy system control.

PSO is easy to implement, and has a well-balanced and flexible performance to improve the local and global search capacities. PSO method performs search using particles of a population, where each particle characterizes a possible solution.

In the PSO structure, particles vary their locations by moving in a multi-dimensional search domain having two important reasoning capabilities: their memory of their own best position - local best (lb) and knowledge of the global best (gb). This is based on the evaluation of a specific objective function, in anticipation of a relatively unvarying position has been met, or awaiting computational limits to be reached. Velocity of the particle influences its position.

Let $xi(t)$ indicate the location of particle i in the exploration domain at an instance of time; if not then identified, t represents discrete time units. Particle location is varied by addition of a velocity $vi(t)$ to its present location, which is essentially a calculated displacement based on information about the swarm varied with each iteration hence the name velocity [14]:

$$xi(t+1) = xi(t) + vi(t+1) \quad (14)$$

where [15]:

$$vi(t) = vi(t-1) + c1r1(\text{ localbest }(t) - xi(t-1)) + c2r2(\text{ globalbest }(t) - xi(t-1)) \quad (15)$$

With $xi(0)$ being the initial particle position (random) between the minimum and maximum particle position limits (predefined), acceleration coefficient c1 and c2 and random vector $r1$ and $r2$. Each new particle position is then used to evaluate an objective function, which is either minimized or maximized. The input particles in our case were the Q and$R$ matrix elements which were fed to the PSO algorithm and were allowed to vary within a certain range.

Technically, the involved particles will fly around with the updated velocity (15) towards a new position (14). They will keep on competitively flying until they find the best value for the objective function. After each iteration, the stability of the state feedback system was evaluated using the following objective function: max(real($eig(A - B * K)$)). The values for Q and R matrices that resulted in the best objective function were saved and updated within the PSO algorithm.

IV. Results

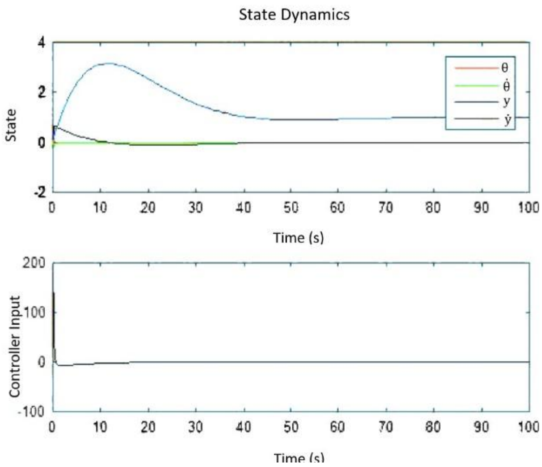

Figure 2. Unit step response (state dynamics).

Figure 2 shows the output system state dynamics when a unit step input was applied at the input terminal of y, meaning the electric unicycle should travel one meter. The gains were calculated based on $Q = \text{eye}$ (4) and $R = 1$. It takes approximately 50 seconds to track the input and reach the desired target. The choice of Q and R greatly influences the settling time of the system and its performance quality.

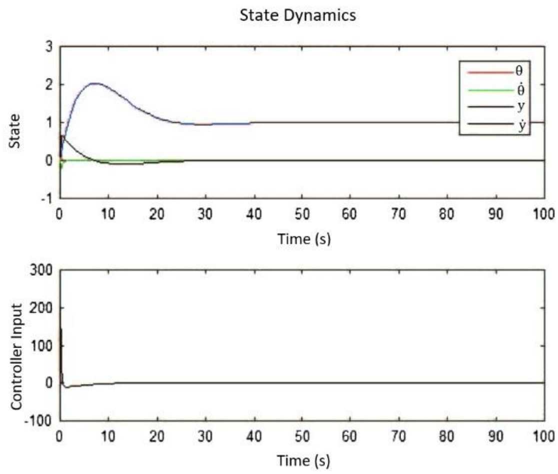

Figure 3. Unit step response (system dynamics).

In Figure (3) the Q matrix was arbitrary set to:

$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } R = [1]$$

It is observed that the system takes about 40 second to reach the desired target. Making slight improvement when compared to the previous case.

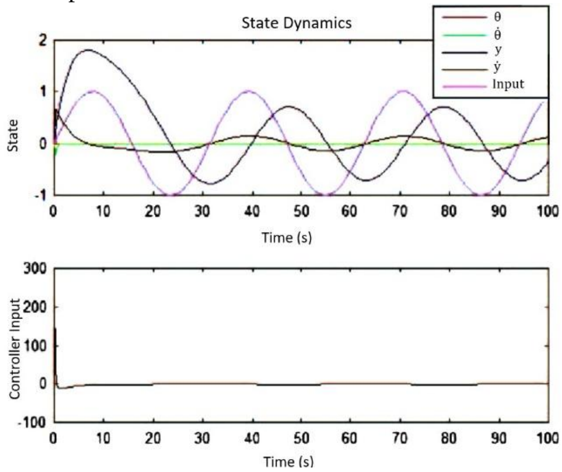

Figure 4. System state dynamics when tracking a sinusoidal input.

Here the system's ability to track a time varying input displacement $y = \sin(0.2t)$, the system was clearly lagging by approximately 9 seconds with an 80% overshoot.

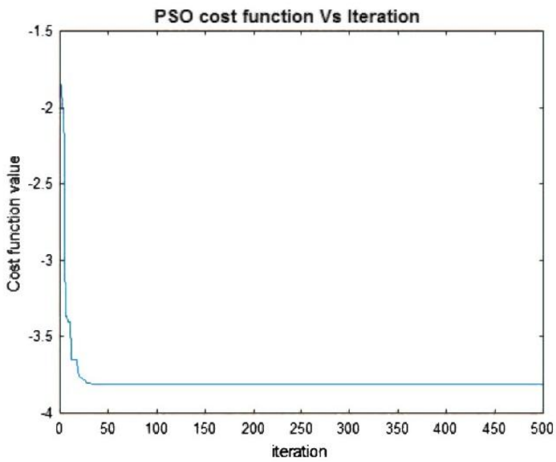

Figure 5. Objective function vs.Iteration number.

Figure (5) shows the convergence of the Particle Swarm Optimization technique in achieving the minimal objective function reflecting the stability of the resulting system which is evaluated and updated on each run of the algorithm. The cost becomes constant after the 50^{th}iteration. For future runs the number of iterations can be limited to 50 and save time.

The system dynamic results on Figure (6) are a result of applying the optimal system LQR gains which were calculated based on the Q and R matrices that were selected by the PSO algorithm and set to

$$Q = \begin{bmatrix} 344 \times 10^6 & 0 & 0 & 0 \\ 0 & 30.9 \times 10^6 & 0 & 0 \\ 0 & 0 & 1 \times 10^6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and} R = [1]$$

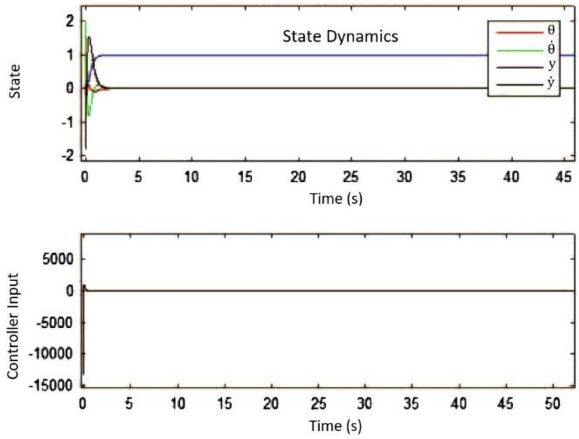

Figure 6. Unit step response (system dynamics).

The optimal feedback gain matrix K was found to equal: $K = [-97404 - 30879 - 316233 - 24958]$ The settling time was found to be approximately 2 seconds, which shows a significant improvement in performance.

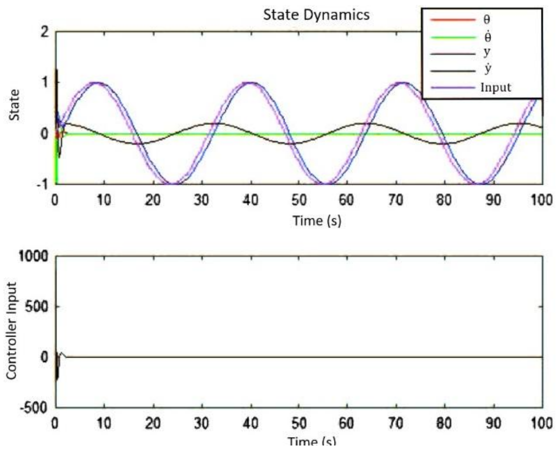

Figure 7. System dynamics as a result of tracking a sinusoidal input.

The resulting system dynamics in Figure(7) show the capability of the unicycle to track a variable input command with high accuracy only lagging by 0.9 seconds while maintaining attitude balance.

V. Conclusions

In this paper a systematic mathematical model of a twowheeled unicycle, based on the theory of dynamic mechanics was simulated for controller implementation. First an LQR controller was designed by selecting $Q = \text{eye}\ (4)$ the identity matrix and $R = 1$ was kept at a constant value equal to one, which yielded a slow response controller. The PSO algorithm was then utilized to overcome the blind selection of Q and R matrices, and it resulted in the minimal objective function value. When implemented the results showed high system response and high accuracy with low latency for tracking variable input command signals, functioning efficiently and at a fast rate as required of the system. The outcomes clearly demonstrate the ability of the LQR controller attaining balance control of the two-wheeled unicycle effectively whilst traveling a prescribed distance, achieving the control targets, and attaining an excellent quality performance of state dynamics.

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