



Gaussian Mixture of Several Components (Machine Learning) of Daily Ozone and Temperature Maximums in Mexico City Trend 2010-2023

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Abstract: In this study we study the trend of maximum Ozone concentrations in Mexico City and Maximum Temperatures, based on the Bivariate Analysis methodology, subsequently using algorithms related to the topic of Data Clustering, especially K-means to be able to observe the classification of groups and subsequently the trend of the different groups of each year of Ozone concentrations and Maximum Temperatures in the City, subsequently we use the Pattern Recognition Gaussian mixture model algorithm.

Keywords: Bivariate Analysis, K-means, Pattern Recognition Gaussian mixture model, Ozone, Maximum Temperatures

I. Introduction

The trend of maximum Ozone concentrations in Mexico City and Maximum Temperatures will be based on the Bivariate Analysis methodology, which, as we have seen in previous studies, will see the behavior for a couple of years of the Maximum Ozone concentrations. Daily ozone in Mexico City, a treatment will also be done with the K-means methodology to be able to observe the classification of groups if in any case the concentrations of each year are part of an already background of Ozone in the city at that same time. group or if it has quite notable variations each year, subsequently the trend of the different groups of each year of Ozone concentrations and Maximum Temperatures in the City, the Pattern Recognition Gaussian mixture model algorithm is used to see the trend or pattern of these concentration data and the maximum daily temperatures of Mexico City and their relationship between both

II. Bivariate Analysis

It is useful to determine if there is a correlation between variables and, if so, the strength of the connection, to find trends and patterns in the data. It is useful for making predictions about the value of a dependent variable based on changes in the value of an independent variable. in this case the behaviors of the daily Ozone Maximum against the Daily Temperature Maximum in Mexico City, [See <http://www.ijlret.com/Papers/Vol-07-issue-03/1.B2021164.pdf>] so the Analysis of its Bivariate probability distribution functions is used, this study has already been done in past articles to see the relationship between both variables, but now it will be done for each group of bone variable of only the Maximum Ozone concentrations in Mexico City.

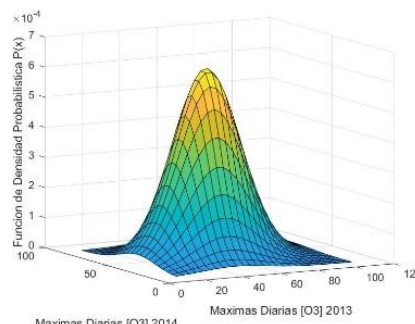


Figure (1) Gaussian Distribution 2D

III. K-means

K-means is also a partitional clustering method in which we need to specify the number of clusters before starting the clustering process. Suppose the number of clusters is m , then we can define an objective function as the sum of squared distances between a data point and its nearest cluster centers. We can follow a procedure to



minimize the objective function iteratively by finding a new set of cluster centers that can reduce the value of the objective function in each iteration. Here we use this methodology to see if there are differences between the 2 groups formed or formed one united group.

K-means clustering (k-means for short), is one of the most well-known methods for data clustering. The goal of k-means is to find k points in a data set that can best represent the data set in a certain mathematical sense.

Data compression: We can use these cluster centers to represent the original data set. Since the number of centers is much smaller than the size of the original data set, the goal of data compression can be achieved.

Data Classification: We can use these cluster centers for data classification so that the calculation load and the influence of noisy data are reduced.

K-means is also a partitional clustering method in which we need to specify the number of clusters before starting the clustering process.

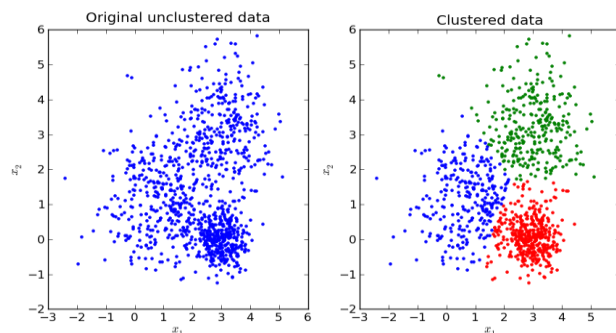


Figure (2)

There are some other k-medias facts that we should keep in mind:

K-means iteration can only guarantee non-increment of the objective function. However, it cannot guarantee the finding of the global minimum of the objective function, there are no efficient methods that can guarantee the finding of the global minimum of the objective function. Therefore, it is advisable to run k-means several times starting from different initial centers and then keep the best result.

There are some other k-medias facts that we should keep in mind:

A better set of initial centers will have a positive influence on the final results of the grouping.

IV. Stochastic Gaussian Mixture

The Gaussian Mixture Model (GMM for short) is an effective tool for data modeling and pattern classification. GMM assumes that the modeled data are generated by a probability density distribution that is the weighted sum of a set of Gaussian PDFs. By using EM (expectation maximization), we can identify the optimal set of parameters for GMM iteratively.

Characteristics they have is that they are highly flexible for complicated data sets. There is no guarantee of convergence to the general optimum, we will use the 2D form.

$$gmm(x; \alpha_1, \mu_1, \Sigma_1, \alpha_2, \mu_2, \Sigma_2, \alpha_3, \mu_3, \Sigma_3) =$$

$$\alpha_1 g(x; \mu_1, \Sigma_1) + \alpha_2 g(x; \mu_2, \Sigma_2) + \alpha_3 g(x; \mu_3, \Sigma_3), \text{ where } \sum_{i=1}^3 \alpha_i = 1.$$

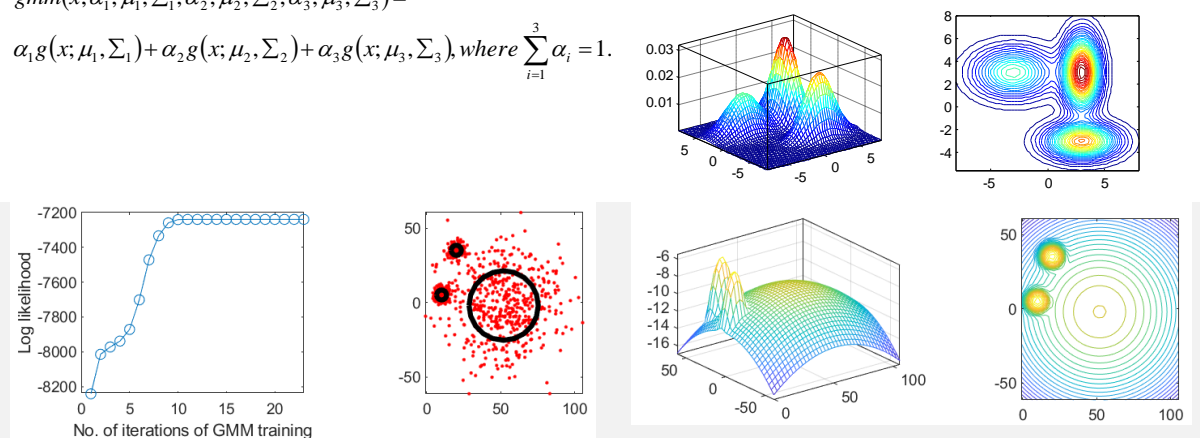


Figure (3)

Source (<http://mirilab.org/jang/books/dcpr/>)

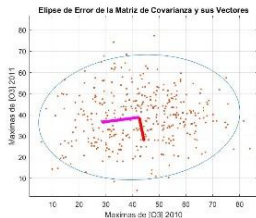
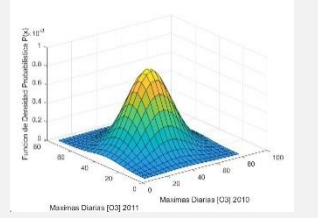


$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho_{12} & \rho_{13} \\ \rho_{21} & \sigma_2^2 & \rho_{23} \\ \rho_{31} & \rho_{32} & \sigma_3^2 \end{bmatrix}$$

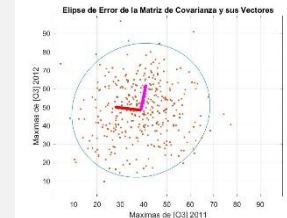
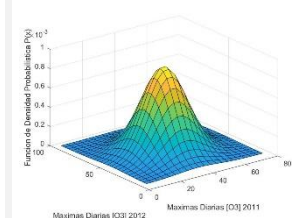
Let's see the following Results

Table 1. Bivariate Analysis of Ozone Maximum from year to year in Mexico City.

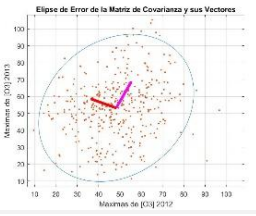
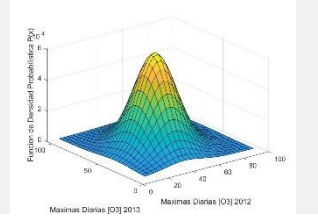
2010-2011



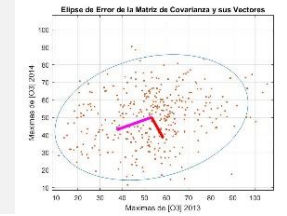
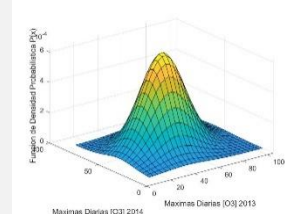
2011-2012



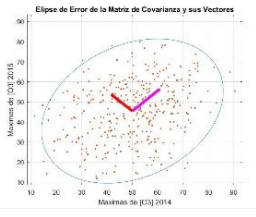
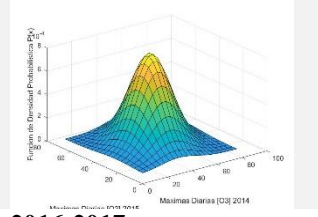
2012-2013



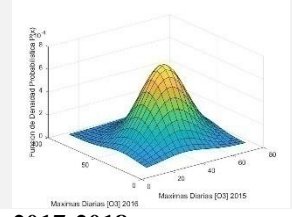
2013-2014



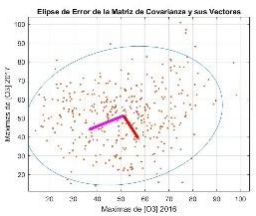
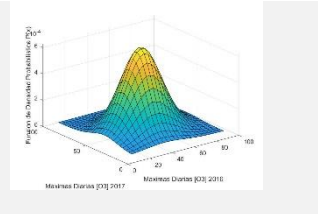
2014-2015



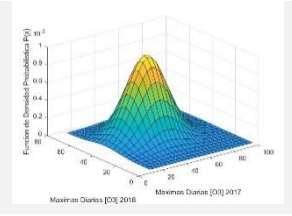
2015-2016



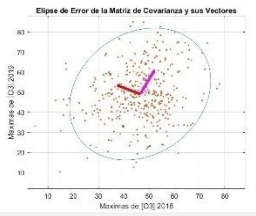
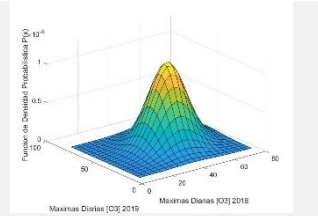
2016-2017



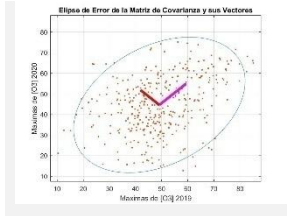
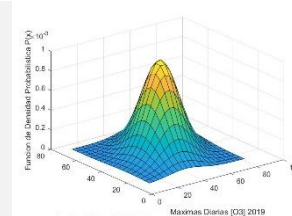
2017-2018



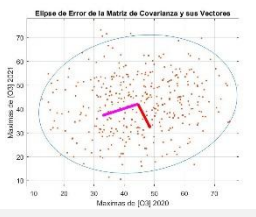
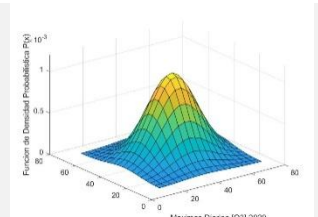
2018-2019



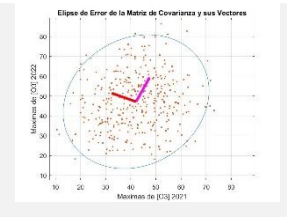
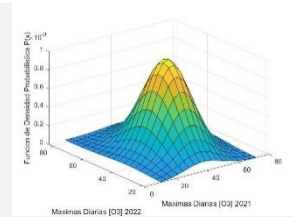
2019-2020



2020-2021



2021-2022





2022-2023

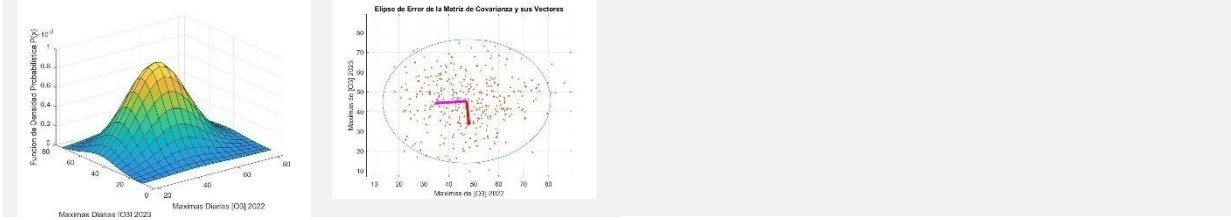


Table 2. Data from the Covariance Matrix and its Adjustment

2010-2011				2011-2012			
Sigma =		R =		Sigma =		R =	
236.7015	15.2581	1.0000	0.0820	146.2184	12.2280	1.0000	0.0680
15.2581	146.2184	0.0820	1.0000	12.2280	221.3347	0.0680	1.0000
2012-2013				2013-2014			
Sigma =		R =		Sigma =		R =	
221.3347	54.9333	1.0000	0.2073	317.3343	58.4664	1.0000	0.2241
54.9333	317.3343	0.2073	1.0000	58.4664	214.4227	0.2241	1.0000
2014-2015				2015-2016			
Sigma =		R =		Sigma =		R =	
214.4227	59.5023	1.0000	0.2762	216.3687	62.9052	1.0000	0.2525
59.5023	216.3687	0.2762	1.0000	62.9052	286.7594	0.2525	1.0000
2016-2017				2017-2018			
Sigma =		R =		Sigma =		R =	
286.7594	40.1880	1.0000	0.1571	228.3284	16.2849	1.0000	0.0943
40.1880	228.3284	0.1571	1.0000	16.2849	130.5380	0.0943	1.0000
2018-2019				2019-2020			
Sigma =		R =		Sigma =		R =	
130.5380	31.3080	1.0000	0.2040	180.4835	63.6961	1.0000	0.3526
31.3080	180.4835	0.2040	1.0000	63.6961	180.8123	0.3526	1.0000
2020-2021				2021-2022			
Sigma =		R =		Sigma =		R =	
180.8123	19.6424	1.0000	0.1231	140.7614	26.9536	1.0000	0.1649
19.6424	140.7614	0.1231	1.0000	26.9536	189.8730	0.1649	1.0000
2022-2023							
Sigma =		R =					
189.8730	2.1219	1.0000	0.0121				
2.1219	163.3078	0.0121	1.0000				



By Clusters to be able to observe or classify which group the data belongs to

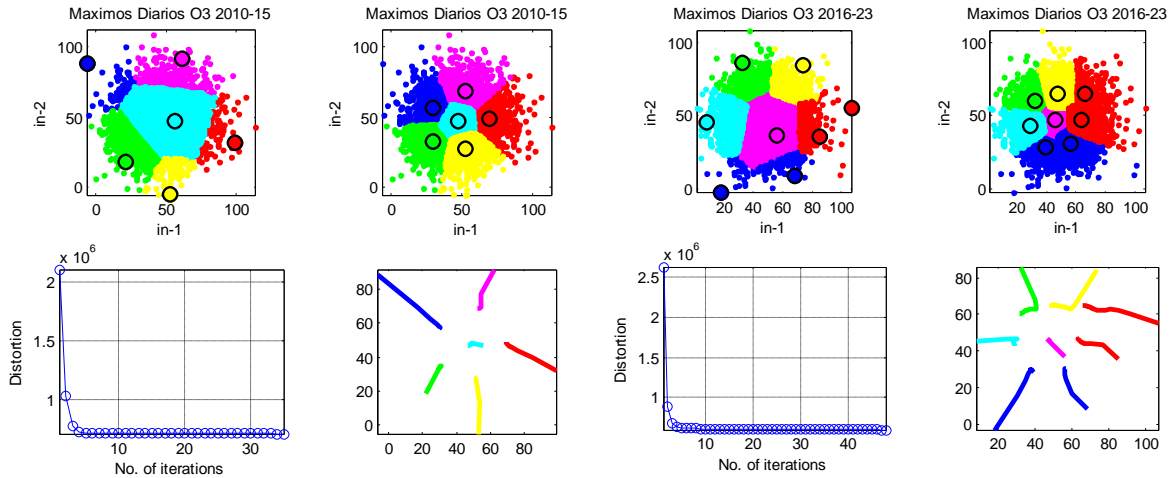
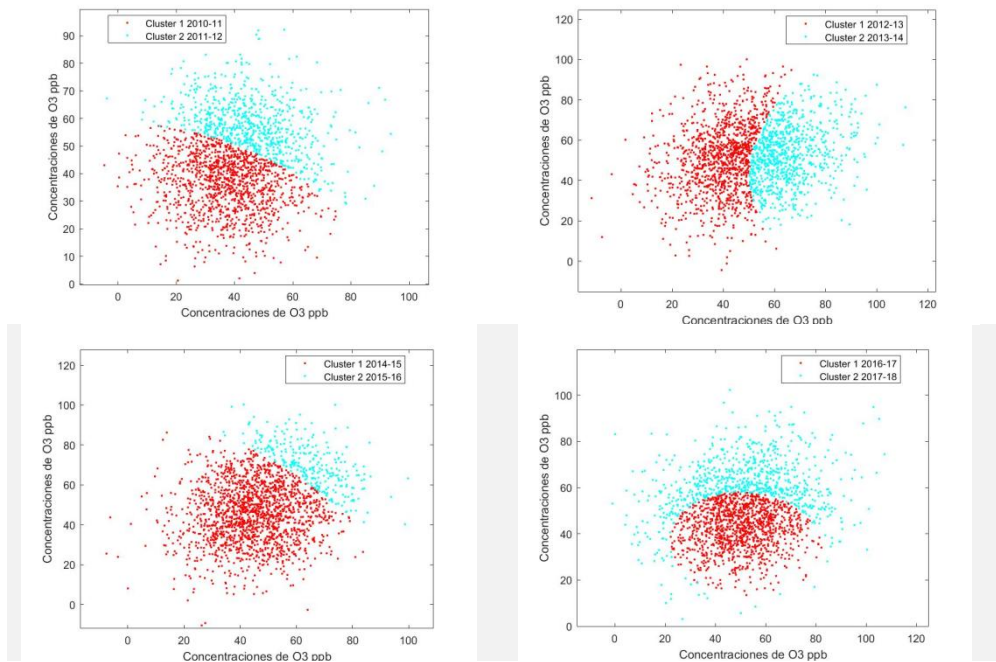


Figure (4)

The upper left graph is the initial centers and the corresponding groups, the top right graph is the final centers and corresponding groups. The bottom left graph is the distortion with respect to the number of iterations. The lower right graph is the trajectories of the centers during the grouping process.

In the image above, we can clearly identify the groups by visual inspection. If we set the number of groups to 6 to run k-means, the result is satisfactory. However, if there is no way to perform visual inspection (for example, when the data dimensions are more than 3), then we must use cluster validation methods to identify the optimal number of clusters.



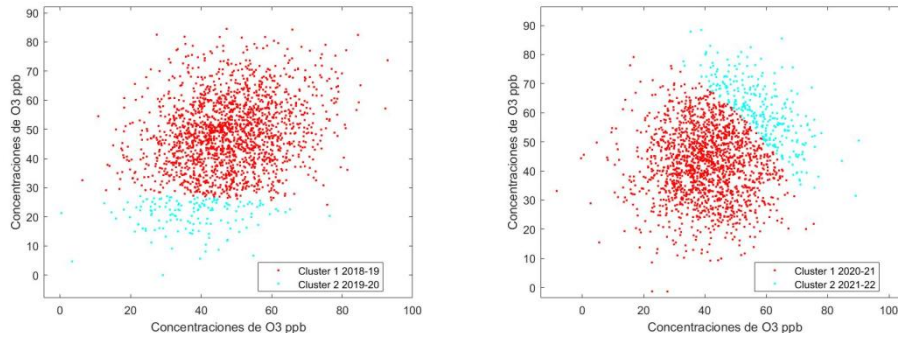
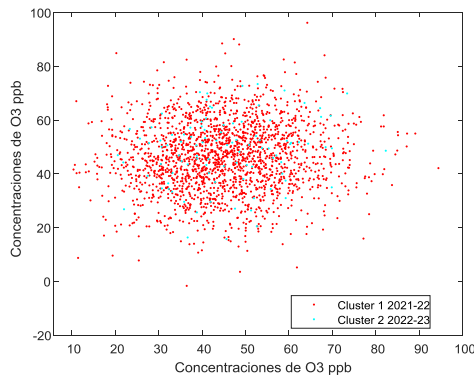
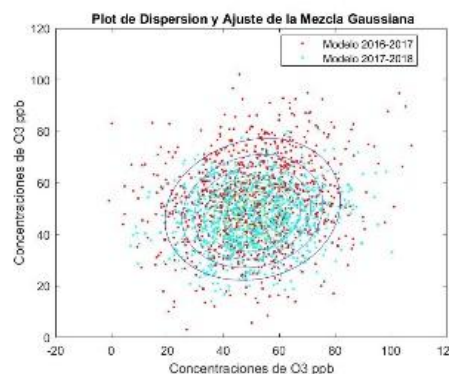
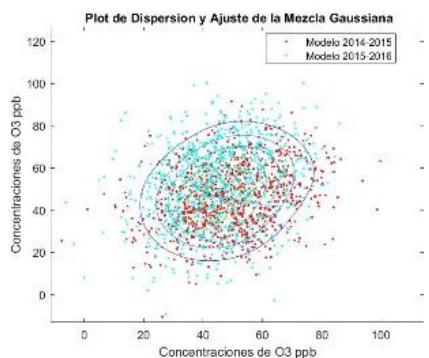
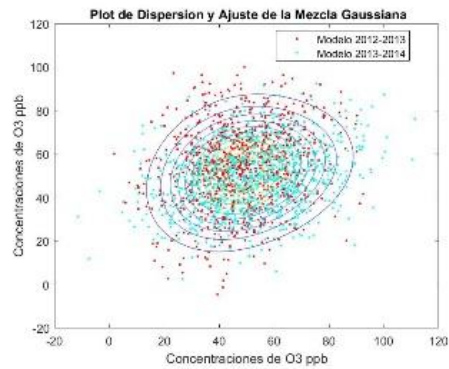
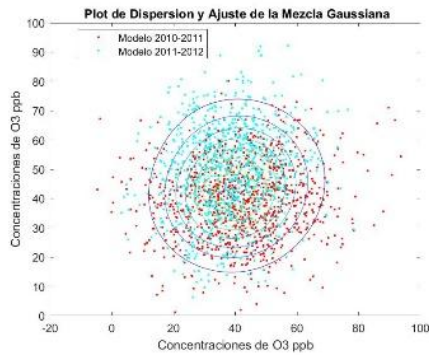


Figure (5) By Clusters by pairs of years of the Bivariate Distribution



Gaussian Mixture and its Scatter Plot



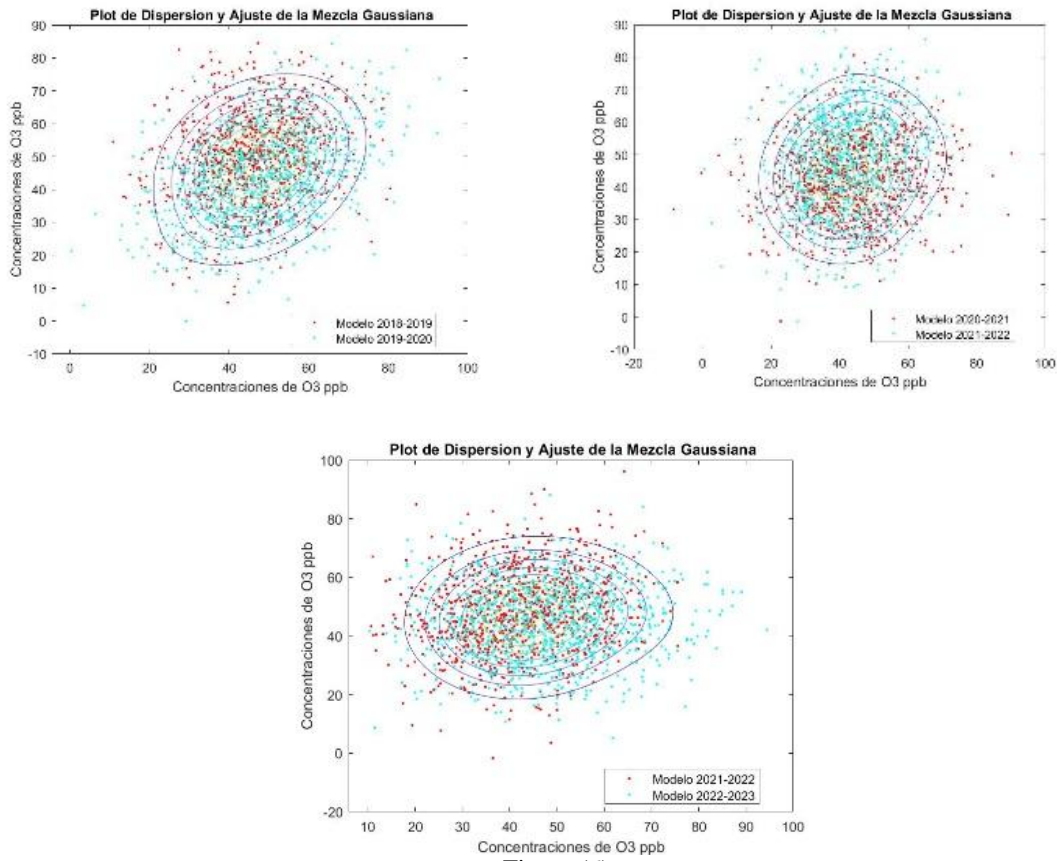
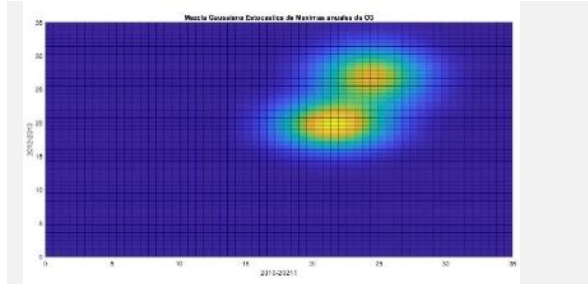
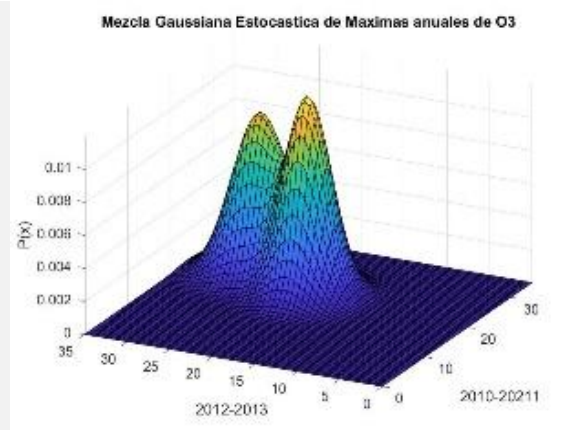


Figure (6)

Table 3. 2D Gaussian Mixture of Daily Ozone Maximum

2010-11 2012-13

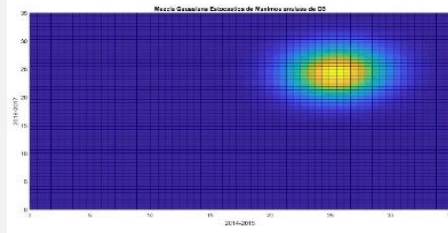
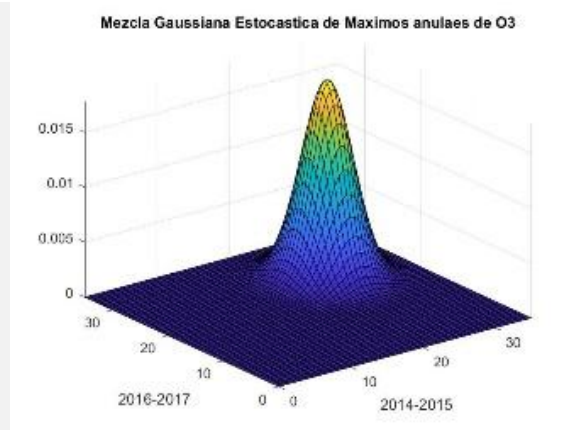


Component 1:
 Mixing proportion: 0.500000
 2010-2011
 Mean: 21.2381 19.4059
 Real Mean: 85 78

Component 2:
 2012-2013
 Mixing proportion: 0.500000
 Mean: 24.1283 26.6116
 Real Mean: 77.18 85.15



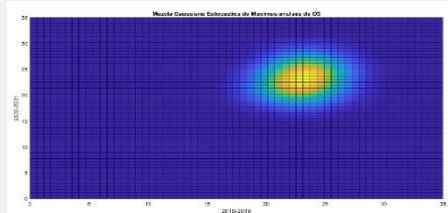
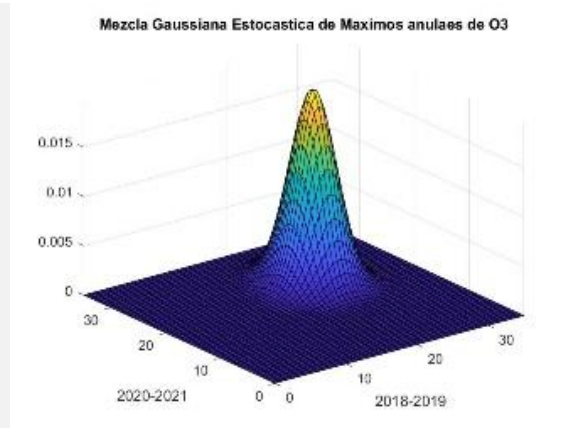
2014-15 2016-17



Component 1:
 Mixing proportion: 0.500000
 2014-2015
 Mean: 25.0293 22.7009
 Real Mean: 80 72.64

Component 2:
 Mixing proportion: 0.500000
 2016-2017
 Mean: 25.5082 25.6616
 Real Mean: 81.6 82.11

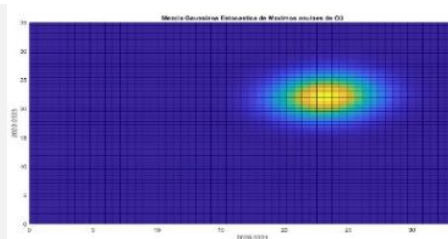
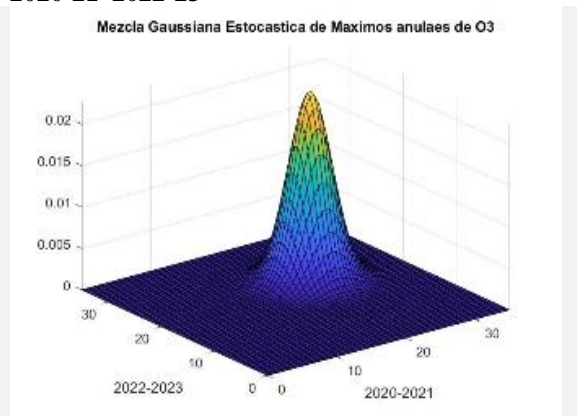
2018-19 2020-21



Component 1:
 Mixing proportion: 0.500000
 2018-2019
 Mean: 23.2529 24.5596
 Real Mean: 75 80

Component 2:
 2020-2021
 Mixing proportion: 0.500000
 Mean: 22.2855 21.0674
 Real Mean: 71 65

2020-21 2022-23



Component 1:
 Mixing proportion: 0.500000
 2020-2021
 Mean: 22.2855 21.0674
 Real Mean: 71.29 70

Component 2:
 2022-2023
 Mixing proportion: 0.500000
 Mean: 23.5835 22.6229
 Real Mean: 84.8 82



Gaussian mixture of n components

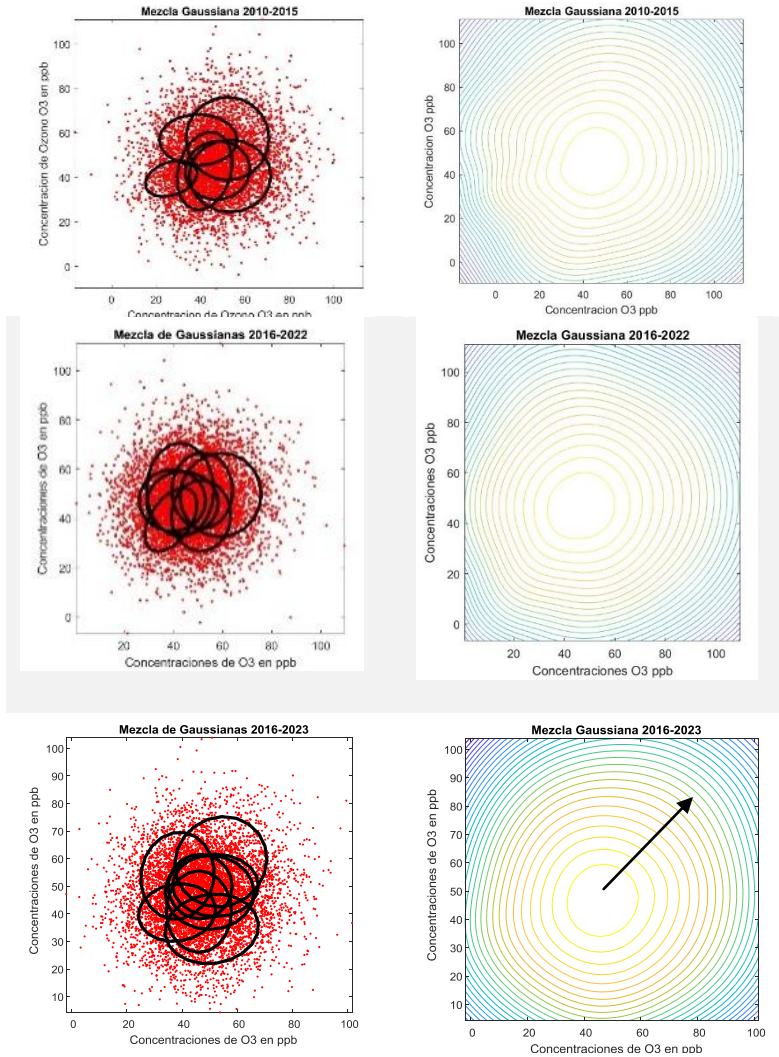


Figure (7)

In these results with this Gaussian mixture of various components for years, we can see a slight elevation pattern, but very slight and it is seen in the slightly more intense colors in the 2016-23 mixture, but with a downward trend of Ozone concentrations

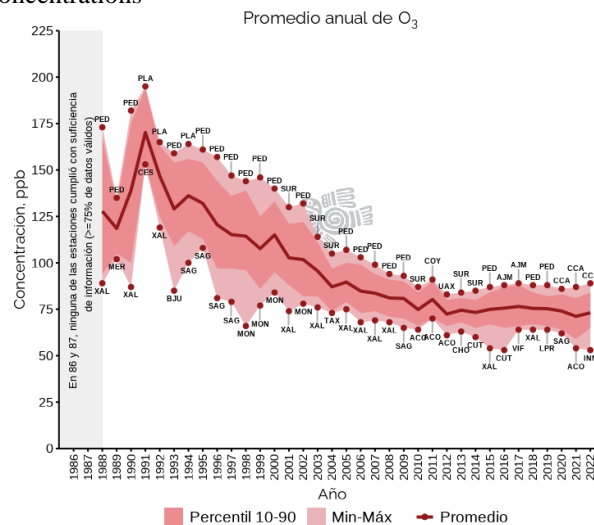


Figure (8)



In the official Ozone trend figure for Mexico City we can see that it only reaches 2022, it is not updated and is the one on the official portal, but we can see that it coincides with that slight trend shown in the Gaussian mixture of n components.

Maximum Daily Temperatures in México City.

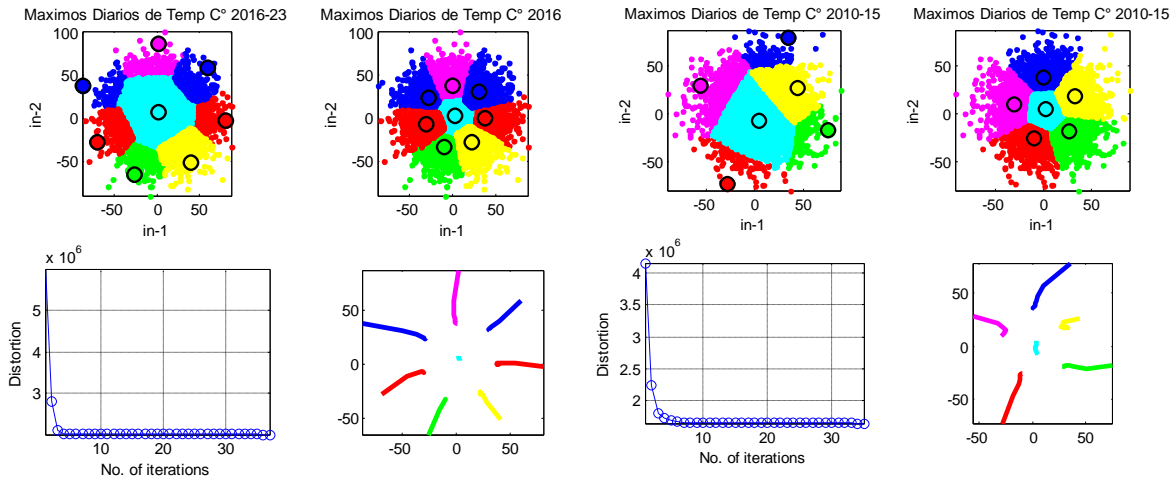


Figure (9)

By Clusters for Daily Maximum Temperatures in CDMX

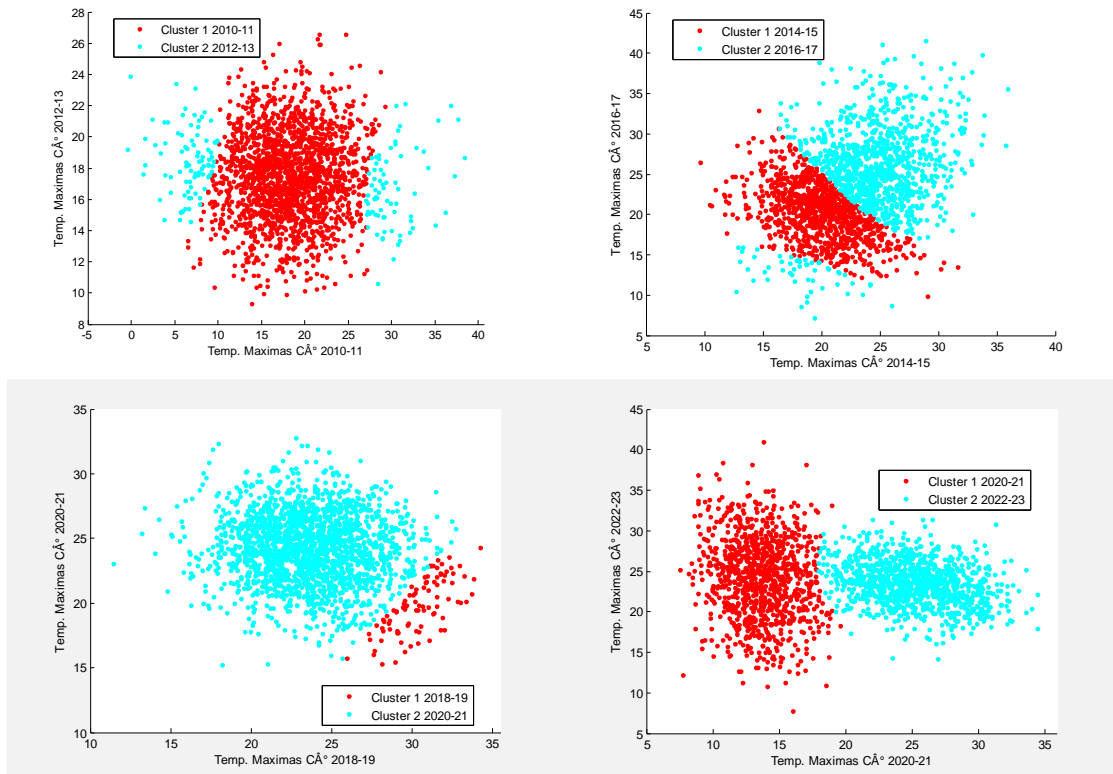


Figure (10)



By Cluster by Pairs of years

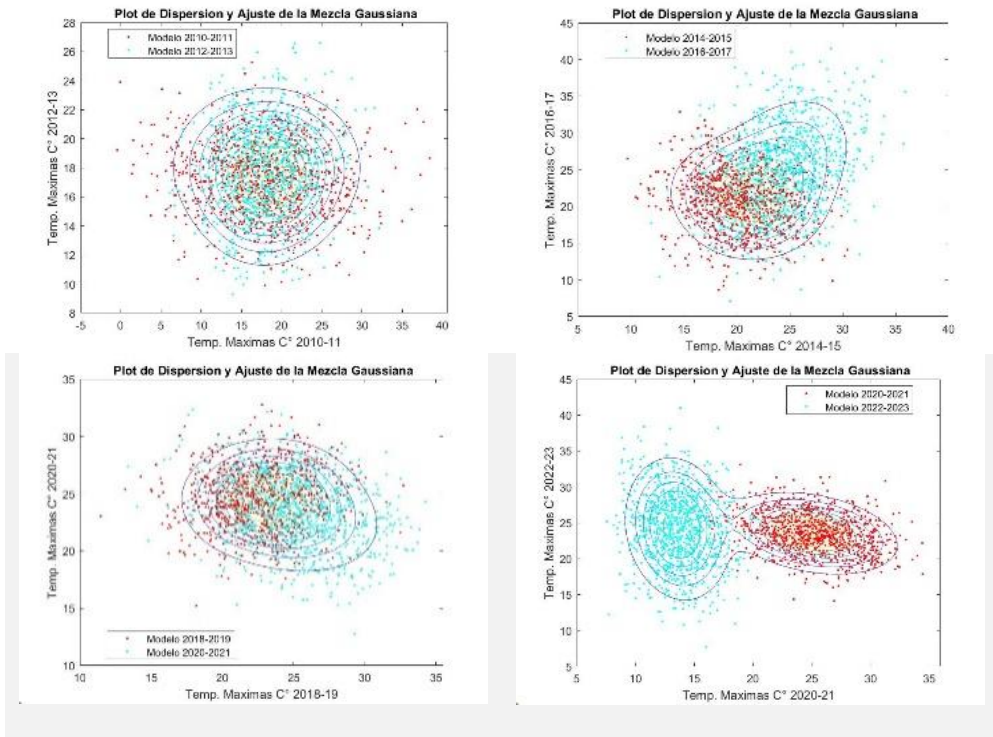
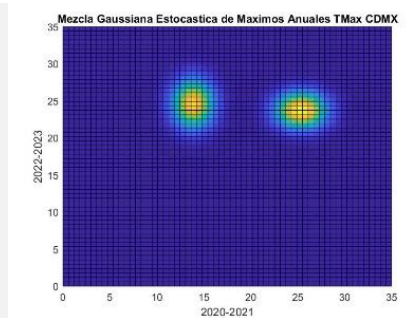
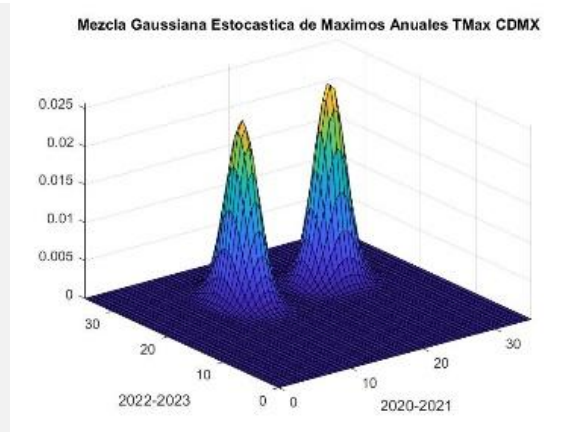


Figure (11)

Gaussian Mixture and its Scatter Plot

Table 4. 2D Gaussian Mixture of Daily Temperature Maximums

2020-21 2022-23

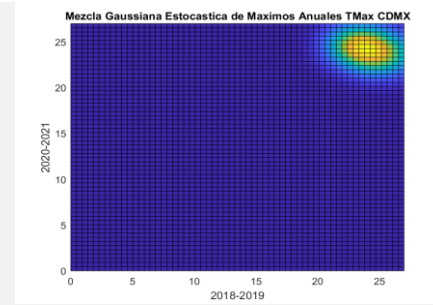
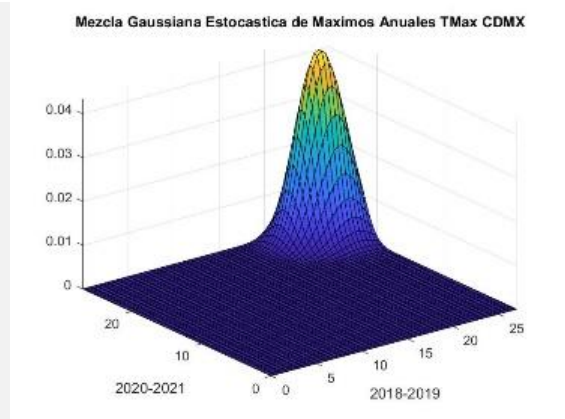


Component 1:
 Mixing proportion: 0.500000
 2020-2021
 Mean: 25.0684 23.4198

Component 2:
 Mixingproportion: 0.500000
 2022-2023
 Mean: 13.5900 24.3035



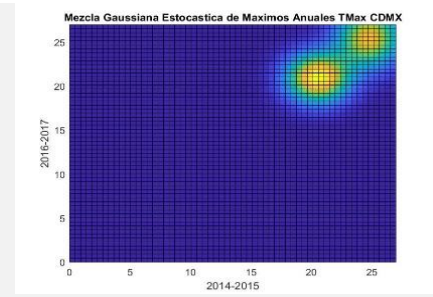
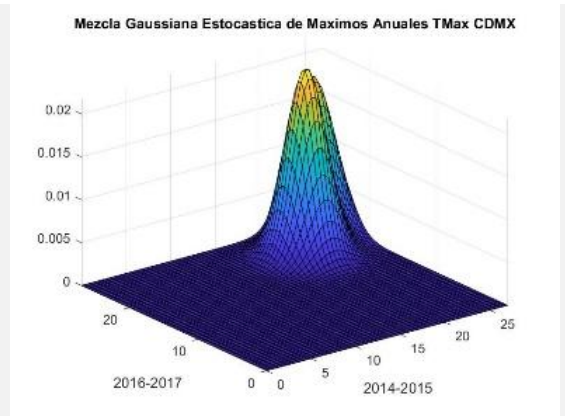
2018-19 2020-21



Component 1:
Mixing proportion: 0.500000
2018-2019
Mean: 22.8712 24.4113

Component 2:
Mixing proportion: 0.500000
2020-2021
Mean: 25.0684 23.4198

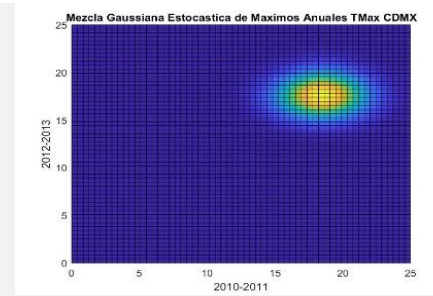
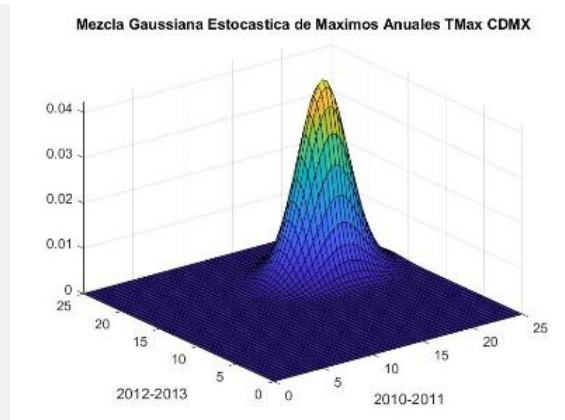
2014-15 2016-17



Component 1:
Mixing proportion: 0.500000
2014-2015
Mean: 20.3195 20.7225

Component 2:
Mixing proportion: 0.500000
2016-2017
Mean: 24.5593 25.2841

2010-11 2012-13



Component 1:
Mixing proportion: 0.500000
2010-2011
Mean: 18.1940 17.0605

Component 2:
Mixing proportion: 0.500000
2012-2013
Mean: 18.3277 17.7526



Gaussian mixture of n components

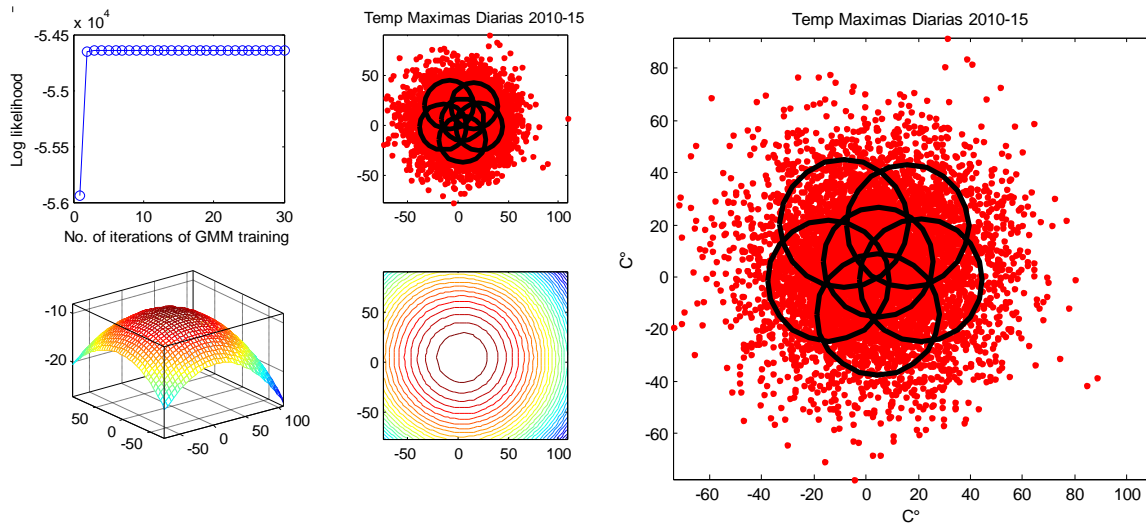


Figure (12)

GMM (Gaussian mixture model) for 2-D "Unequal" data. Number of Gaussians is 6, with K means = 6

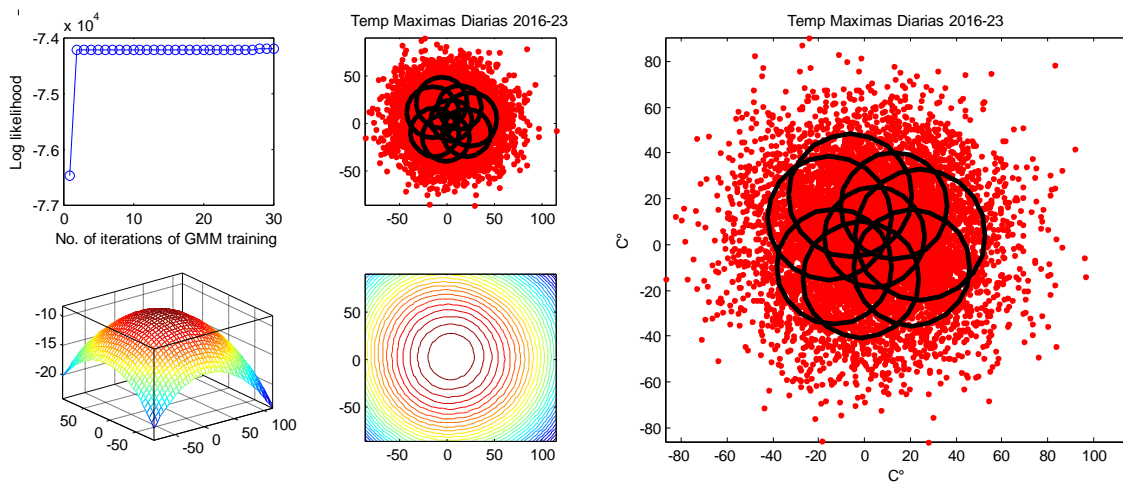


Figure (13)

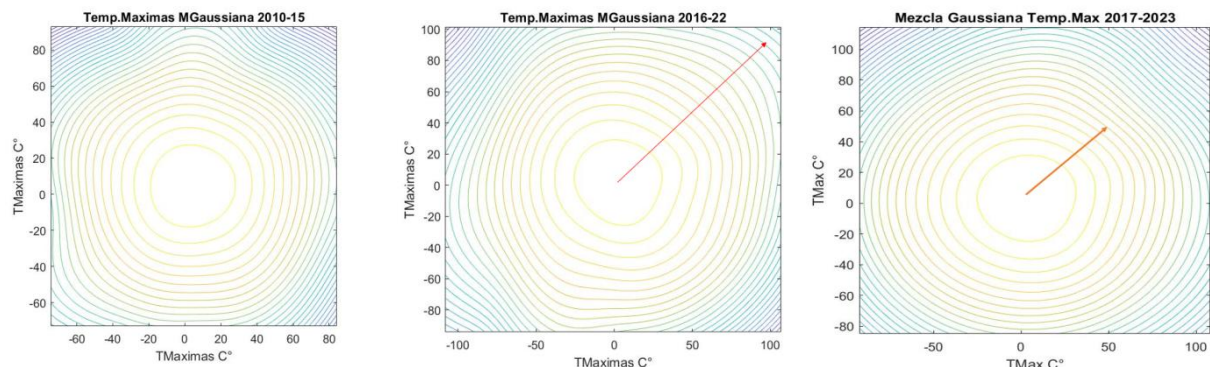


Figure (14)

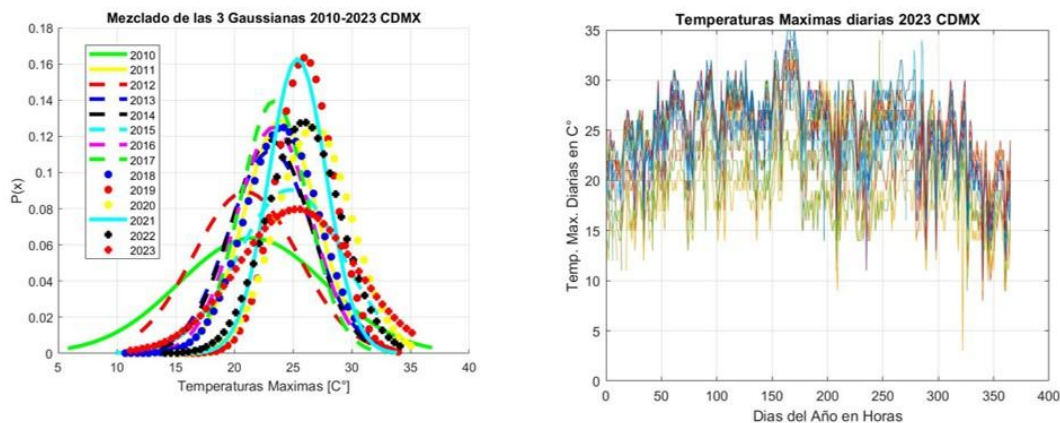


Figure (15)

The average Maximum Temperature until June 2023 is 26.12 C° of the 1D Gaussian Mixture and the CONAGUA of México is 25.8 C° and now the Maximum average Temperature until November 12, 2023 is 25.34 C° and until October the CONAGUA is 25.5 C° on average, now that of the full year of 2023 is 25.3 C° of the 1D Gaussian Mixing and that of CONAGUA until November 30 is 25.4 C°

Conclusions

It can be concluded with this more complete analysis that the methodology gives us the behavior or trend of the maximum ozone concentrations of each year taken from the database of the Air Secretary of Mexico City, as well as the temperatures. Daily maximums When starting with the cluster analysis, when making the groups, a certain symmetry is observed, then when taking the clusters by pairs of years, a similar group pattern is observed in the daily ozone maximums except in the 2021-22 clusters and 2022-23 are totally united without distinction and in the clusters by pairs of years of the daily maximum temperatures in the years 2021-22 and 2022-23 totally separated except for a slight union, now in the Gaussian mixture you can see the patterns of daily maximum temperatures, in the 2021-22 and 2022-23 mixture of maximum daily ozone concentrations is a single Gaussian with a slight pattern to the right of the x-axis, having a slight rebound beyond 85 ppb, but In general I feel the downward trend and showing average values approximately coinciding with the official trend graph.

Now, by further combining the mixture of several Gaussians per year for ozone, a slight trend is shown, shown with the arrow on the image, but it is very slight. For daily maximum temperatures, this effect is seen a little more and with the mixture of With the 3 Gaussians you can see the trend of the last one in 2023 a little better, Mexico City is also a heat island and more like the effect of the child that hovers over the entire globe, this effect of high temperatures increases a little more plus the heat waves that have been observed.

We can say that there is already a background concentration of ozone in Mexico City which perhaps the effect could be observed more or those maximum daily temperatures could be seen with a more geostatistical treatment.

There are also the days of environmental contingencies due to ozone for Mexico City in 2023 and so far in 2024, with a very similar count so far.

Índice de calidad del aire mayores a 150					
Fecha	NOO3	NEO3	CEO3	SOO3	SEO3
2023-02-23	124	123	151	167	142
2023-03-25	157	131	137	132	125
2023-03-26	136	130	143	153	122
2023-11-20	123	154	104	138	49

Índice de calidad del aire mayores a 150					
Fecha	NOO3	NEO3	CEO3	SOO3	SEO3
2024-02-22	109	127	161	172	172
2024-02-23	131	183	134	138	129
2024-02-24	120	133	162	140	136
2024-03-06	175	164	121	119	90
2024-03-23	158	151	136	105	69



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