International Journal of Latest Research in Engineering and Technology (IJLRET)
ISSN: 2454-5031
www.ijlret.com || Volume 08 - Issue 04 || April 2022 || PP. 10-18

Panel Method for Flow Problems with the Ideal Flows

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Abstract: Panel methods are well-known methods for solving potential fluid flow problems which have been applied to solve the Laplace equation for aerodynamics ([2], [4]). Thin solid flow problems with the negative sub-current were studied carefully by analysic method for aerodynamics. If flow velocity is smaller than 120m/s, then incompressible flow models are applied. In addition, if the motion is not non-vertox then problems lead to Laplace equation with respective conditions. To be flow current with $0.3 \leq M \leq 0.6$ as well as the asumptions above, then compressible flows will be converted to incompressible flows by Prandtl – Glauert transformation.

Keywords: Aerodynamics, compressible flows, incompressible flows, vertox motion.

I. Introduction

To begin, we consider the problem of enclosing a solid by a negative sub-current. Assume the flow is two dimension, stagnation as shown in figure 1:

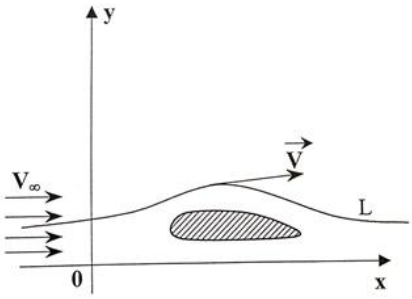


Figure 1: the flow current of the plane's wings

Where V_∞ is the velocity far from the solid
$\vec{V} = (u, v)$ is the velocity field near the surface of solid
L is the current
Determine the velocity field distribution and the pressure distribution on the solid surface.

II. Panel Method

Discreting the solid by N sub-panels linked together as show in figure 2 ([3]):

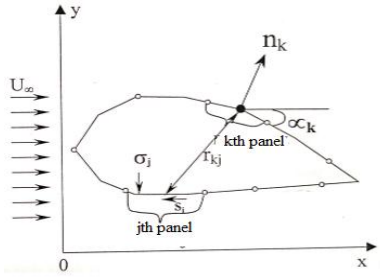


Figure 2: sub – panels on airfoil surface

On each panel, it is placed a source point with wattage σ. If the wattage is defined for every panel that the condition of non-permeation through solid surface is satisfied, then the fluid flow with velocitypotential is the sum of source points coressponding to $\varphi = V_\infty x$. Thevelocity potential of such flow is of the form:

$$\varphi(x_k, y_k) = U_\infty x_k + \frac{1}{2\pi}\sum_{j=1}^{N} \sigma_j \int lnr_{kj}\, ds_j \quad (1)$$

where $r_{kj} = \sqrt{(x_k - x_j)^2 + (y_k - y_j)^2}$

$\sigma_j \int ds_j$ is the source intensity of the jth panel

subject to

$$V_n|_{-}s = \frac{\partial\varphi}{\partial n_k}(x_k, y_k) = -U_\infty sin\alpha_k + \frac{1}{2\pi}\sum_{j=1}^{N}\sigma_j \int lnr_{kj}\, ds_j = 0 \tag{2}$$

$$\frac{1}{2\pi}\sum_{j=1}^{N}\sigma_j \int lnr_{kj}\, ds_j = U_\infty sin\alpha_k \tag{3}$$

Where$\alpha_k$is the slope of the kth panel relative to Ox

If $k = j$, the integral in (2) is estimated as follow:

$$\int_k \frac{\partial}{\partial n_k}(lnr_{kj})ds_k = \pi \tag{4}$$

for$k \neq j$, the integral is eatimated as a function of knot points (x_k, y_k, x_j, y_j). Therefore, equations (3) are an algebraic generally system of linear equations

$$A\sigma = B \tag{5}$$

Where $\sigma$are unkown variables

The entries of A are

$$A_{kj} = 0.5\delta_{kj} + \frac{1}{2\pi}\int \frac{\partial}{\partial n_k}(lnr_{kj})ds_j \tag{6}$$

The entries of B are

$$B_k = U_\infty sin\alpha_k \tag{7}$$

To solve Eq. (5) we use the decay method. After finding σ_j, the velocity components will be found by the following fomulas

$$\begin{cases} u(x,y) = \frac{1}{2\pi}\sum_{j=1}^{N}\sigma_j \int \frac{x-x_j}{(x-x_j)^2+(y-y_j)^2} ds_j \\ v(x,y) = \frac{1}{2\pi}\sum_{j=1}^{N}\sigma_j \int \frac{y-y_j}{(x-x_j)^2+(y-y_j)^2} ds_j \end{cases} \tag{8}$$

The pressure distribution on the surface of a solid is determined directly from the Bernoulli's integral:

$$c_p = \frac{p-p_\infty}{0.5\,\rho U_\infty^2} = 1 - \left(\frac{q}{U_\infty}\right)^2 \tag{9}$$

along with the complete velocity field $\vec{q} = (U_\infty + u, v)$

To calculate the velocity components u and v, the more efficient numerical method was introduced by Hess.

From the distribution ofEq.(8) Hess estimated$V_{kj} = (u_{kj}, v_{kj})$, where$u_{kj}, v_{kj}$are called the induced velocity components at the point(x_k, y_k)which generated by the source density on the jth panel ([1]). Thus, equation (8) can be written as follow:

$$\begin{cases} u(x_k, y_k) = \sum_{j=1}^{N} u_{kj}\,\sigma_j \\ v(x_k, y_k) = \sum_{j=1}^{N} v_{kj}\,\sigma_j \end{cases} \tag{10}$$

$u_{kj}, v_{kj}$are expressed in term of the respective components in the local coordinate system:

$$\begin{cases} u_{kj} = q_{kj}^t\, cos\alpha_j - q_{kj}^n\, sin\alpha_j \\ v_{kj} = q_{kj}^n\, cos\alpha_j + q_{kj}^t\, sin\alpha_j \end{cases} \tag{11}$$

Eqs. (11) are the coordinate system transformation formulas

Where$q_{kj}^t, q_{kj}^n$are the induced velocity components at (x_k, y_k)which generated by the source density on the jth panel in the local coordinate system along with the tangent direction and normal direction and they are given by

$$\begin{cases} q_{kj}^t = ln\left(\frac{(\xi_k+0.5\Delta s_j)^2+\eta_k^2}{(\xi_k-0.5\Delta s_j)^2+\eta_k^2}\right) \\ q_{kj}^n = 2\tan^{-1}\left(\frac{\eta_k \Delta s_j}{\xi_k^2+\eta_k^2-\left(\frac{\Delta s_j}{2}\right)^2}\right) \end{cases} \tag{12}$$

where$\Delta s_j$is the length of the jth panel

$(\xi, \eta)$is the local coordinate system on the jth panel as show in figure 3.

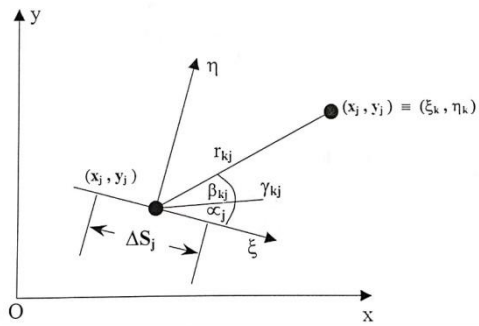


Figure 3

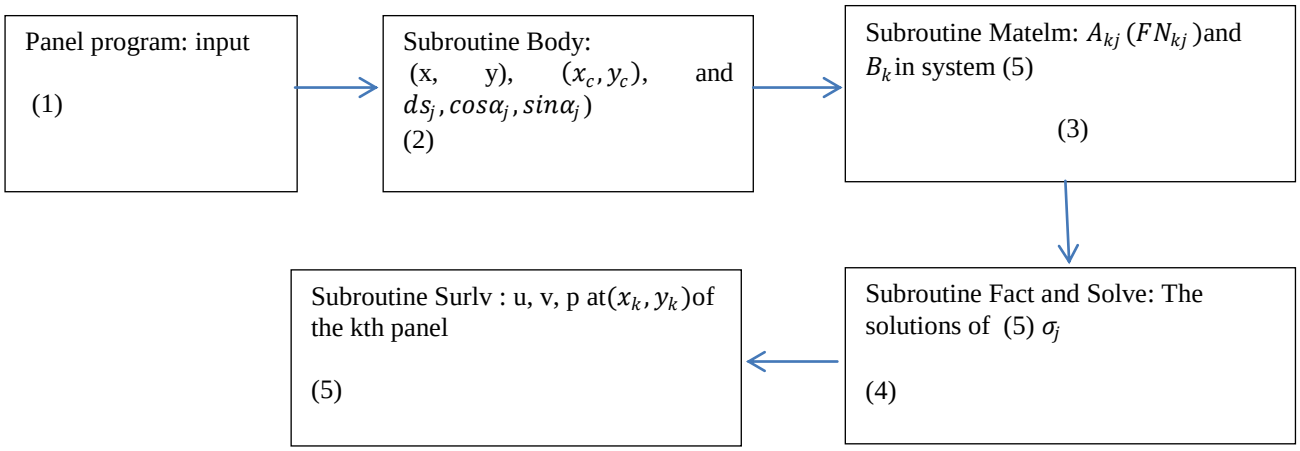
If$n_k$is the unit normal vector of the kth panel, then A_{kj} is given by

$$A_{kj} = n_k V_{kj} \tag{13}$$

Since, Eq. (5) with A and B is defined by Eqs. (6), (7), (13) and solved by the decay method. When we have σ, u, v are clasified byEq. (10).

III. Diagram, Program, and Results

IV.1. Algorithmic diagram



IV.2. Program

The program is tested by applying the flow problem with unit circle (b = 1) by imcompressible flow. The number of panels is 20.

```
        PROGRAM PANEL METHOD
        Dimension x(50), y(50), xc(50), yc(50), ds(50), fn(50),
*       ft(50, 50), rhs(50), sde(50), ci(50), si(50), aa(50), iks1(50)
        common x, y, xc, yc, ds, fn, ft, rhs, pi, cpi, ci, si, unif, vinf, sde
        open(1,file='panel.dat')
        open(6,file='panel.out')
        write(*,*)'input: n, ipr, uinf, vinf,fmn,b'
        read(*,*) n, ipr, uinf, vìn, fmn, b
1       format(2i5,4e10.3)
        write(6,2)n,b
                write(6,3)uinf,vinf,fmn
2       format(1x,'so phan tu',i2,'phan tu',5x,'nua truc b=',e10.3,//)
3       format(1x,'cac thanh phan van toc=',2f6.3,2x,'so march',f6.3,//)
        m=n+1
                                        pi=3.14159265
                                        cpi=2./pi
                        call body(n,m,ipr,fmn,b)
                        call matelm(n,m,ipr)
```

```
                    do 5 k=1,n
                    do 4 j=1,n
4       aa(k,j)=fn(k,j)
5       sde(k)=rhs(k)
        call fact(n,aa,iks1)
        call solve(n,aa,iks1,sde)
        call survl(n,b,fmn)
stop
end
                SUBROUTINE BODY(N,M,IPR,FMM,B)
        dimension x(50),y(50),xc(50),yc(50),ds(50),fn(50,50),ft(50,50),rhs(50),sde(50),ci(50),si(50)
        common x,y,xc,yc,ds,fn,ft,rhs,pi,cpi,ci,si,uinf,vinf,sde
1               fac=sqrt(1.-fmn**2)
                    nhlff=n/2+1
                    nhh=nhlff+1
                    an=nhlff-1
        dth=pi/an
        do 2 i=1,nhlff
ai=i-1
th=ai*dth
th=pi-th
x(i)=cos(th)
y(i)=b*sin(th)
2       y(i)=y(i)*fac
do 3 i=nhh,n
3       x(i)=x(n+2-i)
        y(i)=-y(n+2-i)
x(m)=x(1)
y(m)=y(1)
do 4 i=1,n
xc(i)=(x(i)+x(i+1))*0.5
4       yc(i)=(y(i)+(y(i+1))*0.5
do 5 i=1,n
                sx=x(i+1)-x(i)
                sy=y(i+1)-y(i)
                ds(i)=sqrt(sx**2+sy**2)
                ci(i)=(x(i+1)-x(i))/ds(i)
5               si(i)=(y(i+1)-y(i))/ds(i)
if(ipr.eq.0)return
        Write(6,6)
6 format(2x, 'cac thong so phan tu')
        Do 7 i=1,n
7 write(6,8)i,x(i),y(i),xc(i),yc(i),ds(i),ci(i),si(i)
8 format(2x,'i=',i2,3x,'x,y=',2f8.4,3x,'xc,yc=',2f8.4,3x,'span=',f8.4,3x,'ci,si=',2f8.4)
return
end
                SUBROUTINE MATELM(N,M,IPR)

        dimension x(50),y(50),xc(50),yc(50),ds(50),fn(50,50),ft(50,50),rhs(50),sde(50),ci(50),si(50)
        common x,y,xc,yc,ds,fn,ft,rhs,pi,cpi,ci,si,uinf,vinf,sde
                do 2 k=1,n
                    do 1 j=1,n
                if(k.eq.j) fn(k,j)=2.*pi
                if(k.eq.j) ft(k,j)=0.0
                    if(k.eq.j) goto 1
                            dyj=si(j)*ds(j)
                            dxj=ci(j)*ds(j)
                            sph=ds(j)*0.5
```

```
        xd=xc(k)-xc(j)
                                        yd=yc(k)-yc(j)
                                        rkj=sqrt(xd**2+yd**2)
                                        bkj=atan2(yd,xd)
                                        alj=atan2(dyj,dxj)
                                        gkj=alj-bkj
                                        zik=rkj*cos(gkj)
                                        etk=-rkj*sin(gkj)
                                        r1s=((zik+sph)**2)+etk**2
                                        r2s=((zik-sph)**2)+etk**2
                                        qt=alog(r1s/r2s)
                                        den=zik**2+etk**2-sph**2
                                        gmn=etk*ds(j)
                                        qn=2.0*atan2(gnm,den)
                                        ukj=qt*ci(j)-qn*si(j)
                                        vkj=qt*si(j)+qn*ci(j)
                                        fn(k,j)=-ukj*si(k)+vkj*ci(k)
                                        ft(k,j)=ukj*ci(k)+vkj*si(k)
1 continue
        rhs(k)=uinf*si(k)-vinf*ci(k)
2 continue
        if(ipr.1e.1)return
3 write(6,4)
4 format(2x,'cac thanh phan van toc phap')
        do 5 k=1,n
5 write(6,8) k,(fn(k,j),j=1,n)
write(6,6)
6 format(2x,'cac thanh phan van toc tiep')
        do 7 k=1,n
7 write(6,8)k, (ft(k,j),j=1,n)
8 format(2x,i5,(10f110.5))
write(6,9)
9 format(2x, 'cac phan tu cua B')
write(6,10)(rhs(k),k=1,n)
10 format(2x,10f10.5)
Return
End
SUBROUTINE FACT(N,A,JPVT)
        dimension a(50,50), jpvt(50)
        nm=n-1
        do 5 k=1,nm
                                kp=k+1
                                l=k
                                do 1 i=kp,n
                                if(abs(a(i,k)).gt.abs(a(l,k)))l=i
1 continue
        Jpvt(k)=l
                        s=a(l, k)
                        a(l,k)=a(k,k)
                        a(k,k)=s
                        if(abs(s).lt.10e-15) go to 6
                        do 2 i=kp,n
                                a(i,k)=-a(i,k)/s
2 continue
        do 4 j=kp,n
                s=a(l,j)
                a(l,j)=a(k,j)
                a(k,j)=s
```

```
                if(abs(s).lt.10e-15) go to 4
                do 3 i=kp,n
                a(i,j)=a(i,j)+a(i,k)*s
3 continue
4 continue
5 continue
return
6 jpvt(n)=-1
return
end
SUBROUTINE SOLVE(N,A,JPVT,B)

        dimension a(50,50), jpvt(50), b(50)
                nm=n-1
        do 2 k=1,nm
                kp=k+1
                1=jpvt(k)
                                        s=b(1)
                                                b(1)=b(k)
                                                b(k)=s
                                do 1 i=kp,n
                                b(i)=b(i)+a(i,k)*s
1 continue
2 continue
        do 4 ka=1,nm
                                km=n-ka
                                k=km+1
                                b(k)=b(k)/a(k,k)
                                s=-b(k)
                        do 3 i=1,km
                        b(i)=b(i)+a(i,k)*s
3 continue
4 continue
                b(1)=b(1)/a(1,1)
        return
                end
SUBROUTINE SURVL(N,B,FMN)

        dimension x(50),y(50),xc(50),yc(50),ds(50),fn(50,50),ft(50,50),rhs(50),sde(50),ci(50),si(50)
        common x,y,xc,yc,ds,fn,ft,rhs,pi,cpi,ci,si,uinf,vinf,sde
                fac=sqrt(1.-fmn**2)
                        gam=1.4
                        c1=0.5*(gam-1.)*fmn**2
                        c2=0.5*gam*fmn**2
                        gmp=gam/(gam-1.)
                        write(6,1)
1 format(2x,'van toc va ap suat tai diem giua')
                do 4 k=1,n
                                qts=0.0
                                qns=0.0
                                do 2 j=1,n
        qts=qts+ft(k,j)*sde(j)
2       qns=qns+fn(k,j)*sde(j)
                        qnk=qns+vinf*ci(k)-uinf*si(k)
                        qtk=qts+vinf*ci(k)+uinf*si(k)
                        uu=uinf-qns*si(k)+qts*ci(k)
                        vv=vinf+qns*ci(k)+qts*si(k)
                        uu=uu/fac/fac
```

```
                                    vv=vv/fac
                                    pp=1.-uu**2-vv**2
                                    if(fmn.gt.0.05) pp=(1.+c1*pp)**gmp-1.)/c2
        dum=b*b*xc(k)
                                    dum=yc(k)*yc(k)+dum*dum
                                    qex=(1.+b)*yc(k)/sqrt(dum)
write(6,3) xc(k),yc(k),qnk,qtk,uu,vv,pp,qex
3 format(1x,'xc, yc=',2f6.3,3x,'qn,qt=',2f6.3,3x,'u,v=',2f6.3,3x,'p=',f6.3,3x,'qex=',f6.3)
4 continue
return
end
```

IV.3. Results

This program was tested by applying for the flow problem with unit circle ($b = 1$) and incompressible flow. The number of panels is 20. Computational results and theorical results are shown in figure 4, 5, 6, and 7

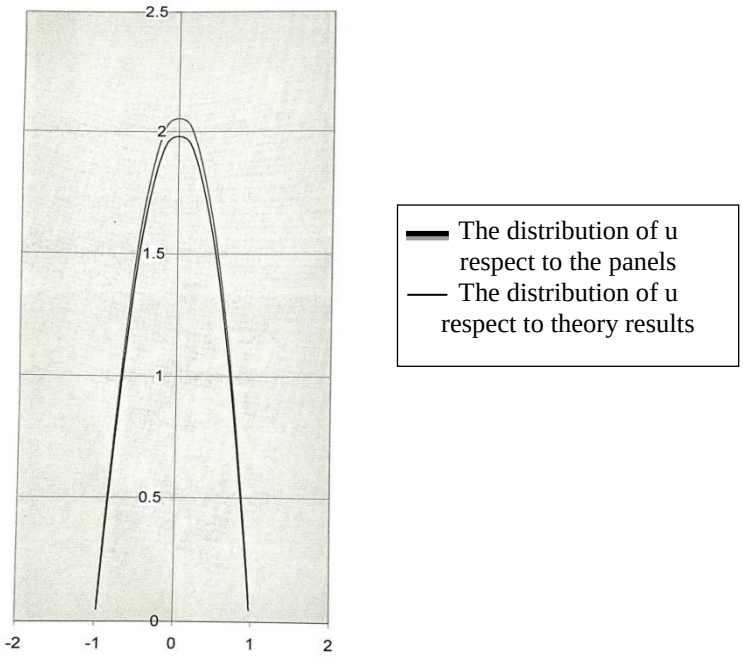


Figure 4: The distribution of u on the surface of unit circle

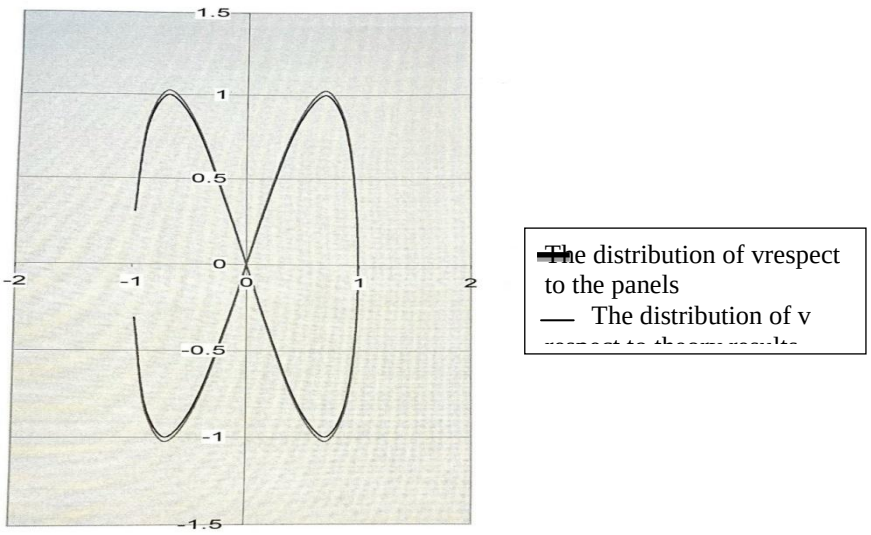


Figure 5: The distribution of v on the surface of unit circle

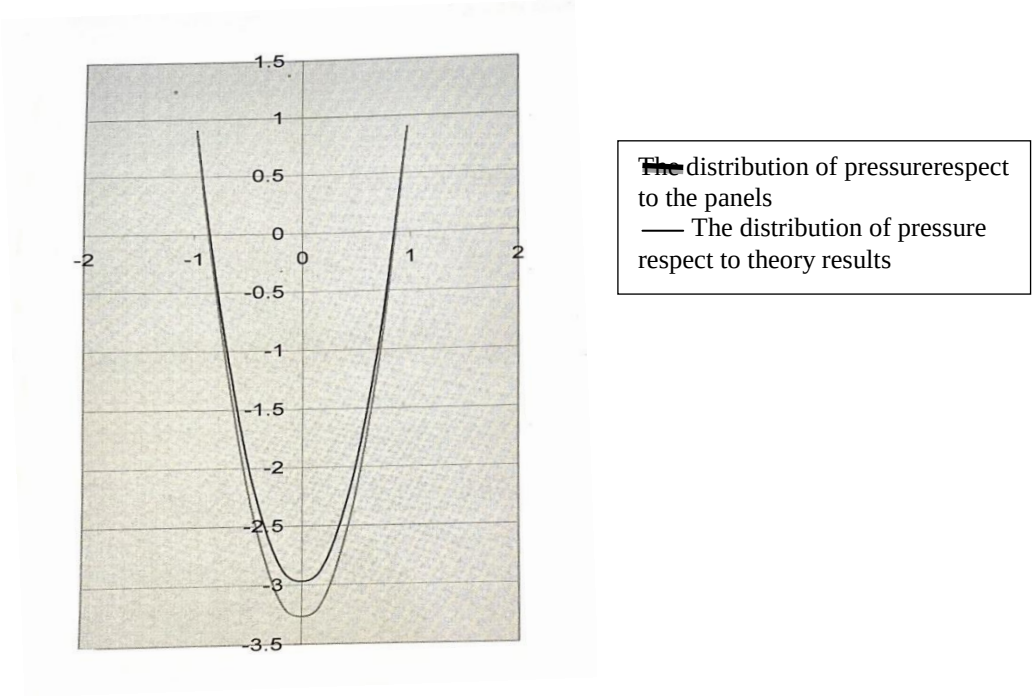


Figure 6: The distribution of pressure on the surface of unit circle

To be profile having ellipse shape ($b = 0.3$), then panel method was used to calculate for incompressible flow and compressible flow ($M = 0.4$). The number of panels is 20. The calculation result of pressure coefficient for two cases is shown in figure 7:

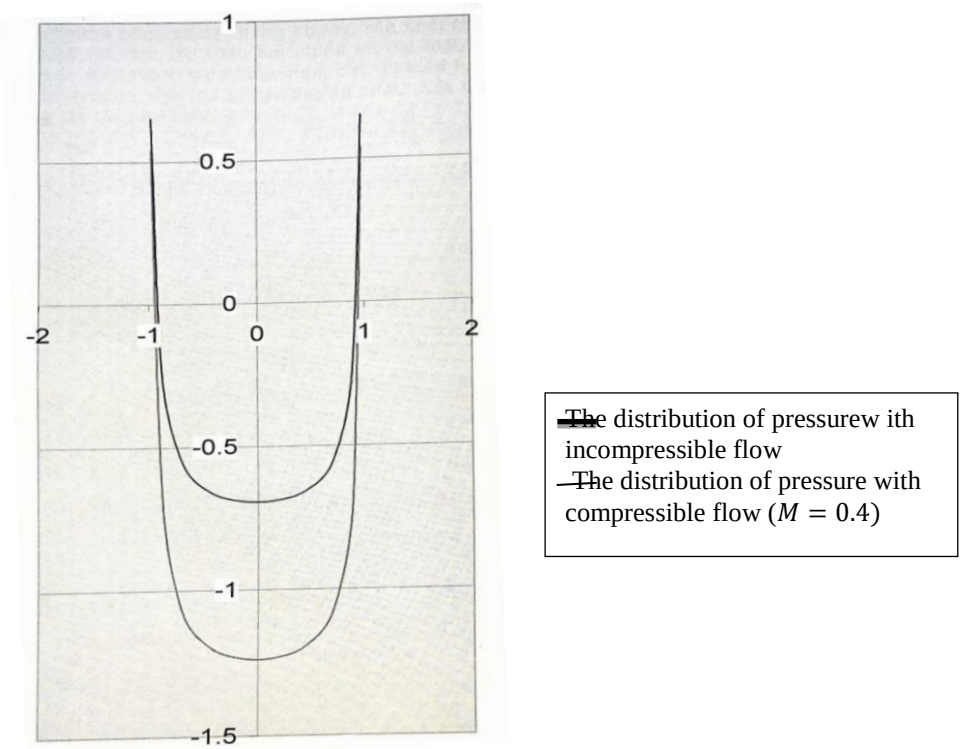


Figure 7: The distribution of pressureof incompressible flow and compressible flow

IV. Conclusion

Panel method is one of the methods are used widely in many fields such as industry, aviation, car manufacturing, etc. This method can convert two dimensional problem to three dimensional one. In this paper, we focus on two dimensional problem with respect data calculated the distribution of velocity fields and pressure fields.The results are shown in figure 4, 5, 6, and 7. We saw that the results calculated by panel method are very suitable for the theory results.

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