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ZJ Transform and Its Application to ODE with Variable Coeffients

M. Sc. Zenteno Jiménez José Roberto
National Polytechnic Institute, México City
ESIA-Ticóman Unit Gustavo A. Madero Mayor's Office
Email: jzenteno@ipn.mx

Abstract: The following article is a study, application and results on the ZJ Transform to solve Differential Equations with Variable Coeffients.

Summary: The following article is a study, application, properties and results on the ZJ Transform to solve partial Differential Equations with Variable Coeffients.

Keywords: Laplace Transform, ODE Variable Coeffients, ZJ Transform, Elzaki Transform, Aboodh Transform.

I. Introductión

Among the methods to solve Ordinary Differential Equations with variable coefficients there are many methods, among them is the Laplace Transform, one of the most well-known and powerful tools, this variation of the Transformation is born from the Relationship with the Laplace Transform. , by Elzaki and Sumudu [Elzaki Tarig The New Integral Transform "Elzaki Transform" (2011)] in [Fethi Bin Muhammed Belgacem and Ahmed Abdullatif Karaballi. Sumudu Transform fundamental properties research and applications, 2006] it is possible to obtain this variant of Transformation and in this it is proposed to obtain the Inverse ZJ Transform with the following definition of [Zenteno Jiménez José Roberto November 2022], now in this new article it is applied to the Ordinary Differential Equations with Variable Coefficient with the relationship of two other important Transforms such as Aboodh and Elzaki.

Transformed

$$ZJ[f(t),\beta Z)] = ZJ(\beta Z) = \frac{\beta^n}{Z}\int_0^\infty e^{\frac{-Zt}{\beta}} f(t)dt \qquad (1)$$

Inverse Transform

$$ZJ^{-1}[f(Z\beta),t)] = \frac{\beta^n Z}{2\pi i}\int_{-\infty}^{\infty} e^{Z\beta t} f\left(\frac{1}{\beta}\right) d\beta Z \qquad (2)$$

Relating to other Transforms

Elzaki, Aboodh, Transform [See References]

The general Transform would be like, where p and s depend on s in their variation

$$T[f(t),s)] = T(s) = A(s)\int_0^\infty e^{-B(s)t} f(t)dt \qquad (3)$$

With

$$|f(t)| < \begin{Bmatrix} Me^{-B(s)t} & t \leq 0 \\ Me^{B(s)t} & t \geq 0 \end{Bmatrix}$$

Transformations of the Derivative and the Transformation of the term with the Variable Coefficient of the Differential Equation

First Derivative	$ZJ\left[\frac{dy}{dt}\right] = -y(x,0)\frac{\beta^n}{z} + \frac{z}{\beta}\hat{\varphi}$
Second Derivative	$ZJ\left[\frac{d^2y}{dt^2}\right] = -\frac{\beta^n}{z}y'(x,0) - \frac{\beta^n}{\beta}y(x,0) + \frac{z^2}{\beta^2}\hat{\varphi}$
Third Derivative	$ZJ\left[\frac{d^3y}{dt^3}\right] = -\frac{\beta^n}{z}y''(x,0) - \frac{\beta^n}{\beta}y'(x,0) - \frac{\beta^n z}{\beta^2}y(x,0) + \frac{z^3}{\beta^3}\hat{\varphi}$

Theorems. Now with the Transformation we have the following, in this case we must see n = 1 and so then we have:

1) ty	$ZJ[ty] = -\frac{dZJ[y]}{d\left(\frac{z}{\beta}\right)} - \frac{\beta}{z} ZJ[y]$
2) ty'	y = y' $ZJ[ty'] = -\frac{dZJ\left[-y(x,0)\frac{\beta}{z} + \frac{z}{\beta}\hat{\varphi}\right]}{d\left(\frac{z}{\beta}\right)} - \frac{\beta}{z} ZJ\left[-y(x,0)\frac{\beta}{z} + \frac{z}{\beta}\hat{\varphi}\right]$
3) ty''	y'=y'' $ZJ[ty''] = -\frac{dZJ\left[-\frac{\beta}{z}y'(x,0) - y(x,0) + \frac{z^2}{\beta^2}\hat{\varphi}\right]}{d\left(\frac{z}{\beta}\right)} - \frac{\beta}{z} ZJ\left[-\frac{\beta}{z}y'(x,0) - y(x,0) + \frac{z^2}{\beta^2}\hat{\varphi}\right]$
4) t^2y'	$ZJ[t^2y'] = \frac{z}{\beta}\frac{d^2ZJ[\hat{\varphi}]}{d\left(\frac{z}{\beta}\right)^2} + 2\frac{dZJ[\hat{\varphi}]}{d\left(\frac{z}{\beta}\right)} - 2y(x,0)\frac{\beta^3}{z^3}$
5) t^2y''	$ZJ[t^2y''] = \frac{z^2}{\beta^2}\frac{d^2ZJ[\hat{\varphi}]}{d\left(\frac{z}{\beta}\right)^2} + 4\frac{z}{\beta}\frac{dZJ[\hat{\varphi}]}{d\left(\frac{z}{\beta}\right)} + 2ZJ[\hat{\varphi}] - 2y'(x,0)\frac{\beta^3}{z^3}$

II. Demonstrations Of 1,2,3 and 4

According to [7] the article, explain why n = 1

$$T\left(tf^n(t)\right) = -\frac{p(s)}{q'(s)}\frac{d}{ds}\left(\frac{T(f^n(t))}{p(s)}\right)$$

With p = $\frac{\beta^n}{z}$ and q = $\frac{z}{\beta}$ therefores = $\frac{z}{\beta}$ making the corresponding substitution we obtainwith n = 1

$$ZJ\left(tf(t)\right) = -\frac{\beta^n}{z}ZJ\left(f(t)\right) - \frac{dZJ\left(f(t)\right)}{d(\frac{z}{\beta})}$$

Now with f = f'

$$ZJ(tf') = -\frac{\beta^n}{z}ZJ(f') - \frac{dZJ(f')}{d(\frac{z}{\beta})}$$

Now remembering the f' of the zj transform we have the following, with n = 1

$$f' = -y(x,0)\frac{\beta^n}{z} + \frac{z}{\beta}\hat{\varphi}$$

When replacing it we are left with the following, an EDO. Linear, which can be solved with the initial condition

$$ZJ(tf') = \frac{-y(x,0)\beta^2}{z^2} + y(x,0)\frac{\beta^2}{z^2} - \frac{z}{\beta}\hat{\varphi}' - 2\hat{\varphi}$$

$$ZJ(tf') = -\frac{z}{\beta}\hat{\varphi}' - 2\hat{\varphi}$$

Now with the second derivative in f

$$ZJ(tf(t)'') = -\frac{\beta^n}{z}ZJ(f(t)'') - \frac{dZJ(f(t)'')}{d(\frac{z}{\beta})}$$

$$ZJ(tf'') = -\frac{z^2}{\beta^2}\hat{\varphi}' - 3\frac{z}{\beta}\hat{\varphi} + y(x,0)\frac{\beta}{z}$$

With the solutionthe ODE that is observed

$$\hat{\varphi} = y(x,0)\frac{\beta^2}{z^2}$$

Now with the following expression 4

$$ZJ(t^2 f') = \left(\frac{z}{\beta}\right)\frac{d^2 ZJ(f(t)')}{d\left(\frac{z}{\beta}\right)^2} + 2\frac{dZJ(f(t)')}{d\left(\frac{z}{\beta}\right)} - 2y(x,0)\frac{\beta^3}{z^3}$$

We proceed to derive the Transformation twice.

$$\frac{dZJ(f)}{d\left(\frac{z}{\beta}\right)} = \frac{d}{d\left(\frac{z}{\beta}\right)}\left(\frac{\beta}{Z}\int_0^\infty e^{\frac{-Zt}{\beta}} f(t)dt\right)$$

So, with f = f'

$$\frac{d^2 ZJ(f')}{d\left(\frac{z}{\beta}\right)^2} = ZJ(t^2 f') + 2\left(\frac{z}{\beta}\right)ZJ(f') + 2\left(\frac{\beta}{z}\right)^3 ZJ(f')$$

Now substituting

$$ZJ(tf') = -\frac{\beta}{z}ZJ(f') - \frac{dZJ(f')}{d(\frac{z}{\beta})}$$

With

$$ZJ\left[\frac{dy}{dt}\right] = -y(x,0)\frac{\beta}{z} + \frac{z}{\beta}\hat{\varphi}$$

We finish with this expression by arranging the terms

$$ZJ(t^2 f') = \left(\frac{z}{\beta}\right)\frac{d^2 ZJ(f(t)')}{d\left(\frac{z}{\beta}\right)^2} + 2\frac{dZJ(f(t)')}{d\left(\frac{z}{\beta}\right)} - 2y(x,0)\frac{\beta^2}{z^2} + 2ZJ\big(f(t)\big) + \frac{2\beta}{z}ZJ(f(t)) - 2y(x,0)\frac{\beta^3}{z^3}$$

See these terms, solving the expression

$$-2y(x,0)\frac{\beta^2}{z^2} + 2ZJ\big(f(t)\big) + \frac{2\beta}{z}ZJ\big(f(t)\big) = 0$$

We have the solution is

$$ZJ\big(f(t)\big) = \frac{y(x,0)\beta^2}{(\beta+z)z}$$

And finally we have the proven expression

$$ZJ(t^2 f') = \left(\frac{z}{\beta}\right)\frac{d^2 ZJ(f(t)')}{d\left(\frac{z}{\beta}\right)^2} + 2\frac{dZJ(f(t)')}{d\left(\frac{z}{\beta}\right)} - 2y(x,0)\frac{\beta^3}{z^3}$$

Proving 5, let's use this expression from [7]

$$ZJ\big(t^2 f^n(t)\big) = \frac{p(s)}{q'(s)}\frac{d}{d\,s}\left(\left(\frac{1}{q'(s)}\right)\left(\frac{d}{d(s)}\left(\left(\frac{1}{p(s)}\right)T(f(t)^n)\right)\right)\right)$$

You have the following

$$ZJ\big(t^2 f^2(t)\big) = \frac{\beta}{z}\frac{d}{d\left(\frac{z}{\beta}\right)}\left(\frac{d}{d\left(\frac{z}{\beta}\right)}\left(\frac{z}{\beta}\right)ZJ(f(t)'')\right)$$

Which proceeds as follows

$$= \left(\frac{z}{\beta}\big(ZJ(f(t)'')\big)'' + 2\frac{z}{\beta}ZJ(f(t)'')'\right) * \frac{\beta}{z}$$

A form of Equation is obtained as follows

$$= \frac{z^2}{\beta^2}\hat{\varphi}'' + \frac{4z}{\beta}\hat{\varphi}' + \left[\frac{2z^2}{\beta^2}\hat{\varphi}' + \frac{4z}{\beta}\hat{\varphi}\right] + 2\hat{\varphi} + \frac{2y'(x,0)\beta^2}{z^2} - \frac{2y'(x,0)\beta^3}{z^3}$$

Now seeing these terms and seeing this form

$$\left[\frac{2z^2}{\beta^2}\hat{\varphi}' + \frac{4z}{\beta}\hat{\varphi}\right] = \frac{2y(x,0)'\beta^2}{z^2}$$

Giving as a solution and substituting it we can eliminate those terms

$$\hat{\varphi} = -\frac{y'(x,0)\beta^3}{z^3}$$

Thus the expression is reduced as

$$= \frac{z^2}{\beta^2}\hat{\varphi}'' + \frac{4z}{\beta}\hat{\varphi}' + 2\hat{\varphi} - \frac{2y(x,0)'\beta^3}{z^3}$$

$$ZJ[t^2y''] = \frac{z^2}{\beta^2}\frac{d^2ZJ[\hat{\varphi}]}{d\left(\frac{z}{\beta}\right)^2} + 4\frac{z}{\beta}\frac{dZJ[\hat{\varphi}]}{d\left(\frac{z}{\beta}\right)} + 2ZJ[\hat{\varphi}] - 2y'(x,0)\frac{\beta^3}{z^3}$$

Now let's make the comparison with the Aboodh Transform, if we observe for the ZJ transform it is taken to S = $\frac{z}{\beta}$ or in Aboodh's case it would be v = $\frac{z}{\beta}$

Aboodh Transform

$$A(f(t)) = \frac{1}{v}\int_0^\infty f(t)e^{-vt}dt$$

With v = z/β we have the ZJ Transform with n different from 1 and the solution method for the ODE with Variable Coefficient procedes by substituting the previous expressions, which are identical to those of the Aboodh Transform

$$ZJ(f(t)) = \frac{\beta^n}{z}\int_0^\infty f(t)e^{-\frac{zt}{\beta}}dt$$

In the case of the Elzaki Transform, we take With v = β/z in the v term outside the Integral with n = 1 and here with z=1, which would proceed with the solution in the same way

$$E(f(t)) = v\int_0^\infty f(t)e^{-\frac{t}{v}}dt = \frac{\beta}{z}\int_0^\infty f(t)e^{-\frac{t}{\frac{\beta}{z}}}dt = \beta\int_0^\infty f(t)e^{-\frac{t}{\beta}}dt$$

The beta number is the complex number and with n=1, just as z is a constant, it is equal to 1 and thus the ZJ transform = Aboodh with $v = \frac{z}{\beta}$ and for Elzaki is $v = \frac{\beta}{z}$

III. Applications of the ZJ transform some examples

Equation 1 $y'' + ty' - y = 0 y(0) = 0, y'(0) = 1$

Making the substitution we have the form with the ODE, remember n = 1

$$-\frac{z}{\beta}\hat{\varphi}' + \left[\frac{z^2}{\beta^2} - 3\right]\hat{\varphi} = \frac{z}{\beta}$$

Thus the solution is obtained as

$$\hat{\varphi} = \frac{\beta^3}{z^3}$$

With C=1, now applying the Inverse $f\left(\frac{1}{\beta}\right)$ and with $Z\beta^n$ with n = 1

$$= \frac{1}{Z^2\beta^2} = \frac{1}{S^2} = t$$

Now if we put the transformation function with n other than 1 we get like this

$$-z\beta\hat{\varphi}' + \left[\frac{z^2}{\beta^2} + (n - z - 1)\right]\hat{\varphi} = \frac{\beta^n}{z}$$

When solving it we are left with extra terms that prevent some terms from being canceled and other terms from being incomplete, now we eliminate the terms inside the parentheses and arrange the equation

$$\hat{\varphi}' - \frac{z}{\beta^3}\hat{\varphi} = -\frac{\beta^{n-1}}{z^2}$$

With the following solution

$$\hat{\varphi} = -e^{\left(\frac{z}{\beta^3}\right)} \int \frac{\beta^{n-1}}{z^2} e^{\left(-\frac{z}{\beta^3}\right)\left(\frac{z}{\beta}\right)} d\left(\frac{z}{\beta}\right) + Ce^{\left(\frac{z}{\beta^3}\right)}$$

With solution and C =0

$$\hat{\varphi} = \frac{\frac{\beta^{n-1}}{z^2}}{\frac{z}{\beta^3}} = \frac{\beta^{n+2}}{z^3}$$

Applying the Inverse $f\left(\frac{1}{\beta}\right)$ and with $Z\beta^n$ we get the same

$$= \frac{1}{Z^2\beta^2} = \frac{1}{S^2} = t$$

Now an observation if we ignore the derivative and with n =1

$$-\frac{z}{\beta^3}\hat{\varphi} = -\frac{\beta^{n-1}}{z^2}$$

$$\hat{\varphi} = \frac{\beta^3}{z^3}$$

Applying the Inverse $f\left(\frac{1}{\beta}\right)$ and with $Z\beta^n n = 1$

$$= \frac{1}{Z^2\beta^2} = \frac{1}{S^2} = t$$

Equation $2y'' + 3ty' - 6y = 2y(0) = 0, y'(0) = 0$

By making the substitution we have the form with the ODE and it has a similar form to that of example 1 in the solution, now if we put the transformation function with the n different from 1 it looks like this, once agai neliminating the terms in the parentheses

$$-z\beta\hat{\varphi}' + \left[\frac{z^2}{\beta^2} + (n + z - 6)\right]\hat{\varphi} = \frac{2\beta^{n+1}}{z^2}$$

$$\hat{\varphi}' - \frac{z}{\beta^3}\hat{\varphi} = -\frac{2\beta^n}{z^3}$$

With a solution similar to example 1

$$\hat{\varphi} = \frac{\frac{2\beta^n}{z^3}}{\frac{z}{\beta^3}} = \frac{2\beta^{n+3}}{z^4}$$

Applying the Inverse $f\left(\frac{1}{\beta}\right)$ and with $Z\beta^n$ we get the solution

$$= \frac{2}{Z^3\beta^3} = \frac{2}{2S^3} = t^2$$

Now an observation we see this again, if we do without the derivative again and with n =1

$$\hat{\varphi}' - \frac{z}{\beta^3}\hat{\varphi} = -\frac{2\beta^n}{z^3}$$

$$\frac{z}{\beta^3}\hat{\varphi} = \frac{2\beta^n}{z^3}$$

$$\hat{\varphi} = \frac{2\beta^{n+3}}{z^4}$$

Applying the Inverse $f\left(\frac{1}{\beta}\right)$and with $Z\beta^n n = 1$

$$= \frac{1}{Z^3\beta^3} = \frac{1}{S^3} = t^2$$

Equation $3y' + ty'' - 2ty' - 2y = 0y(0) = 1, y'(0) = 2$

Using the solutions found previously, which are the solutions of the equations that result from the transformation, we have as

$ty' = \frac{\beta^2}{z^2}$ and $ty'' = \frac{y(x,0)\beta^2}{z^2}$ substituting we have

$$\left[\frac{\beta^2}{z^2}\right] + \left[-\frac{\beta}{z} + \frac{z}{\beta}\hat{\varphi}\right] - \frac{2\beta^2}{z^2} - 2\hat{\varphi} = 0$$

Now there are two expressions

$$= \frac{\beta^3}{z^3 - 2z^2\beta} + \frac{\beta^2}{z^2 - 2z\beta}$$

Applying the Inverse $f\left(\frac{1}{\beta}\right)$ and with $Z\beta^{n=1}$ we have the solution for each of them

$$y = \frac{1}{2}(e^{2t} - 1)\text{and}y = e^{2t}$$

The solution being the expression $\boldsymbol{y = e^{2t}}$

Equation $4t^2y'' + 4ty' + 2y = 12t^2y(0) = 0, y'(0) = 0$

Using the expressions found, we have

$$t^2y'' = \frac{z^2}{\beta^2}\frac{d^2ZJ[\hat{\varphi}]}{d\left(\frac{z}{\beta}\right)^2} + 4\frac{z}{\beta}\frac{dZJ[\hat{\varphi}]}{d\left(\frac{z}{\beta}\right)} + 2ZJ[\hat{\varphi}] - 2y'(x,0)\frac{\beta^3}{z^3}$$

$$\left(\frac{z^2}{\beta^2}\hat{\varphi}'' + 4\frac{z}{\beta}\hat{\varphi}' + 2\hat{\varphi}\right) + 4\left(-\frac{z}{\beta}\hat{\varphi}' - 2\hat{\varphi}\right) + 2\hat{\varphi} = 12\left[\frac{2\beta^4}{z^4}\right]$$

$$\frac{z^2}{\beta^2}\hat{\varphi}'' - 4\hat{\varphi} = 12\left[\frac{2\beta^4}{z^4}\right]$$

$$\hat{\varphi}'' - 4\frac{\beta^2}{z^2}\hat{\varphi} = 12\left[\frac{2\beta^6}{z^6}\right]$$

By solving the second-order ODE homogeneously, we have $\hat{\varphi}\left(\frac{z}{\beta}\right) = \hat{\varphi}(0) = 0$, thus C1 y C2 must be 0, and:

$$\frac{z^2}{\beta^2}\hat{\varphi}'' = \left[\frac{24\beta^4}{z^4}\right]$$

Applying the Inverse $f\left(\frac{1}{\beta}\right)$ and with $Z\beta^{n=1}$ we get the solution

$$y = \frac{3}{5}t^2$$

Conclusion

We can conclude that the ZJ transformation applied to ODE with Variable Coefficient Works correctly by setting n = 1, without this step more terms are obtained when making the change and there is no complete cancellation of several terms, it is also related to the transformation of Aboodh and Elzaki when making z = 1, basically it has more the relationship with the Aboodh transform being in some way its inverse way of using z/β and with n=1, also the observations when dispensing with the derivative, there we find Once the solution is already enclosed, the general solutions of the ODE that arise when making the transformation are also used and respond to finding the solution in a certain way when transforming it.

ZJ transform

$$ZJ[f(t), \beta Z)] = ZJ(\beta Z) = \frac{\beta^n}{Z}\int_0^{\infty} e^{\frac{-Zt}{\beta}} f(t)dt$$

Inverse Transform

$$ZJ^{-1}[f(Z\beta), t)] = \frac{\beta^n Z}{2\pi i}\int_{-\infty}^{\infty} e^{Z\beta t} f\left(\frac{1}{\beta}\right) d\beta Z$$

Table 1 of some transformations

1	$\frac{\beta^{n+1}}{z^2}$
t	$\frac{\beta^{n+2}}{z^3}$
$\sqrt{t}$	$\frac{\sqrt{\pi}\beta^{n+3/2}}{2z^{5/2}}$
e^{at}	$\frac{\beta^{n+1}}{z(z-a)}$

$sen(bt)$	$\frac{b\beta^{n+2}}{z(z^2+\beta^2)}$
$cos(bt)$	$\frac{b\beta^{n+1}}{z^2+b^2\beta^2}$
$t^n e^{kt}$	$\frac{n!\,\beta^n}{\left(k-\frac{z}{\beta}\right)^{n+1} z}$
$senh(bt)$	$\frac{b\beta^{n+2}}{z(z^2-z^2\beta^2)}$
$cosh(bt)$	$\frac{\beta^{n+1}}{(z^2-z^2\beta^2)}$
$\delta(t-t_0)$	$\frac{\beta^n e^{-\frac{z}{\beta}t0}}{z}$
$e^{at}F(t)$	$f(\frac{z}{\beta}-a)$
$\frac{dy}{dt}$	$-y(x,0)\frac{\beta^n}{z}+\frac{z}{\beta}\hat{\varphi}$
$\frac{d^2y}{dt^2}$	$-\frac{\beta^n}{z}y'(x,0)-\frac{\beta^n}{\beta}y(x,0)+\frac{z^2}{\beta^2}\hat{\varphi}$
$\frac{d^3y}{dt^3}$	$-\frac{\beta^n}{z}y''(x,0)-\frac{\beta^n}{\beta}y'(x,0)-\frac{\beta^n z}{\beta^2}y(x,0)+\frac{z^3}{\beta^3}\hat{\varphi}$

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