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Summary of the new ZJ Transform and its application to Differential Equations, Partial Equations and Integral Equations

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Summary: The following article is a summary and application with results on the ZJ Transform to solve Differential, Partial, Integro Differential and Integral Equations, we use the Integral Transform to solve by a simpler algebraic way the reduction to simpler differential equations, the integral transform as such "maps" an equation in its original domain to another suitable domain (for example, the Fourier Transform, Laplace, Mellin, etc ...). subsequently the solution is then re- mapped to the original domain with the inverse transform of said transform. The ZJ integral transform is based on that concept of changing the original domain to a suitable one in order to reduce the expression.

Keywords: Laplace Transform, ZJ Transform, Integral Transform, Partial Differential Equations, Ordinary Differential Equations, Integral Equations

Introducción

Integral transforms are derived from the classical integral. An integral transform T of f(τ) within a given interval is defined as follows:

$$T[f(\tau)] = \int_{\tau_2}^{\tau_1} K(s,\tau) f(\tau) d\tau$$

Where K (s, τ) is the Kernel of the transformation or core of the transform, there are numerous integral transforms, such as Laplace, Fourier, Mellin, Hankel, Sumudu, etc., classified according to the choice of the kernel and the range of the interval, in the real world there are many physical phenomena and processes that are described by principles or laws that are expressed in the form of relations or statements that involve rates of change, the rates of change are derivatives and the relations or statements are equations depending on the law or physical process observed and therefore we have the differential equations that describe that dynamic system, now within the methods to solve ordinary differential equations, partial differential equations, integral equations etc ... there are several methods mentioned above, and among these forms is the ZJ Transform that is born from the Relation with the Laplace Transform, and the other transforms such as Elzaki and Sumudu.

A summary, properties and examples of its use will be givenbelow.

Methodology and Important Mathematical Properties

Definition 1.
The Transform is defined as:

$$\boldsymbol{ZJ\left[\left(f(t),\frac{z}{\beta}\right)\right] = ZJ\left(\frac{z}{\beta}\right) = \frac{\beta^n}{z}\int_0^\infty e^{\frac{-zt}{\beta}} f(t)dt} \quad (1)$$

Inverse Transform

$$\boldsymbol{ZJ^{-1}[f(Z\beta),t)] = \frac{z\beta^n}{2\pi i}\int_{-\infty}^{\infty} e^{z\beta t} f\left(\frac{1}{\beta}\right) d\beta Z} (2)$$

Where z is a positive integer constant, β is the complex number and n is also a positive integer.

Definition 2.
The space W of exponentially decaying test functions is the space of complex-valued functions φ(t) that

Satisfy the following properties:

(i) φ(t) is infinitely differentiable, i.e., φ(t) $\in C\infty(Rn)$.

(ii) φ(t) and its derivatives of all orders vanish at infinity faster tan the reciprocal of the exponential of order 1/ω; i.e.

$$|e^{\frac{1}{\omega}}Dk\ \varphi(t)| < M,\ \forall\ 1/\omega,\ k \in \mathbb{R}.$$

Then a function f(t) is said to be exponentially growing if and only if f(t) together with all its derivatives grows more slowly than the exponential function of order 1/ω; that is, there exists a real constant 1/ω and M such that $Dk\ \varphi(t) < M\ e^{\frac{1}{\omega}}$.

A continuous linear function on the space W of test functions is called an exponentially growing distribution, and this dual space of W is denoted W.

The above Transform of Definition 1 obeys this definition, see in Applications of the ZJ Transform for Differential Equations http://www.ijlret.com/Papers/Vol-09-issue-02/2.B2023304.pdf Related to other Sumudu, Elzaki, Natural, Aboodh, Pourreza, Mohand, Sawi, Kamal, G and Complex SEE Transforms [See References]

Properties of the ZJ Transform

Linearity Property with C1 and C2 constant

$$ZJ[C1f1(t) + C2f2(t)] = Cf1\left(\frac{z}{\beta}\right) + C2f2\left(\frac{z}{\beta}\right)$$

First Translation Theorem $ZJ[e^{at}f(t)] = f\left(\frac{z}{\beta} - a\right)$

Second Translation Theorem $ZJ[G(t)] = e^{-\frac{z}{\beta}a}f\left(\frac{z}{\beta}\right)$ con a > 0

With

$$G(t) = \left\{\begin{matrix} f(t-a) & t > a \\ 0 & t < a \end{matrix}\right\}$$

Change of Scale

$$ZJ[f(at)] = \frac{1}{a}f\left(\frac{z}{\beta a}\right)$$

Transformation of an Integral

$$ZJ\left[\int_0^{\infty} f(\tau)d\tau\right] = \frac{f\left(\frac{z}{\beta}\right)}{\frac{z}{\beta}}$$

Teorem 1.

We define the Convolution of two functions defined on [0, ∞). The Convolution f∗g is defined as: Let f1 and f2 be a new Integral transformation F1 and F2, then the new integral transformation of the Convolution of f1 and f2 is:

$$ZJ[f1 * f2] = \frac{\beta^n}{z}\int_0^\infty e^{-\frac{z}{\beta}t}\int_0^\infty f1(t)f2(t-\tau)\,dt$$

$$ZJ[f1 * f2] = \frac{\beta^n}{z}\int_0^\infty f1(\tau)d\tau\int_0^\infty e^{-\frac{z}{\beta}t} f2(t-\tau)\,dt$$

$$ZJ[f1 * f2] = \frac{\beta^n}{z}\int_0^\infty e^{-\frac{z}{\beta}t} f1(\tau)d\tau\int_0^\infty e^{-\frac{z}{\beta}t} f2(t)\,dt$$

$$ZJ[f1 * f2] = \frac{f1\left(\frac{z}{\beta}\right)f2\left(\frac{z}{\beta}\right)}{\frac{\beta^n}{z}}$$

The Convolution f∗g is thus defined:

$$f1 * f2 = \int_0^\infty f1(t)f2(t-\tau)d\tau = \frac{f2\left(\frac{z}{\beta}\right)f1\left(\frac{z}{\beta}\right)}{\frac{\beta^n}{z}}$$

Table 1 transformations for some basic functions

Functión	ZJ Transform
1	$\frac{\beta^{n+1}}{z^2}$
t	$\frac{\beta^{n+2}}{z^3}$
t^2	$\frac{2\beta^{n+3}}{z^4}$
t^3	$\frac{6\beta^{n+4}}{z^5}$
t^k With k positive integer	$\frac{k!\,\beta^{n+(k+1)}}{z^{k+2}}$
t^n With n > 0	$\frac{\beta^{n+1}}{z^{n+1}}\,\Gamma(n+1)$
$\ln t$ Where γ is Euler's constant	$\frac{\beta^{n+1}}{z^2}\left[-\gamma+\ln\left(\frac{z}{\beta}\right)\right]$
$\sqrt{t}$	$\frac{\sqrt{\pi}\beta^{n+3/2}}{2z^{5/2}}$
e^{at}	$\frac{\beta^{n+1}}{z(z-a\beta)}$
$\frac{1}{\sqrt{t}}$	$\frac{\sqrt{\pi}\beta^{n+1/2}}{z^{3/2}}$
$cos(bt)$	$\frac{\beta^{n+1}}{z^2+b^2\beta^2}$

$sen(bt)$	$\frac{b\beta^{n+2}}{z(z^2+b^2\beta^2)}$
$t^n e^{kt}$ With n positive integer	$\frac{n!\,\beta^n}{z\left(k-\frac{z}{\beta}\right)^{n+1}}$
$senh(bt)$	$\frac{b\beta^{n+2}}{z(z^2-b^2\beta^2)}$
$cosh(bt)$	$\frac{\beta^{n+1}}{(z^2-b^2\beta^2)}$
$e^{at}\cos(bt)$	$\frac{\beta^n\left(\frac{z}{\beta}-a\right)}{z\left(\left(\frac{z}{\beta}-a\right)^2-b^2\right)}$
$e^{at}\text{sen}(bt)$	$\frac{b\beta^n}{z\left(\left(\frac{z}{\beta}-a\right)^2+b^2\right)}$
$\frac{tsen(at)}{2a}$	$\frac{\beta^{n+3}}{z^4+2a^2\beta^2z^2+a^2\beta^4}$
$\frac{tsenh(at)}{2a}$	$\frac{\beta^{n+3}}{(z-a\beta)^2(z+a\beta)^2}$
$\frac{e^{bt}senh(at)}{a}$	$\frac{\beta^{n+2}}{z(z^2-2\beta bz+b^2\beta^2-a^2\beta^2)}$
$\frac{e^{bt}\cosh(at)}{a}$	$\frac{\beta^{n+1}(z-\beta b)}{az(z^2-2\beta bz+b^2\beta^2-a^2\beta^2)}$
$tcos(at)$	$\frac{\beta^{n+2}(z^2-a\beta^2)}{z(z^4+2a^2\beta^2z^2+a^4\beta^4)}$
$\delta(t-t_0)$	$\frac{\beta^{n+1}e^{-\frac{z}{\beta}t0}}{z^2}$
$e^{at}F(t)$	$f(\frac{z}{\beta}-a)$
$\frac{\text{erf}(\sqrt{at})}{\sqrt{a}}$	$\frac{\beta^{n+3}}{z^2}\left(\frac{1}{\sqrt{(z+a\beta)}}\right)$
$\frac{e^{at}\text{erf}(\sqrt{at})}{\sqrt{a}}$	$\frac{\beta^{n+3}}{z}\left(\frac{1}{\sqrt{z}(z-a\beta)}\right)$
$J_0(t)$	$\frac{\beta^{n+1}}{z\sqrt{\beta^2+z^2}}$
$J_1(t)$	$\frac{\beta^n\left(\sqrt{\beta^2+z^2}-z\right)}{z\sqrt{\beta^2+z^2}}$
$J_n(at)$	$\frac{\beta^n\left(\sqrt{a^2\beta^2+z^2}-z\right)^n}{a^2z\sqrt{a^2\beta^2+z^2}}$
$\frac{dy}{dt}$	$-y(x,0)\frac{\beta^n}{z}+\frac{z}{\beta}\hat{\varphi}$
$\frac{d^2y}{dt^2}$	$-\frac{\beta^n}{z}y'(x,0)-\frac{\beta^n}{\beta}y(x,0)+\frac{z^2}{\beta^2}\hat{\varphi}$
$\frac{d^3y}{dt^3}$	$-\frac{\beta^n}{z}y''(x,0)-\frac{\beta^n}{\beta}y'(x,0)-\frac{\beta^n z}{\beta^2}y(x,0)+\frac{z^3}{\beta^3}\hat{\varphi}$
$f(t)^{(n)}$	

Where n is the nth derivative.	$\frac{z^n\hat{\varphi}}{\beta^n}-\frac{\beta^n}{z}\left(\sum_{k=1}^{n-1}\left(\frac{z}{\beta}\right)^{n-1-k}y^k(0)\right)$
$\frac{\partial^2 y}{\partial x^2}$	$\frac{d^2\hat{\varphi}}{dx^2}$
$\frac{\partial y}{\partial x}$	$\frac{d\hat{\varphi}}{dx}$

Table 2 with the transformation expressions for differential equations with variable coefficient, with the Transformation we have the following, in this case we must see n = 1 and thus we have:

$\boldsymbol{ty}$	$ZJ[ty]=-\frac{dZJ[y]}{d\left(\frac{z}{\beta}\right)}-\frac{\beta}{z}ZJ[y]$
$\boldsymbol{ty'}$	$ZJ[ty']=-\frac{dZJ\left[-y(x,0)\frac{\beta}{z}+\frac{z}{\beta}\hat{\varphi}\right]}{d\left(\frac{z}{\beta}\right)}-\frac{\beta}{z}ZJ\left[-y(x,0)\frac{\beta}{z}+\frac{z}{\beta}\hat{\varphi}\right]$
$\boldsymbol{ty''}$	$ZJ[ty'']=-\frac{dZJ\left[-\frac{\beta}{z}y'(x,0)-y(x,0)+\frac{z^2}{\beta^2}\hat{\varphi}\right]}{d\left(\frac{z}{\beta}\right)}-\frac{\beta}{z}ZJ\left[-\frac{\beta}{z}y'(x,0)-y(x,0)+\frac{z^2}{\beta^2}\hat{\varphi}\right]$
$\boldsymbol{t^2y'}$	$ZJ[t^2y']=\frac{z}{\beta}\frac{d^2ZJ[\hat{\varphi}]}{d\left(\frac{z}{\beta}\right)^2}+2\frac{dZJ[\hat{\varphi}]}{d\left(\frac{z}{\beta}\right)}-2y(x,0)\frac{\beta^3}{z^3}$
$\boldsymbol{t^2y''}$	$ZJ[t^2y'']=\frac{z^2}{\beta^2}\frac{d^2ZJ[\hat{\varphi}]}{d\left(\frac{z}{\beta}\right)^2}+4\frac{z}{\beta}\frac{dZJ[\hat{\varphi}]}{d\left(\frac{z}{\beta}\right)}+2ZJ[\hat{\varphi}]-2y'(x,0)\frac{\beta^3}{z^3}$

Examples using the ZJ transform with this type of Improper Integrals with the numerator as a Trigonometric function.

Example 1

$$\int_0^\infty \frac{2\cos(x)\,dx}{1+x^2}$$

Taking the ZJ transform and putting a t in the cosine argument

$$\frac{\beta^n}{z}\int_0^\infty\left(\int_0^\infty\frac{2\cos(tx)\,dx}{1+x^2}\right)e^{\frac{-zt}{\beta}}dt$$

Now there are two functions which are the ones that appear below the Integral operator after solving the Integral it results as

$$\int_0^\infty\left(\frac{2}{1+x^2}\right)\left(\frac{\beta^{n+1}}{z^2+x^2\beta^2}\right)dx=\frac{\pi\beta^{n+1}}{2z(z+\beta)}$$

Now taking the Inverse of the Transform we have the solution and remember that it is $0 < x < \infty$ so with t = 1

$$\int_0^\infty \frac{2\cos(x)\,dx}{1+x^2} = \frac{\pi}{e}$$

Example 2
Now we will do the following exercise, prove if m > 0

$$\int_0^\infty \frac{\cos(mx)\,dx}{(1+x^2)^2} = \frac{\pi\, e^{-m}(1+m)}{4}$$

Let's see with m =1

$$\int_0^\infty \frac{\cos(x)\,dx}{(1+x^2)^2} = \frac{\pi\, e^{-1}}{2}$$

Taking the ZJ transform into the trigonometric function, putting at in the cosine argument

$$\frac{\beta^n}{z}\int_0^\infty \left(\int_0^\infty \frac{\cos(tx)\,dx}{(1+x^2)^2}\right) e^{\frac{-zt}{\beta}}\,dt$$

There are two functions which are those that appear below the Integral operator after solving the Integral it results as

$$\int_0^\infty \left(\frac{1}{(1+x^2)^2}\right)\left(\frac{\beta^{n+1}}{z^2+x^2\beta^2}\right)dx = \frac{\pi\beta^{n+1}(z+2\beta)}{4z(z^2+2\beta z+\beta^2)}$$

Now taking the Inverse of the Transform we have the solution and remember that it is 0<x<∞ so with t = 1

$$\int_0^\infty \frac{\cos(x)\,dx}{(1+x^2)^2} = \frac{\pi\, e^{-1}}{2}$$

Let's see with m = 2

$$\int_0^\infty \frac{\cos(2x)\,dx}{(1+x^2)^2} = \frac{3\pi\, e^{-2}}{4}$$

Taking the ZJ transform into the trigonometric function, putting a t in the cosine argument

$$\frac{\beta^n}{z}\int_0^\infty \left(\int_0^\infty \frac{\cos(2tx)\,dx}{(1+x^2)^2}\right) e^{\frac{-zt}{\beta}}\,dt$$

There are two functions which are those that appear below the Integral operator after solving the Integral it results as

$$\int_0^\infty \left(\frac{1}{(1+x^2)^2}\right)\left(\frac{\beta^{n+1}}{z^2+4x^2\beta^2}\right)dx = \frac{\pi\beta^{n+1}(z+4\beta)}{4z(z^2+4\beta z+4\beta^2)}$$

Now taking the Inverse of the Transform we have the solution and remember that it is 0<x<∞ so with t = 1

$$\int_0^\infty \frac{\cos(x)\,dx}{(1+x^2)^2} = \frac{3\pi\, e^{-2}}{4}$$

If we observe we can see how the solution behaves, if we proceed with a larger m each time the result will be similar except for the coefficient and exponent of the exponential, therefore, this is how we can demonstrate it.

$$\int_0^\infty \frac{\cos(mx)\,dx}{(1+x^2)^2} = \frac{\pi\, e^{-m}(1+m)}{4}$$

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Example 3

Now we will do the following exercise.

$$\int_0^\infty \frac{x\,\text{sen}(2x)\,dx}{3+x^2} = \frac{\pi e^{-2\sqrt{3}}}{2}$$

Taking the ZJ transform into the trigonometric function, putting a t in the sine argument

$$\frac{\beta^n}{z}\int_0^\infty \left(\int_0^\infty \frac{x\,\text{sen}(2tx)\,dx}{3+x^2}\right) e^{\frac{-zt}{\beta}}\,dt$$

Taking the ZJ transform into the trigonometric function, putting a t in the sine argument

$$\int_0^\infty \left(\frac{1}{3+x^2}\right)\left(\frac{2x^2\beta^{n+2}}{z(z^2+4x^2\beta^2)}\right)dx = \frac{\pi\beta^{n+1}(z-2\sqrt{3}\beta)}{2z(z^2-12\beta^2)}$$

Now we proceed in the same way taking the Inverse of the Transform we have the solution and remember that it is 0 < x < ∞ so with t = 1

$$\int_0^\infty \frac{x\,\text{sen}(2x)\,dx}{3+x^2} = \frac{\pi e^{-2\sqrt{3}}}{2}$$

Examples now for Integro Differential Equations
Ejemplo 4

$$\frac{dy}{dt} - \int_0^t y(\tau)\cos(t-\tau)\,d\tau = \cos(t)$$

With $y(0) = 1$ Using the Transformation we have

$$ZJ\left[\frac{dy}{dt}\right] - ZJ\left[\int_0^t y(\tau)\cos(t-\tau)\,d\tau\right] = ZJ[\cos(t)]$$

$$\left[-y(x,0)\frac{\beta^n}{z}+\frac{z}{\beta}\hat{\varphi}\right] - \left(\frac{z}{\beta^n}\right)\left[\frac{\beta^{n+1}}{z^2+\beta^2}\right]\hat{\varphi} = \frac{\beta^{n+1}}{z^2+\beta^2}$$

$$\hat{\varphi} = \frac{\beta^{n+2}}{z^3}+\frac{\beta^{n+1}}{z^2}+\frac{\beta^{n+3}}{z^4}$$

Now taking the Inverse Transform

The solution is a vailable $y(t) = 1 + t + \frac{t^2}{2}$

Example 5

$$\frac{dy}{dt} + \int_0^t y(\tau)d\tau = 1$$

With $y(0) = 1$ Using the Transformation we have

$$ZJ\left[\frac{dy}{dt}\right] + ZJ\left[\int_0^t y(\tau)d\tau\right] = ZJ[1]$$

$$\left[-y(x,0)\frac{\beta^n}{z}+\frac{z}{\beta}\hat{\varphi}\right] - \left(\frac{\beta}{z}\right)\hat{\varphi} = \frac{\beta^{n+1}}{z^2}$$

$$\hat{\varphi} = \frac{\beta^{n+2} + z\beta^{n+1}}{z(z^2+\beta^2)}$$

Now taking the Inverse Transform
The solution is available $y(t) = sen(t) + \cos(t)$

Example 6

$$\frac{d^3y}{dt^3} = \int_0^t y(\tau)d\tau$$

With $y(0) = y''(0) = 0 \quad and \quad y'(0) = 2$ Using the Transformation we have

$$ZJ\left[\frac{d^3y}{dt^3}\right] = ZJ\left[\int_0^t y(\tau)d\tau\right]$$

$$\left(\frac{z^3}{\beta^3} - \frac{\beta}{z}\right)\hat{\varphi} = \frac{2\beta^n}{\beta}$$

$$\hat{\varphi} = \frac{2z\beta^{n+2}}{(z^4 - \beta^4)}$$

Now taking the Inverse Transformation. We have the solution $y(t) = sen(t) + \mathbf{senh}(t)$

Examples for Integral Equations

Example 7

$$u(x) = \frac{13}{3}x - \frac{1}{4}\int_0^1 x\,t\,u(t)dt$$

Using the Convolution Theorem we have the following

$$ZJ[u(x)] = ZJ\left[\frac{13}{3}x\right] - \frac{1}{4}\left(\frac{z}{\beta^n}\right)ZJ[xt]\,ZJ[u(t)]$$

Thus

$$\hat{\varphi} = \frac{(52x\beta^{n+1})}{3(4z^2 + x\beta^2)}$$

Applying the inverse $f\left(\frac{1}{\beta}\right)$ and with $Z\beta^n$ we have and by power series expansion of the hyperbolic cosine with the first term

$$= 3.46x$$

Example 8

$$\frac{e^{2x}}{2} = \int_0^{1/2} e^{2x-2t}u(t)dt$$

Using the Transformation we have the following:

$ZJ\left[\frac{e^{2x}}{2}\right] = ZJ\left[\int_0^{1/2} e^{2x-2t}u(t)dt\right]$ this is $\frac{e^{2x}}{2}\left(\frac{\beta^{n+1}}{z^2}\right) = \frac{\beta}{z}\hat{\varphi}$ Now taking the Inverse $f\left(\frac{1}{\beta}\right)$ and with $Z\beta^n$ one has $u(t) = \frac{e^{2x}}{2}$ Now as you can see the solution is on the left side as well, therefore the general solution is $u(t) = e^{2x}$, by substituting in the Integral Equation with e^{2t}, gives usthedesiredequality.

Example 9

$$\frac{dy}{dt} + 4\int_0^t y(\tau)d\tau = \mathrm{t} - \mathbf{sen}\,t$$

With $y(0) = 2$ Using the ZJ Transformation we have

$$ZJ\left[\frac{dy}{dt}\right] + ZJ\left[4\int_0^t y(\tau)d\tau\right] = ZJ[\mathbf{t} - \mathbf{sen}\,t]$$

$$\left[-y(x,0)\frac{\beta^n}{z} + \frac{z}{\beta}\hat{\varphi}\right] - 4\left(\frac{\beta}{z}\right)\hat{\varphi} = \frac{\beta^{n+2}}{z^3} - \frac{\beta^{n+2}}{z(z^2+\beta^2)}$$

We get

$$\hat{\varphi} = \frac{2\beta^{n+3}}{z^2(z^2+4\beta^2)} - \frac{2\beta^{n+3}}{(z^2+\beta^2)(z^2+4\beta^2)} + \frac{4\beta^{n+1}}{(z^2+4\beta^2)}$$

Now we reduce terms and taking the Inverse ZJ Transform we obtain the desired solution

$$y(t) = \frac{1}{4} - \frac{1}{3}cos(t) + \frac{25}{12}\cos(2t)$$

Example 10

$$\frac{dy}{dt} + 2y + 5\int_0^t y(\tau)d\tau = u(\mathbf{t})$$

With $y(0) = 0$ Using the ZJ Transformation, taking u(t) as a constant, we have;

$$ZJ\left[\frac{dy}{dt}\right] + ZJ[2y] + ZJ\left[5\int_0^t y(\tau)d\tau\right] = ZJ[\mathbf{u}(\mathbf{t})]$$

$$\left[-y(x,0)\frac{\beta^n}{z} + \frac{z}{\beta}\hat{\varphi}\right] + 2\hat{\varphi} + 5\left(\frac{\beta}{z}\right)\hat{\varphi} = u(t)\frac{\beta^{n+1}}{z^2}$$

$$\hat{\varphi} = \frac{u(t)\beta^{n+2}}{z(z^2+5\beta^2+2z\beta)}$$

Taking the Inverse ZJ Transform we get

$$= \frac{1}{2}\left[\frac{2u(t)}{(z^2\beta^2+2z\beta+5)}\right] = \frac{1}{2}\left[\frac{2u(t)}{(z\beta+1)^2+4}\right] = \frac{u(t)}{2}[e^{-t}sen(2t)]$$

With the desired solution, taking u(t) constant, check the solution yourself which is the one obtained.

$$y(t) = \frac{u(t)}{2}[e^{-t}sen(2t)]$$

Conclusions

With these examples we can see that the transformation helps to solve some types of trigonometric functions under the sign of the improper integral, integral equations, integro-differential equations, the relevance of the transform is the similarity with the Laplace transform and the other variants that come out of it, in some mathematical aspects it supports to solve more quickly the expression in question.

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