



Spatial Genius: How 3D Packing Transforms Shipping

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Abstract: The article covers how three-dimensional packing affects the shipping cost under dimensional weight pricing, and the algorithmic plus engineering solutions through which packing geometry is converted into a direct economic effect. Its objective is to make a 3D packing algorithm e-commerce ready, compare constructively with metaheuristic approaches, estimate ML and DRL roles in online formulations, justify integration of stability validators and compatibility rules, and sketch an end-to-end scheme for embedding into a WMS with early rating of candidates. The relevance is driven by rising parcel volumes and carrier practices that charge for the maximum of actual and dimensional weight, which turns every extra centimeter into an overpayment and raises the bar for millisecond decisions on the fulfillment line. The novelty lies in formulating the objective as a forecasted cost that accounts for carrier divisors and zonal rates, and in describing a hybrid cascade in which fast heuristics of the EB-AFIT class and their implementations in 3DContainerPacking and Sharp3DBinPacking provide a baseline plan, while ALNS and GA improvers, including a GAN+GA hybrid, as well as DRL policies and CP-SAT validators, bring the layout to a technologically feasible and economically best online solution. The traditional 3D case is NP-hard, hence the domination of very pointwise candidate constructive heuristics usable at predictable runtimes in operation improved by metaheuristics and learnable policies under tight time budgets, explicit stability and compatibility checking for physical feasibility changing the objective from proxy metrics-boxes or fill rate-to forecasted cost instead removes choice paradoxes and minimizes payments integration into a WMS with early rating, telemetry of actual dimensions and closed verification loop material consumption and trips which lowers carbon footprint while keeping throughput. The article will be helpful to WMS and fulfillment system architects, warehouse and transportation operations managers, combinatorial optimization researchers, e-commerce packing solution developers, and .NET stack integrators.

Keywords: three-dimensional packing, dimensional weight, online packing, EB-AFIT, 3DContainerPacking, Sharp3DBinPacking, ALNS, genetic algorithms

1. Introduction

Modern shipping pays for volume rather than kilograms; therefore, any “air” in the box converts into direct costs. Carriers compute dimensional weight as length times width times height divided by a coefficient, then compare it with actual mass and charge the larger of the two values. FedEx guides for parcels in inches specify a divisor of 139 (FedEx, 2024), while DHL materials for centimeters use a divisor of 5000 (DHL, 2023). This makes size and packing the first levers for cost control, since extra centimeters quickly turn into payable “kilograms” in the carrier’s rate grid, even if the parcel is light.

The parcel market itself sets the scale of the problem; order flow is growing, and competitive pressure on rates forces more precise accounting of cubic centimeters. According to the annual Pitney Bowes index, 22.37 billion parcels were shipped in the United States in 2024, which is 3.4 percent more than in 2023, with shares shifting among operators (Pitney Bowes, 2024). This dynamic makes inefficient packing too expensive, since with mass unchanged, every extra liter of volume increases the billed weight and, consequently, the payment.

Against this backdrop, three-dimensional packing has ceased to be an academic curiosity and has become a mandatory fulfillment technology. In e-commerce, it is often necessary to select from many fixed container sizes and to do so online, when items arrive sequentially without knowledge of future items. Contemporary research describes precisely such formulations, online 3D packing with stability checks and orientation constraints, and demonstrates practical applicability in warehouses where seconds of computation and predictability of results matter (Nguyen & Nguyen, 2023). Development proceeds along two lines: fast heuristics for guaranteed response times and hybrids with deep learning to raise fill, which is confirmed by systematic articles and by new packages for comparing algorithms on real flows.

The criticality of 3D packing manifests not only in shipping price but also in chain resilience. Properly chosen containers and tight packing reduce material consumption and the number of trips at the same revenue, which lowers the carbon footprint per parcel. Notably, major players link packing optimization to measurable effects, for example, Amazon reports a 43 percent reduction in average packaging weight per shipment since 2015 and the avoidance of more than 3 million tons of packaging materials, including accelerating the shift from



plastic fillers to paper (Amazon, 2023). For the market, this signals that spatial packing optimization, including algorithmic cartonization, is becoming the norm rather than a one-off initiative.

2. Materials and Methodology

The materials and methodology of the study were based on a multilayer analysis of algorithmic, engineering, and economic aspects of applying three-dimensional packing to reduce shipping costs. The theoretical part was formed from systematic reviews of offline and online formulations of the problem (Ali et al., 2022; Wu et al., 2023), including the specifics of e-commerce, where items arrive in a streaming mode and the solution must fit millisecond budgets. Key algorithmic features in the three-dimensional case, six allowable orientations, stack stability, and compatibility by weight and shape were validated through comparative analysis with two-dimensional cutting and packing problems (Guo et al., 2022). Examples of constructive heuristics include EB-AFIT, which is implemented as a part of the 3DContainerPacking library (Nugent, 2022) and used here as baseline technology together with alternatives available in Sharp3DBinPacking, both for benchmarking purposes and for protection against worst cases.

Methodologically, it was carried out as a combination of several lines of research. First, there was a comparative analysis between heuristics and metaheuristics: First Fit and Best Fit with different sorting profiles were tested together with GA, HALNS, and hybrid approaches GAN+GA (Zhang et al., 2024) to increase fill density as well as reduce the number of boxes used. The second is an assessment highlighted in integrating machine learning and reinforcement learning, whereby DRL models capable of online formulations of predicting the order and orientation of items under physical constraints are reviewed (Murdivien & Um, 2023; Xiong et al., 2024). Third, the potential of constraint programming was studied, used to formalize complex packing rules and quickly filter inadmissible configurations, relying on CP-SAT models (Wróbel, 2023; Liu et al., 2025).

3. Results and Discussion

The classical three-dimensional packing formulation is simple: given a set of rectangular items and one or more rectangular containers, pack all items without overlaps or boundary violations, with the goal usually to minimize the number of containers or unused volume, see figure 1. Under these minimum constraints, the problem is NP-Hard. This rules out a general polynomial algorithm that can be used for significant inputs; therefore, heuristics and approximate methods are what have to be used in practice. It is stated unambiguously by rigorous reviews and fundamental works on 3D packing that this property, together with typical constraint set objective functions and canonical tests, makes a stable foundation when engineering solutions in logistics (Wu et al., 2023).

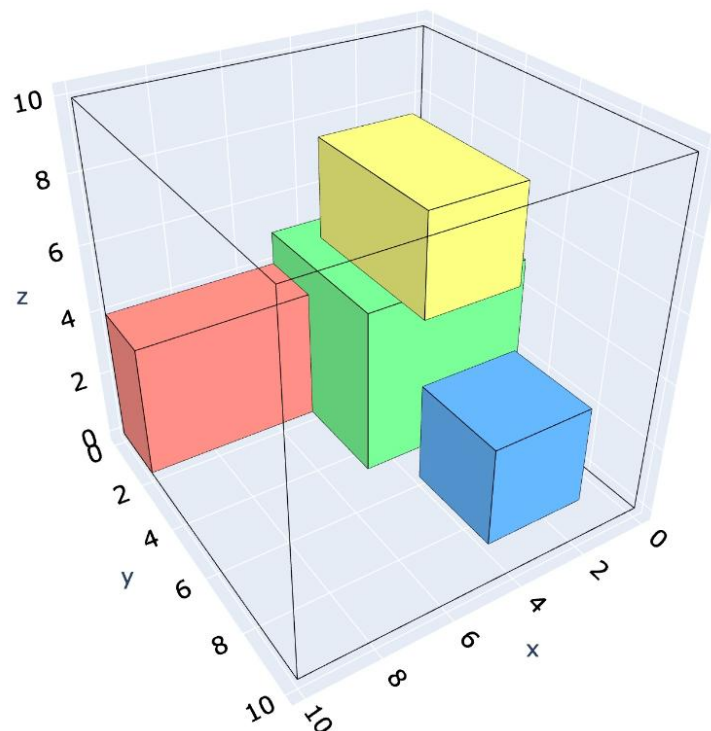


Figure 1: Example of 3D BPP (Wu et al., 2023)



The differences between the two-dimensional and three-dimensional cases are not just a matter of the additional coordinate. In three dimensions, for example, there are six feasible orthogonal orientations per item to be considered. Gravity stack stability has to be checked by layer load distribution per layer, including fragility and compatibility rules (i.e., heavy objects cannot be placed on light or deformable ones). Guillotine cutting constraints and strategies play significant roles in 2D, but must control contact surfaces and centers of mass in 3D; otherwise, geometrically correct packing may not be physically feasible. Current practice in 3D packing with stacking and fragility constraints already shows practically functional decompositions into stacking and planar subproblems. A comparative review in 2D shows that the jump from two to three dimensions makes the problem much more complex when orientation and stability need to be considered (Guo et al., 2022).

In a real warehouse, the distinction between offline and online modes is key. Offline means the entire batch is known in advance, and a globally best configuration can be sought. Online means positions arrive sequentially, and a decision for the current item must be made without knowledge of future ones, which is typical for streaming cartonization, robotic kitting, and wave sorting. A per-item time is valued for the online mode. Greedy with stability check and fast computation of admissible extreme points preferred, offline allowing heavier multistart searches and metaheuristics. Record quality improvements within millisecond time budgets directly fit to industrial fulfillment, where any mistake immediately turns into billable volume, and rework on the line has recently been enabled by the newest reviews on online 3D packing and new work on fast stability validation (Gao et al., 2025). The Online bin packing framework is illustrated in Figure 2.

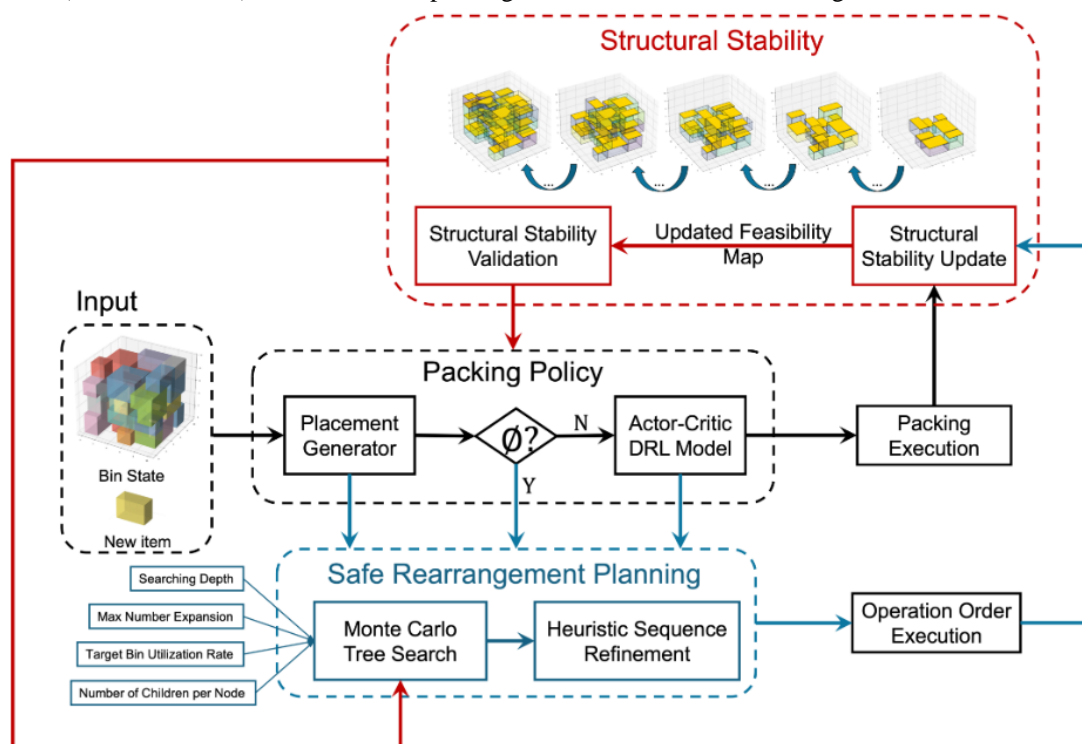


Figure 2: Online bin packing framework (Gao et al., 2025)

In real packing systems, constructive heuristics come to the fore. They provide predictable runtimes and acceptable fill for fixed container sizes. Classical First Fit and Best Fit choose a position for the following item by simple occupancy and residual volume criteria, and their effectiveness depends strongly on the sorting order by size, weight, or side ratios. More sophisticated plans use many potential placement spots, for example, extreme points and container corners, so that unsuitable positions can be disposed of quickly. What came out is what is being used in EB-AFIT, a heuristic enumerator of orthogonal orientations. It supports fixed box sizes with a relatively good tradeoff between time utilization and the number of boxes used. Public implementations prove these properties as handy extension points for orientation as well as compatibility constraints (read 3DContainerPacking for .NET and forks on GitHub), plus the C# port of Kris' family-of-heuristics in Sharp3DBinPacking (Nuget, 2022). Review articles on online and offline 3D packing identify such greedy, extremely pointwise methods as the industrial standard for streaming cartonization, where milliseconds and solution stability are crucial (Ali et al., 2022).



When constructive rules are not enough, metaheuristics are used. They add search over the permutation space and improve initial layouts through local and global mutations. Genetic algorithms and their hybrids with domain solution generators show a consistent gain in the number of containers and average occupancy, especially on heterogeneous size sets. A recent paper in Scientific Reports, combines GAN with GA. Less containers used than pure baseline algorithms on the same set of benchmarks proves the value of generating diverse yet structurally meaningful initial populations, as seen in Figure 3 (Zhang et al., 2024).

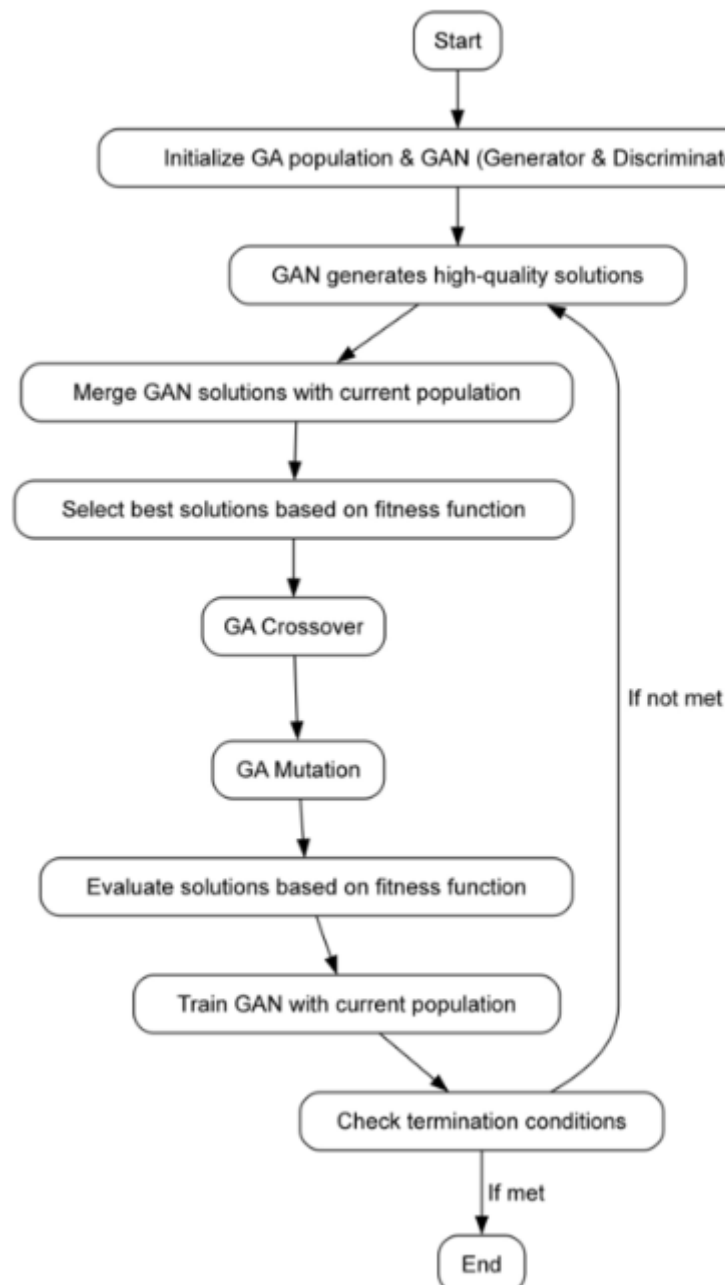


Figure 3: The interplay between GAN and GA (Zhang et al., 2024)

Popular for e-commerce problems with multiple types of box sizes and soft time limits are hybrid adaptive large neighborhood search methods, HALNS. They run by smashing and patching the solution in alternation and on a scale of thousands of positions under small time windows. Multidimensional bin packing benchmarking confirms near-optimal utilization by ALNS, GA, and their hybrids with moderate time if wisely choosing the operator set and selection criteria (Zhang et al., 2024).

Machine Learning enhances both types by training to predict the packing order and direction, either as a straightforward choice policy or by hints on which applicant point to pick next. In online formulations, meaning by the consecutive arrival of items, in the environment of reinforcement learning, packing can be defined as a



series of agent actions that need maximization of fill under the constraints of physical feasibility and stacking rules. There exist DRL approaches within millisecond budgets that maintain stack stability in two-stage schemes that have to jointly select item index and orientation (Murdivien & Um, 2023). Another location is much more robust against lousy sequences; adjustable robust optimization gets much closer to average and worst-case performances and reduces quality dropouts under input permutation. Generalization towards new size distributions adopts transformer policies pre-trained on earlier order data fine-tuned with just a very few steps on the newer SKU groups. A domain heuristic brings the solution to a technologically usable form (Xiong et al., 2024).

Packing rules that are complex and hard to embed into constructive heuristics can be expressed via constraint programming. In this paper, we explicitly state constraints (non-overlap, pairwise bans, vertical support, mass centering, layer balance) solved by hybrid CP-SAT methods. CP-SAT in OR-Tools is based on merging the strengths of CP and SAT. It allows search scale to the size of the problem, indicator constraints available only at the full strength of Boolean propagation, and reasonable control over quality-time tradeoffs. Most models draw integer variables for coordinates and orientation; non-overlap maps as a set of disjunctions. Recent work on container loading proves that the CP model with block structure and decomposition to stacking subproblems solves realistic instances and is in competition with MIP and metaheuristics when stability rules plus joint optimization over several objectives are essential (Wróbel, 2023). CP models find selective use in practice, validation of heuristic plans, and solving small but strictly regulated batches. As a component in hybrids with ALNS and DRL, they can be used for providing very fast pruning of inadmissible configurations (Liu et al., 2025).

Practical cartonization on .NET is conveniently built around two libraries, each covering its part of the tasks, and together they provide flexibility under warehouse constraints. 3DContainerPacking runs the EB-AFIT heuristic, which uses a static catalog of box sizes and generates a placement from extreme points by enumerating all allowed orthogonal orientations of the item. This method works very well in online packing when a predictable time, milliseconds per item, and stable volume utilization are a requirement. The fill percentage very seldom degrades with moderate assortment and controlled size variability. In heterogeneous flows, it is convenient to maintain multiple sorting profiles, by descending longest edge, by base face area, by mass density, and to switch them by order type or by operational telemetry.

Sharp3DBinPacking fills in this view with a set of other plans, helping to check and protect against the worst cases. Different quick methods react in different ways to size spreads and limits; hence, it makes sense to cover both sets in one link, where the plan choice, sorting order, and candidate filtering rules are set as parts. In real-life systems, it helps to keep a flow: a quick building method acts as the first plan, then, if there is time left, a finder is started, for instance, local search or changes of the last k spots. This reduces the chance of opening an extra box on complex combinations.

Orientation constraints and compatibility rules are more conveniently specified at the item model and placement validator levels. For orientations, an allowable subset of the six orthogonal poses is set, or a trick with equal sizes along two axes is used to exclude undesired rotations without branching. Compatibility rules include stacking bans, heavy under light is not allowed, minimum support surface areas, center of mass constraints, offsets from walls, keep-out zones for labels and flaps. It throws out positions that are not physically possible, then checks physical feasibility, vertical support, and layer load before applying the cost function. This saves time, thus making behavior predictable for operators. In typical use, both libraries fall into either the milliseconds or seconds category for dozens and sometimes hundreds of positions. Memory consumption is low because there is no storage of large state populations when using constructive methods. Pros include high speed, simple interpretability of solutions, natural support for fixed boxes, and fine-tuning of selection criteria. Cons include sensitivity to sorting order and to unfavorable size ratios, possible opening of an additional box in some instances, and a limited search horizon without a pluggable refiner. These limitations are substantially mitigated by heuristic cascades, adaptive changes of sorting profiles, and targeted local permutations of the last items.

Integration into a WMS relies on a simple data flow. The system stores a catalog of containers with internal dimensions and weight limits, an SKU directory with sizes in standardized units, weights, a set of allowable orientations and flags, and a set of global rules, stacking constraints, minimum supports, and compatibility. In the packing part of assembling an order, the system requests the packer by choosing one or more possible boxes, sending back a list of items with their quantities and constraints, and then receives a packing plan plus attributes for further usage in label printing, pick-by-light station indication, as well as robot gripper control. It also sends back errors and exceptions, such as unpacked leftovers, which come in as diagnostic codes for scenarios like automatic escalation box selection, larger-box splitting-the-batch-and-manual-top-off-packing.

The packing result is used simultaneously in several subsystems. Coordinates and orientations feed a 3D viewer and operator prompts. Layer and per-layer mass go to stability control. The packing order is translated



into step-by-step instructions and robot commands if the line is automated. Based on the chosen box, the WMS generates packing slips and shipping labels and passes dimensions and weight to the rating module, which in turn returns the final cost, service class, and carrier-specific constraints. It is vital to record actual sizes and weights via telemetry to correct the catalog and reduce discrepancies between plan and fact.

Connecting carrier rate calculations is best done at the candidate evaluation stage, not afterward. Then the objective function minimizes forecasted cost rather than the number of boxes. To this end, the rating module computes billed weight from box dimensions and the carrier divisor, compares it with actual mass, applies zonal rates, minimum billable weights, and service class surcharges, adds the cost of consumables, and, if necessary, oversize fees. The packer receives the price for each candidate from the rater and chooses the configuration with the minimum cost subject to constraints. This provides a direct solution to the business metric, as it does not allow a smaller box to be more expensive due to an unfavorable aspect ratio and divisor. The direct solution optimizes the actual business metric, not its proxy thereby quickening the payback period and reducing exceptions on the line.

Eventually, the engineer gets a coherent set of tools. Constructive heuristics cover streaming cases with fixed bins. Fill-improving metaheuristics for difficult batches are provided by ML in the loop which provides assortment shift and disorder penalties as well as constraint programming support for explicit rule connections to safety and physical possibility of packing.

This mixture gives the needed steadiness between quickness, quality, and meeting constraints; thus, it directly impacts the expense, sustainability, and dependability of the supply chain.

4. Conclusion

Three-dimensional packing confirms its practical significance: under dimensional weight pricing, every extra centimeter transforms into payable kilograms; therefore, geometry optimization directly reduces payments and material intensity. The market amplifies this effect, growing parcel flow makes inefficient packing too expensive, and leaders' practices link cartonization to measurable outcomes. A reduction in average packaging weight per shipment and the saving of millions of tons of materials indicate a steady trend toward embedding such technologies as a fulfillment norm.

On the algorithmic side, limits and solutions are fixed: the classical 3D formulation is an NP-hard problem, so in real-world usage, fast constructive heuristics with predictable time dominate operational flows plus refiners and learnable policies. Major differences from 2D to 3D, six allowed orientations, stability checks, fragility, and compatibility impose a set of placement rules, extreme point generation, and physical feasibility validation. The mode has to make decisions in milliseconds when online; offline permits heavier searches. The tool portfolio has extremely pointwise heuristics of the EB-AFIT class, plus their implementations found inside 3DContainerPacking as well as Sharp3DBinPacking, GA, together with HALNS metaheuristics for higher fill, GAN+GA hybrids, not forgetting ML and DRL for order plus orientation selection. CP-SAT is applied to formalize hard constraints as a point solver, validator, or component inside hybrid cascades.

Integration into a WMS turns theory into economics: a catalog of container sizes and weight limits, standardized SKU sizes and weights, allowable orientations and compatibility rules, a geometry and stability validator, coordinates and packing orders for operators and robots form a closed loop with telemetry of actual dimensions. The key decision, early rating of candidates, shifts the objective from proxy metrics to forecasted cost that accounts for carrier divisors, zonal rates, minimum billable weights, and surcharges, which eliminates paradoxes of choosing a smaller but more expensive box. As a result, a hybrid stack, fast constructive strategies at the input, metaheuristic refiners, and learnable policies as time allows, and CP-SAT for complex rules, simultaneously provides speed, quality, and constraint compliance; therefore, it makes spatial packing optimization a mandatory standard of modern e-commerce.

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