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Motion of MHD Micropolar Fluid Toward a Stagnation Point Under the Influence of Induced Magnetic Field

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Abstract: A steady two-dimensional Magnetohydrodynamic (MHD) mixed convection stagnation point motion of an incompressible, viscous and electrically conducting micropolar fluid toward a stretching and or shrinking vertical surface with surface heat flux is investigated. The effects of induced magnetic field and the heat flux radiation are considered. The transformed differential equations are solved numerically by a finite-differencemethod. The results for skin friction, heat transfer and induced magnetic field coefficients are obtained. The velocity, micro-rotation and temperature distribution for various parameters are shown. The results are in good agreement with earlier studies. The present results are compared with existing results in the literature and shown to be in good conformity as shown in the resulting table and curves.
Key words: Micropolar fluid motion, Stagnation point, Stretching and Shrinking sheet,Induced magnetic field.

1. Introduction

Microrotation fluids theory has been studied by several authors as seen in literature.Eringen A.C. (1966), displays that the effects of local rotary inertia and couple stresses can explain the flow behavior due to the microscopic effects arising from the local structure and micromotions of the fluid elements in which the classical Newtonian fluids theory is inadequate. The behaviors of non-Newtonian fluids such as polymeric fluids, liquid crystals, paints, animal blood, colloidal fluids, ferro-liquids etc. can be described with the help of a mathematical model using this theory. Several researchers have investigated the theory and its applications such as Ariman T., Turk M.A. and Sylvester N.D., (1974), Lukaszewick (1999), Eringen A.C. (2001), Ishak A., Nazar R. and Pop I., (2007, 2008).

The stagnation point flow is important in many practical applications such as cooling of nuclear reactors, cooling of electronic devices, extrusion of plastic sheets, paper production, glass blowing, metal spinningand drawing plastic films and manyhydrodynamic processes. Laminar mixedconvection in two-dimensional stagnationflows around heated surfaces in the case ofarbitrary surface temperature and heat flux variations was examined by Ramachandran N., Chen T.S. and Armaly B.F., (1988). They established a reverse flow developed in the buoyancy opposing flow region and dual solutions are found to exist for a certain range of the buoyancy parameter. Devi C.D.S., Takhar H.S., andNath G.(1991) extended this work for unsteady case. Lok Y. Y., Amin N., CampeanD. and Pop I.,(2005) studied the case for a vertical surface immersed in a micropolar fluid. Chin K.E., Nazar R., Arifin N. andPop I.,(2007), Ling S.C., Nazar R. and Pop I., (2007) and Ishak A., Nazar R. and Pop I.,. (2007, 2008) reported the existence of dual solutions in the opposing flow case. The study of the boundary layer flow under the influence of a magnetic field with the induced magnetic field was considered by few authors. Raptis and Perdikis (1984) studied the MHD free convection boundary layer flow past an infinite vertical porous plate. Later, Kumari, M., Takhar, H.S., and Nath, G., (1990) considered prescribed wall temperature or heat flux, and Takhar, H.S., Kumari, M., and Nath, G., (1993) studied the time dependence of a free convection flow. Ali et al. (2011) discussed MHD mixed convection boundary layer flow under the effect of induced magnetic field. Hydromagnetic thermal boundary layer flow of a perfectly conducting fluid was observed by Das (2011). Mukhopadhyay S., Uddin S. and Layek G.C., (2012) discussed Lie group analysis of MHD boundary layer slip flow past a heated stretching sheet in presence of heat source/sink. Shit and Halder (2012) examined thermal radiation effects on MHD viscoelastic fluid flow over a stretching sheet with variable viscosity. Heat transfer effects on MHD viscous flow over a stretching sheet with prescribed surface heat flux was studied by Adhikari and Sanyal (2013). A steady MHD mixed convection stagnation point flow of an incompressible micropola fluid towards a stretching/shrinking vertical surface with prescribed surface heat flux was also studied by Adhidari (2013).

In this work, the motion ofa steady MHD micropolar fluid stagnation pointfor an incompressible fluid towards a stretching and/or shrinking vertical surface with surface radiation heat flux is studied. The effects of induced magnetic field and the heat flux radiation are taken into account to extend the work of Adhikari and other researchers in this field.

2. Mathematical Formulation for MHD flow Analysis

Consider a steady two-dimensional MHD flow of an incompressible electrically conducting micropolar fluid near the stagnation point on a vertical plate with prescribed surface heat flux with a velocity proportional to the distance from the fixed origin O of a stationary frame of reference (x,y), as shown in figure 1. A uniform induced magnetic field of strength H0 is assumed to be applied in the positive y-direction, normal to the vertical plate. The normal component of the induced magnetic field H2 vanishes when it reaches the wall and the parallel component H1 approaches the value of H0. It is assumed that the velocity of the flow external to the boundary layer $U = ax$ and the surface heat flux $qw = bx$ of the plate are proportional to the distance x from the stagnation point, where a, b are constants.

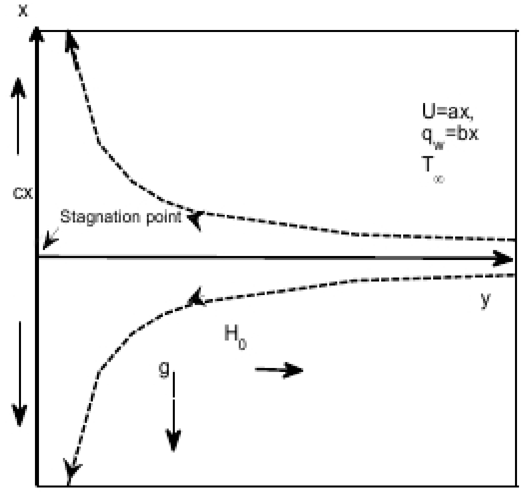

Fig 1: Sketch of the Problem

The magnetic Reynolds number of the flowis taken to be large enough so that theinduced magnetic field is not negligible.Under the Boussinesq and the boundary layer approximations the governing equations are given by

$$\frac{du}{dx} + \frac{dv}{dy} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (1)$$

$$\frac{dH1}{dx} + \frac{dH2}{dy} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad \ldots\ldots.(2)$$

$$\frac{Udu}{dx} + \frac{Vdu}{dy} = \frac{UdU}{dx} + \left(\frac{\mu+\kappa}{\rho}\right).\frac{d2u}{dy2} + \frac{\kappa}{\rho}.\frac{dN}{dy} + \frac{\mu}{\rho}.\left(\frac{H1dH1}{dx} + \frac{H2dH1}{dy}\right) - \frac{\mu}{\rho}.\frac{\hat{H}d\hat{H}}{dx} + g\beta(T - T\infty)\ldots.(3)$$

$$\frac{udH1}{dx} + \frac{vdH1}{dy} - \frac{H1du}{dx} - \frac{H2du}{dy} = \frac{\alpha 1d2H1}{dy2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (4)$$

$$\rho j\left(\frac{udN}{dx} + \frac{vdN}{dy}\right) = \frac{\gamma d2N}{dy2} - \kappa(2N + \frac{du}{dy}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\frac{udT}{dx} + \frac{vdT}{dy} = \frac{\alpha d2T}{dy2} - \frac{1}{\rho Cp}.\frac{dqr}{dy} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Subject to the boundary conditions at y=0: $u=u_w(x) = cx$,

$v = vw\ (x)$, $N = -n\partial u/\partial y$,
$\partial T/\partial y = -qw/k, \partial H1/\partial y = H2 = 0$ ……………………………………… (7)
$at y \to \infty$: $u \to ue\ x = ax$,
$N \to 0$, $T \to T\infty$,
$H1 = He\ x = H0ax/\nu$. …………………………………………………… (8)
where u and v are the velocity componentsalong the x and y-axis respectively, uw(x)the wall shrinking or stretching velocity(c>0 for stretching , c<0 for shrinking andc=0 for static wall), vw(x) the wall mass fluxvelocity, N is the microrotation or angular velocity whose direction of rotation is in thexy plane, μ is the dynamic viscosity, μ0 isthe magnetic permeability, ρ is the densityof the fluid, j is the micro-inertia per unitmass, i.e., micro-inertia density, γ is the spingradient viscosity, κ is the vortex viscosity

or micro-rotation viscosity, T is the fluidtemperature in the boundary layer, β is thethermalexpansioncoefficient, α is thethermal diffusivity, $\alpha 1$ is the magneticdiffusivity, k is he thermal conductivity, qwis the wall heat flux. Note that n is aconstant such that 0≤n≤1. When n=0 thenN=0 at the wall represents concentratedparticle flows in which the microelementsclose to the wall surface are unable to rotate.This case is also known as the strongconcentration of microelements. Whenn=1/2, we have the vanishing of anti-symmetricpart of the stress tensor anddenotes weak concentration ofmicroelements, the case n=1 is used for themodeling of turbulent boundary layer flows.We shall consider here both cases of n=0and n=1/2. Assume $\gamma = \left(\mu + \frac{\kappa}{2}\right) j = \mu\left(1 + \frac{K}{2}\right) j, where\ K = \frac{\kappa}{\mu}$,is the material parameter. This assumption is invoked to allow the field of equations that predicts the correct behavior in the limiting case when the microstructure effects become negligible and the total spin N reduces to the angular velocity [Ahmadi (1976), Yuce(1989)]. By using the Rosseland approximation the radiative heat flux qr in y-direction is given by [Brewster (1992)]:$qr = -(4\sigma s/3ke).(\partial T^4/\partial y)$,...... (9)

whereσs is the Stefan-Bolzmann constant and ke the mean absorption coefficient. It should be noted that by using Rosselandapproximation, the present study is limitedto optically thick fluids.Expanding T^4in a Taylor series about $T\infty$as:

$T^4 = T\infty^4 + 4T^3\infty(T - T\infty)+6T^2\infty(T - T\infty)^2+ \cdots$.

Neglecting higher-order terms beyond the first degree in $T - T\infty$, we get

$T^4 \cong 4T^3\infty T - 3T^4\infty$, ………………………………………………….. (10)

In view of the equations (9) and (10), the equation (6) becomes

$\frac{udT}{dx} + \frac{vdT}{dy} = \frac{\alpha d2T}{dy2} + \frac{16\sigma sT^3\infty}{3ke\rho Cp}.\frac{d2T}{dy2}$,………………………………………… (11)

Introduce a Stream function Ψ as follows u =∂Ψ/∂y, v = −∂Ψ/∂x. ……….. (12)

The momentum, angular momentum and energy equations can be transformed into the corresponding ordinary differential equations by the following transformation:

$$\eta = \sqrt{\left(\frac{a}{v}\right)y}, \qquad f(\eta) = \frac{\psi}{x\sqrt{av}}, \qquad p(\eta) = \frac{N}{ax\sqrt{\frac{a}{v}}},$$

$\theta(\eta) = \frac{k(T-T\infty)}{qw}.\sqrt{\frac{a}{v}}$, $H1 = Ho.\frac{ax}{v}.h'(\eta)$, $H2 = -Ho.\sqrt{\left(\frac{a}{v}\right)h(\eta)}$,……. (13)

Whereη the independent dimensionlesssimilarity variable. Thus u and v are givenby

$u = axf'\eta, v = -av\ f\ \eta$.

Substituting variables (13) into equations (2) to (6), we get the following ordinary differential equations:

1 + K f□ □ □+ ff□ □+ 1 − f□2 + Kp□ +M h□2 − h h□ □− 1 + $\lambda\theta = 0$,……………(14)

α_2 h□ □ □+ f h□ □− h f□ □= 0, ……………………………………………………………… (15)

$\left(1 + \frac{\kappa}{2}\right)p'' + fp' - pf' - K(2p + f'') = 0$ ……………………………. (16)

$\frac{1}{pr}.\left(1 + \frac{4}{3F}\right)\theta'' + f\theta' - \theta f' = 0$ ………………………………………. . (17)

subject to the boundary conditions (7) and (8) which become

$f\ 0 = s, f'0 = e, p\ 0 = -nf''0,\ \theta'0 = -1,\quad 0 = \quad '0 = 0,$

$as\ \eta \to \infty: f'\eta \to 1, p\ \eta \to 0$

$\theta\ \eta \to 0,\quad '\eta \to 1.$ ……………………………………………………………………….. (18)

Here f□ (η) p(η), h□ (η)and θ(η) give (dimensionless) the velocity, the angular velocity, the induced magnetic field and temperature respectively. In the aboveequations, primes denote differentiation with respect to η; $j = v/athe$ characteristic length [Rees &Bassom (1996)], Pr = ν/α the Prandtl number, $M = \mu\ e\ H0^2/\rho\ v^2$the magneticparameter or Hartmann number, $\alpha_2 = \alpha_1/v$ is the reciprocal of the magnetic Prandtlnumber, e=c/a the velocity ratio parameter, $S = \frac{vw(x)}{\sqrt{ax}}$.the constant mass flux with s>0 for suction and s<0 for injection,

$\lambda = \frac{Grx}{Rex5/2}$the Buoyancy or mixed convection parameter,$F = \frac{kek}{4\sigma sT^3\infty}$.

The radiation parameter, $Grx = \frac{g\beta\,(Tw - T\infty)x^3}{v2}$. The local Grashof number and $Rex = Ux/v$ is the local Reynolds number. Here λ is a constant and the negative and positive values of λ correspond to the opposing and assisting flows respectively. When λ=0, i.e., when

Tw=T∞ is for pure forced convection flow. Ramachandran N., Chen T.S. and Armaly B.F., (1988) considered the present problem with M=0 and K=0.

The skin friction coefficient Cf and the local Nusselt number Nux are defined as

$Cf = \frac{\tau w}{\rho U2/2}$, $Nux = \frac{xqw}{k(Tw = T\infty)}$, ………………………………….. (19)

where the wall shear stress τw and the heat flux qw are given by

$\tau w = [(\mu + \kappa)/\partial y + \kappa N] = 0, \quad qw = -k[\partial T/\partial y]y = 0,$…………………… (20)

with k being the thermal conductivity. Usingthe similarity variables (10), we get

$1/2Cf\ Rex1/2 = [1 + (1 - n)/2]\ ''(0)\ , Nu\ x/Rex1/2 = 1/\theta(0)$………………. (21)

3. Numerical Solutions:

The equations (14) – (17) subject to the boundary conditions (18) are solved numerically using an implicit finite difference scheme known as the Keller-box method [Cebeci& Bradshaw (1988)]. The method has following four basic steps:

i) Reduce Equations (14)-(17) to first order equations;
ii) ii) Write the difference equations using central differences;
iii) Linearize the resulting algebraic equations by Newton's method and write them in Matrix-vector form;
iv) iv) Use the Block-tri-diagonal elimination technique to solve the linear system. The details are also described by Adhikariand Sanyal (2013).

Table1: Values of f//(0) and 1/□(0) fordifferent values of Pr
(when λ=1, K=0, n=0.5, M=0, Δ η=0.02)

Pr	Bachok& Ishak(2009)	Bachok& Ishak(2009)	Adhikari and Sanyal (2013)	Adhikari and Sanyal (2013)	New result (for s=0,e=0, n=0)	New result (for s=0,e=0, n=0)
	f//(0)	1/θ (0)	f//(0)	1/θ (0)	f//(0)	1/θ (0)
0.7	1.8339	0.7776	1.8339	0.7776	1.8339	0.7776
1.0	1.7338	0.8781	1.7339	0.8781	1.7338	0.8780
7.0	1.4037	1.6913	1.4037	1.6913	1.4037	1.6913
10.0	1.3711	1.9067	1.3712	1.9072	1.3711	1.9071

4. Results & Discussion:

The step size Δη of η and the edge of the boundary layer η∞ had to be adjusted for different values of parameters to maintain accuracy within the interval 0≤η≤η∞ , where η∞ is the boundary layer thickness, the programme is run in MATLAB up to the desired level of accuracy. The validity of the numerical results have been compared with the results of Bachok and Ishak (2009), Adhikari and Sanyal (2013) and they are found to be in a very good agreement.

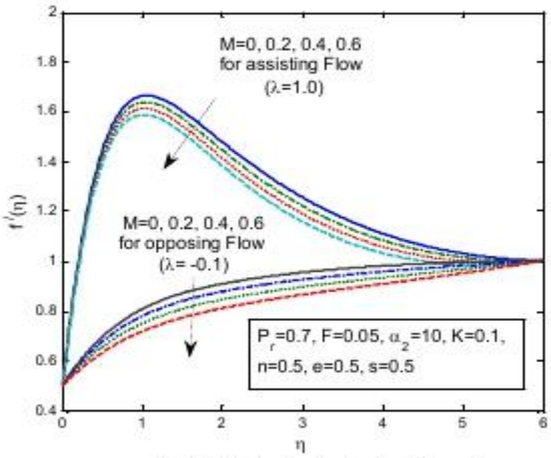

Fig.2. Velocity Distribution for different M in both assisting and opposing motions

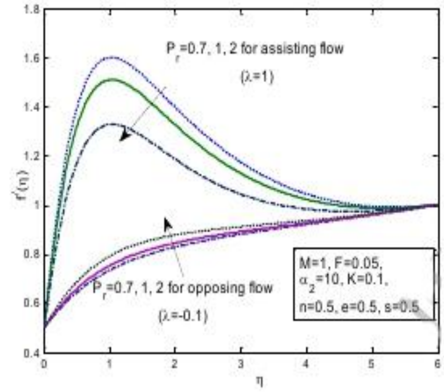

Fig.3. Velocity Distribution for different Pr in both assisting and opposing motions

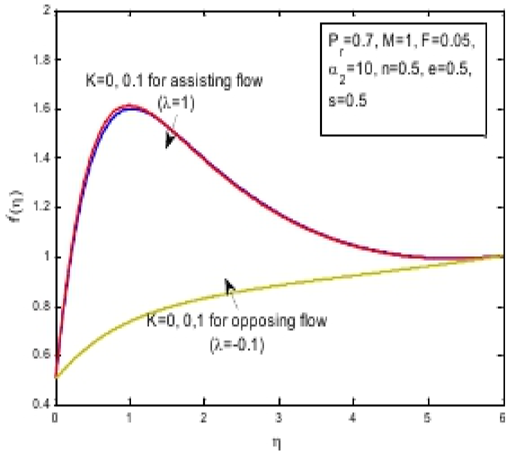

Fig.4. Velocity Distribution for different K in both assisting and opposing motions

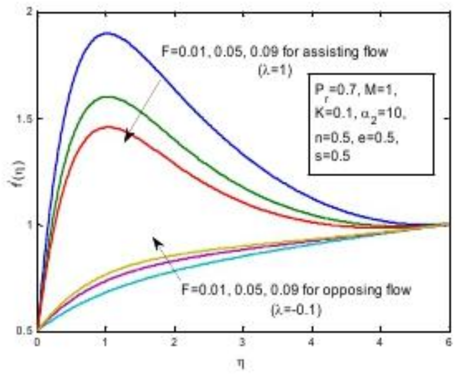

Fig.5. Velocity Distribution for different F in both assisting and opposing motions

The velocitydistribution for different M,Pr, K and F are shown in Figures 2 to 6. The induced magnetic field distribution for different parameters are shown in Figures 7,8 and 9 The Temperature distribution for different M, Pr, and K are also depicted in Figures 10, 11 and 12. The angular velocity distribution for different s parameter andthe Local Nusselt Number for different M were also investigated.

Figures 2 to 6 respectively depict that the velocity profiles for the assisting flowdecrease with the increase of M, Pr, K and F; whereas for the opposing flow the velocity profiles decrease with M, increase with Pr and F but almost no change with K. With the increase of s, Figure 6 describes that the velocity profiles for the assisting flow enhance near boundary and after η=1 it reduce, but for the opposing flow the velocity profiles increase. For both motions the velocity profiles raise with $\alpha 2$. Figures 7 to 9 illustrate that the induced magnetic field distribution for the assisting flow boost with M, Pr, K, F and s; but for the opposing flow it decrease with M, increase with Pr and s, almost no change with K and increase very slowly with F.

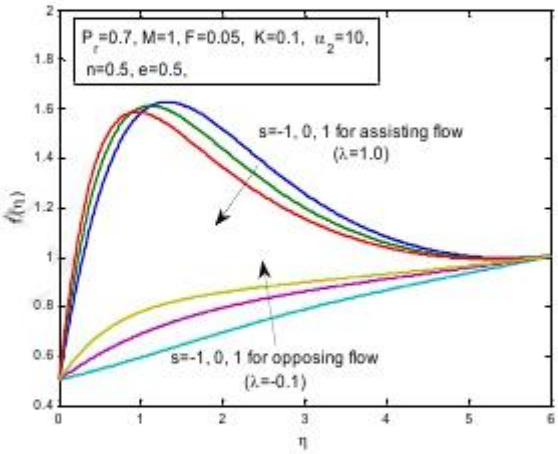

Fig.6. Velocity Distribution for different s parameter in both assisting and opposing motions

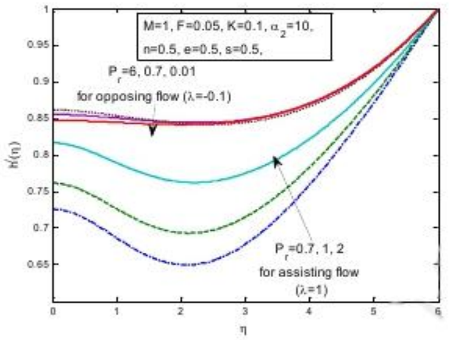

Fig.8. Induced Magnetic Field Distribution for different Pr in both assisting and opposing motions

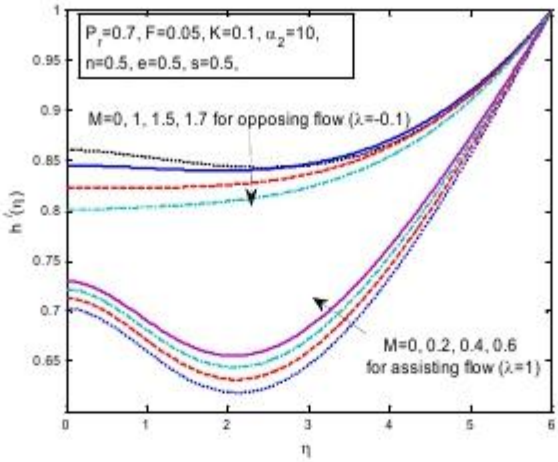

Fig.7. Induced MagneticField Distribution for different M in both assisting and opposing motions

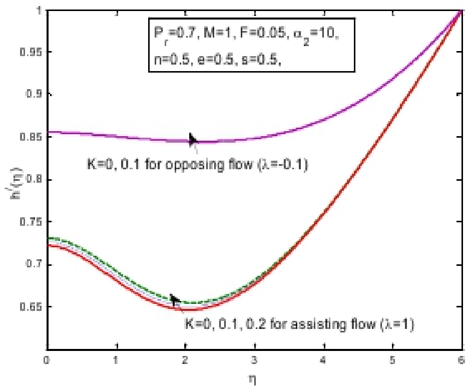

Fig.9. Induced MagneticField Distribution for different M in both assisting and opposing motions

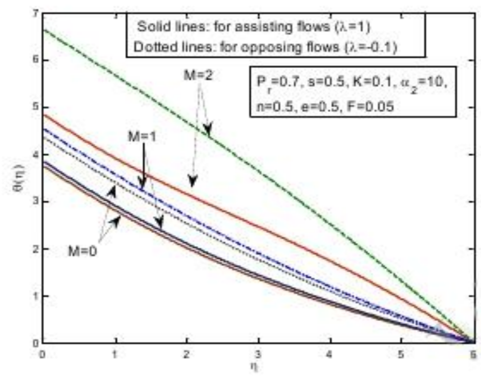

Fig.10. Temperature Distribution for different M in both assisting and opposing motions

Angular velocity profiles increase for both flows with s and M.
Temperature distribution for both flow motionsincrease with M (Fig 10), decrease with F and Pr(Figs.11 and 12). Investigations also revealed that the Skin friction coefficient and the local Nusselt Number decrease with M for the both flows.

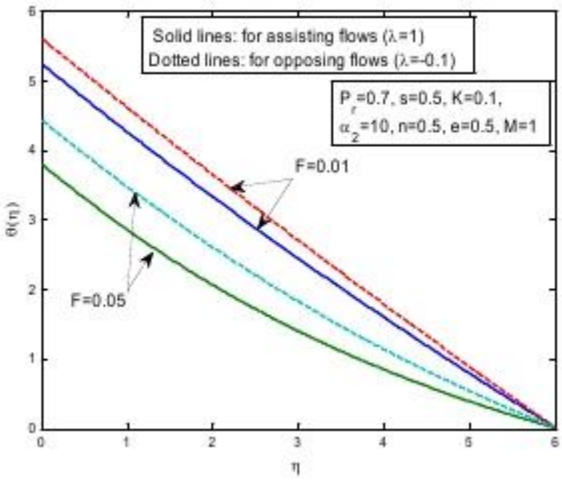

Fig.11. Temperature Distribution for different F in both assisting and opposing

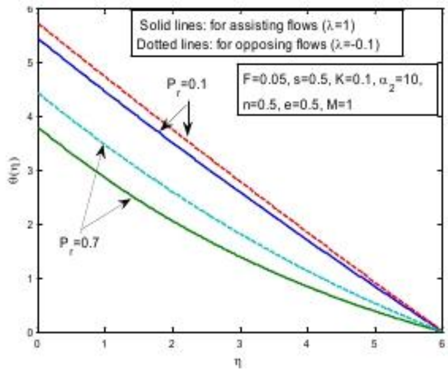

Fig.12. Temperature Distribution for different Pr in both assisting and opposing

Conclusion

The motion of a steady MHD micropolar fluid stagnation point for an incompressible fluid towards a stretching and/or shrinking vertical surface with surface radiation heat flux has been investigated. The effects of induced magnetic field and the heat flux radiation are taken into account. The results depict that the velocity profiles for the assisting flow decrease with the increase of all the parameters investigated while for the opposing motion the velocity profiles decrease with M, increase with Pr and F but almost no change with K. Temperature distribution for both assisting and opposing motions increase with M, decrease with F and Pr. The results are significant because they compare well with those existing in the literature.

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