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OPTIMIZING AN IMPERFECT INVENTORY MODEL WITH RANDOM DEFECTIVE RATE, REWORK, BACKORDER AND INSPECTION PROCESS

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Abstract: In inventory every production batch contains a fraction of defective items. This paper discusses the economic production quantity inventory model with rework process at a single stage production system with planned backorders. The imperfect quality items are detected from the inspection process, so we added the inspection cost in the total system cost along with we developed the inventory model with three different density functions such as uniform, triangular and beta. The numerical examples are given to illustrate the proposed inventory models.
Keywords: EPQ, imperfect production system, rework, inspection cost, planned backorders.

INTRODUCTION

The companies have to choice good decisions regarding to inventories in order to survive and boost in the fierce and competitive businesses. The first inventory model was introduced by Harris [1] in the year 1913. And perhaps the second inventory model best known as Economic Production Quantity (EPQ) or Economic Manufacturing Quantity (EMQ) was developed by Taft [3] in May of 1918. These two models have been extended and further developed by several researchers and scholars. Chiu et al [3] discussed the effect of random defective rate and imperfect rework process in an EPQ model. Jamal et al., [4] explained an EPQ inventory model that determines the optimal lot size in a single-stage production system in which rework is done by addressing two different operational policies immediate rework and rework after N production cycles. In similar way, the following researchers such as Lo et al., [5], Sarkar et al., [6, 11-14], Cardenas Barron [7-10], Chung [15], Khan et al., [16], Sarkar [17, 18, 25], Voros [19].

Wee and Widyadana [20], Widyadana and Wee [21]. Chang et al., [22] and Cardenas – Barron et al., [23, 24], have concentrate their research work under the concept of inventory models with rework. Sana and Chaudhuri [26] explained an EMQ model in an imperfect production process. Roy et al., [27] discussed an economic order quantity model of imperfect quality items with partial backlogging. But Cardenas-Barron et al., [23, 24], assume that the proportion of defective product is a constant and known. Wee and Widyadana [28] using a first in first out (FIFO) rule developed an inventory model with stochastic preventive maintenance time and rework process. Pal et al., [30] derived a Mathematical Model an EPQ with stochastic demand in an imperfect production system. Pasandideh et al., [31] developed two different algorithm to optimize a bi-objective multi-product EPQ model with defective items, rework and limited orders. In this direction several researches like Sana [33-36], Sarkar and Sarkar [37] extended the inventory model with several extensions.

Cardenas – Barron [9] assumed that the defective rate is known and constant. It is well known that in any imperfect production system of real life has random defective rates. Following his Sarkar and Sarkar [38] extends the inventory model to allow random defective rates and consider three different inventory models are developed for three different density functions such as uniform, triangular and beta.

In our proposed model, we extend Sarkar and Sarkar [38] inventory model with following concepts. Usually the imperfect items are detected from the inspection process and also the inventory contains surplus amount of items due to the reworking process of repaired items and backordered items. Therefore, we add the inspection cost and overage cost in the total system cost and also we discussed the impact of the inclusion in this paper.

The outline of this paper is presented as follows. Section 2 we present a notations and assumptions of a Mathematical Model. Section 3 we develop a mathematical model for the three distribution density functions. Numerical examples are done in Section 4. Finally we conclude the paper in Section 5.

2. NOTATIONS AND ASSUMPTIONS

We consider the following notations and assumptions in this model.

2.1. Notations :

The notations that will be used throughout this paper are given below.

Q batch size (units) (decision variable)

B	size of backorders (units) (decision variable)
D	demand rate, units per time
P	production rate, units per time (P > D)
K	cost of production setup (fixed cost) $ per setup
C	Manufacturing cost of a product $ per unit
H	inventory carrying cost per product per unit of time, H = ic + W
i	inventory carrying cost rate, a percentage
w	other inventory costs
W	backorder cost per product per unit of time (linear backorder cost)
F	backorder cost per product (fixed backorder cost)
J	backorder average (units)
T	time between production runs
F′	fixed transportation cost per shipment $ per order
CI	inspection cost $ per unit
c′	cost of an item $ per unit
v	salvage value $ per unit

TC(Q,B)total cost per unit of time

In addition to these notations, we define the following symbols.

$\bar{I}$	inventory average (units)
I_{max}	maximum inventory (units)
R	proportion of defective products in each cycle follows a probability distribution (uniform, triangular and beta)
E[R]	Expected value of proportion of defective products in each cycle.

2.2. Assumptions

We consider the following assumptions to make this model.

1. The model is considered for single type of item.
2. Demand and production rate are constant and known over horizon planning. The production rate is greater than demand rate, hence there is no shortage.
3. All products are screened and the screening cost is added in the total system cost.
4. The proportion of defective products is random variable in each production cycle and it follows three different distribution density functions.
5. There is no scrap items within a cycle and all defective products are reworked to make the perfect quality products.
6. Two types of backorder cost are considered : linear backorder cost (backorder cost is applied to average backorders) and fixed backorder cost (backorder cost is applied to maximum backorder level allowed)
7. There are unlimited inventory storage space and the availability of capital.
8. Inventory holding costs are based on the average inventory.
9. Production and reworking are done in the same manufacturing system at the same production rate.
10. The planning horizon is infinite.

3. MATHEMATICAL MODEL

Based on the assumptions and the above notations we develop the inventory model considering that the proportion of defective products follows. (A) Uniform distribution, (B) triangular distribution and (C) beta distribution.

3.1. Case A: The proportion of defective products follows a uniform distribution

In order to take the randomness of proportion of defective products into account, the expected value of R is used in the development and analysis of inventory model. The inventory behavior through time is represented in Figure 1. According to fig.(1) the maximum inventory I_{max} is simply computed as the sum of I_1 + I_2 from triangle (146), it is easy to see that

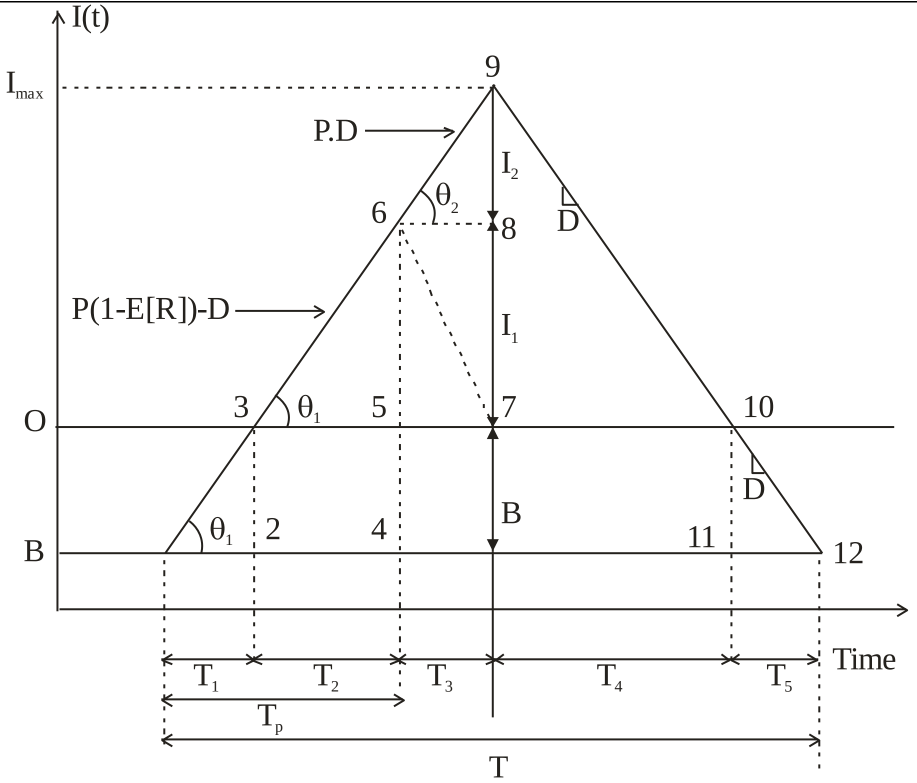

Fig.(1) Inventory behavior for the EPQ with rework at the same cycle and planned backorders

where T_3 is the production time of producing the defective products. Therefore T_3 must be equal to E[R] Q/P. Thus,

$$I_2 = T_3(P - D) = \frac{Q(a+b)(P-D)}{2P} = \frac{Q(a+b)\left(1-\frac{D}{P}\right)}{2}$$

Now the maximum inventory I_{max} can be obtained as summing I_1 and I_2, hence

$$\tan\theta_1 = P(1 - E(R)) - D = \frac{I_1 + B}{(T_1+T_2)}$$

In this case, it is assumed that the R follows a uniform distribution with range {a, b} where parameters a and b are the inferior and superior limits respectively of the uniform distributions. As both limits represent a proportion of defective products, obviously they must satisfy the following constraints : $0 < a < b < 1$. For a uniform distribution it is well known that the expected value for R is given as $E[R] = \frac{(a+b)}{2}$.

Furthermore, the production time of producing Q units is $T_p = T_1 + T_2$. Therefore, $T_1 + T_2$ must be equal to Q/P substituting the expected value E[R] and $T_1 + T_2$.

We obtain

$$P\left(1-\frac{a+b}{2}\right) - D = \frac{I_1+B}{T_1+T_2}$$

(or)

$$I_1 = Q\left(\frac{2-a-b}{2} - \frac{D}{P}\right) - B = Q\left(1-\frac{a+b}{2}-\frac{D}{P}\right)B$$

According to triangle (689), it is easy to see that,

$$\tan\theta_2 = P - D = \frac{I_2}{T_3}$$

i.e., $$T_3 = \frac{E[R]Q}{P} = \frac{Q}{P}\left(\frac{a+b}{2}\right)$$

$$I_{max} = I_1 + I_2$$

$$= Q\left(1-\frac{a+b}{2}-\frac{D}{P}\right)-B+\frac{Q(a+b)\left(1--\frac{D}{P}\right)}{2}$$

$$= Q\left[1-\frac{D}{P}\left(1+\frac{a+b}{2}\right)\right]-B$$

with regard to inventory average $\bar{I}$, this can be calculated by the sum of the area of following triangles : (356), (567), (679) and (7910) divide by T. From Fig.(1) we know that T is the sum of T_1, T_2, T_3, T_4 and T_5. It is well known that T is the time between runs. Furthermore, T is also the time needed to consume all Q units at rate D. Therefore, it is easy to show that T is equal to Q/D. Now, we will determine the area of above mentioned triangles as follows. From triangle (356), one obtains T_2 as

$$T_2 = \frac{Q[(1-E(R)-D/P]-B}{[P(1-E(R)-D]}$$

$$= \frac{Q\left[\left(1-\frac{a+b}{2}\right)-\frac{D}{P}\right]-B}{\left[P\left(1-\frac{a+b}{2}\right)-D\right]}$$

where T_2 is the time needed to build up I_1 units in inventory. As it was stated before, T_3 is equal to E[R]Q/P = ((a + b)/2)Q/P. Then the area of triangles : (356), (567) and (679) are given by

Area of triangle (356) $$= \frac{T_2\ I_1}{2}$$

$$= \frac{\left\{Q\left[\left(1-\frac{a+b}{2}\right)-\frac{D}{P}\right]-B\right\}^2}{2\left[P\left(1-\frac{a+b}{2}\right)-D\right]}$$

Area of triangle (576) $$= \frac{T_3\ I_1}{2}$$

$$= \frac{\left(\frac{a+b}{2}\right)Q\left\{Q\left[\left(1-\frac{a+b}{2}\right)-\frac{D}{P}\right]-B\right\}}{2P}$$

Area of triangle (679) $$= \frac{T_3\ I_{max}}{2}$$

$$= \frac{\left(\frac{a+b}{2}\right)Q\left\{Q\left[\left(1-\frac{a+b}{2}\right)-\frac{D}{P}\right]-B\right\}}{2P}$$

From the triangle 7910, T_4 as

$$T_4 = \frac{Q\left[1-(1+E[R])-\frac{D}{P}\right]-B}{D}$$

$$= \frac{Q\left[1-\left(1+\left(\frac{a+b}{2}\right)\frac{D}{P}\right)\right]-B}{D}$$

where T_4 is the time needed for consumption at hand maximum inventory level I_{max}, then

$$\text{Area of triangle (7910)} = \frac{T_4\ I_{max}}{2}$$

$$= \frac{\left\{Q\left[\left(1-\left(1+\frac{a+b}{2}\right)\frac{D}{P}\right)\right]-B\right\}^2}{2D}$$

Finally the inventory average $\bar{I}$ can be calculated summing the area of triangles (356), (567), (679) and (7910) : and divided by T. Hence, one obtains

$$\bar{I} = \frac{1}{T}\left[\frac{\left\{Q\left[\left(1-\frac{a+b}{2}\right)-\frac{D}{P}\right]-B\right\}^2}{2\left[P\left(1-\frac{a+b}{2}\right)-D\right]}\right]+\left(\frac{a+b}{2}\right)Q\left\{Q\left[1-\frac{a+b}{4}-\left(1+\frac{a+b}{4}\right)\frac{D}{P}\right]-B\right\}$$

$$+\frac{\left\{Q\left[\left(1-\left(1+\frac{a+b}{2}\right)\right)\frac{D}{P}-B\right]\right\}^2}{2D}$$

Simplifying the expression, it reduces

$$\bar{I} = \frac{1}{2Q\left[\left(1-\frac{a+b}{2}\right)-\frac{D}{P}\right]}\left[\left(Q^2+B^2\right)\left(1-\frac{a+b}{2}\right)+\frac{Q^2D^2}{P^2}\left(1+\frac{a+b}{2}+\left(\frac{a+b}{2}\right)^2\right)\right.$$

$$\left.+\frac{Q^2D}{P}\left(\left(\frac{a+b}{2}\right)^3-2\right)+2BQ\left(\frac{D}{P}+\frac{a+b}{2}-1\right)\right]$$

In order to express the above mathematical equation in a more compact expression, let us define the following symbols :

$$A = 1-\frac{a+b}{2}$$

$$E = 1-\frac{a+b}{2}-\frac{D}{P}$$

$$I = \left[1+\frac{a+b}{2}+\left(\frac{a+b}{2}\right)^2\right]\left(\frac{D^2}{P^2}\right)$$

$$O = \left[\left(\frac{a+b}{2}\right)^3-2\right]\left(\frac{D}{P}\right)$$

$$U = \frac{D}{P}+\frac{a+b}{2}-1 = -E$$

then

$$\bar{I} = \frac{1}{2QE}\left[Q^2\left(A+I+O\right)+B^2A - 2BQE\right]$$

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$$\bar{I} = \frac{Q}{2}\left[\left(\frac{A+I+O}{E}\right)\right] + \frac{B^2 A}{2QE} - B$$

with further rearrangement,

$$\bar{I} = \frac{Q}{2}\left[1-\left(1+\frac{a+b}{2}+\left(\frac{a+b}{2}\right)^2\right)\left(\frac{D}{P}\right)\right] + \frac{B^2\left(1-\frac{a+b}{2}\right)}{2Q\left(1-\frac{a+b}{2}-\frac{D}{P}\right)} - B \qquad \text{. . . (1)}$$

if we define L as :

$$L = 1-\left(1+\frac{a+b}{2}+\left(\frac{a+b}{2}\right)^2\right)\left(\frac{D}{P}\right)$$

Finally, the inventory average is given as follows.

$$\bar{I} = \frac{Q}{2}L + \frac{B^2 A}{2QE} - B \qquad \text{. . . (2)}$$

with regard to inventory average of backorders J it can be determined by the sum of the area of triangles : (123) and (101112), and divided by T. From triangle (123), T_1 can be stated as :

$$T_1 = \frac{B}{[P(1-E[R])-D]} = \frac{B}{\left[P\left(1-\frac{a+b}{2}\right)-D\right]} \qquad \text{. . . (3)}$$

where T_1 is the time needed to satisfy the backorders level once production process is started again. Thus,

$$\text{Area of triangle 123} = \frac{T_1 B}{2} = \frac{B^2}{2\left[P\left(1-\frac{a+b}{2}\right)-D\right]}$$

From triangle (101112), T_5 is given by

$$T_5 = \frac{B}{D}$$

where T_5 is the time needed to build up the backorders level of B units. So,

$$\text{Area of triangle 101112} = \frac{T_5 B}{2} = \frac{B^2}{2D}$$

Thus, the inventory average of backorders J can be calculated adding the area of triangles : (123) and (101112), and divided by T.

∴ The average backorder J obtains,

$$J = \frac{1}{T}\left[\frac{B^2}{2\left[P\left(1-\frac{a+b}{2}\right)-D\right]} + \frac{B^2}{2D}\right] \qquad \text{. . . (4)}$$

$$J = \frac{B^2\left(1-\frac{a+b}{2}\right)}{2Q\left[\left(1-\frac{a+b}{2}\right)-\frac{D}{P}\right]} \qquad \text{. . . (5)}$$

$$J = \frac{B^2 A}{2QE} \qquad \text{. . . (6)}$$

Therefore, the total cost of the system by considering setup cost, inventory cost, backorder cost, production cost, transportation cost, overage cost and inspection cost as follows.

$$TC(Q, B) = \frac{KD}{Q} + HG + \frac{FBD}{Q} + WJ + CD(1 + E[R]) + \frac{F'D}{Q} + \frac{(C'-V)D}{Q} + CID \quad \ldots (7)$$

After substituting the value of above expresses in equation (7), we obtain,

$$TC(Q, B) = \frac{KD}{Q} + \frac{HQL}{2} + \frac{HB^2A}{2QE} - HB + \frac{FBD}{Q} + \frac{WB^2A}{2QE}$$

$$+ CD(2-A) + \frac{F'D}{Q} + \frac{(C'-V).D}{Q} + CI\,.\,D \quad \ldots (8)$$

The cost functions consists of two decision variables as Q and B

If we differentiate the eq.(8) with respect to 'B' we obtain the backorder quantity, B as

$$B^* = \frac{(HQ - FD)E}{(W + H)A} \quad \ldots (9)$$

Substituting the values of B in equation (8) and differentiate the equation with respect to 'Q', we get the optimal order quantity, Q as

$$Q^* = \sqrt{\frac{2D\left[K + F + (C'-V)\right](W + H)A \; - \; F^2D^2E}{H\left[AL(W + H) - EH\right]}} \quad \ldots (10)$$

3.2. Case B : The proportion of defective products follows a triangular distribution

In this case, it is assumed that R follows a triangular distribution with parameters [a, b, c] where parameters a and c are the inferior and superior limits respectively and b is the mode of the triangular distribution. As all parameters represent the proportion of defective products, obviously, they must satisfy the following constraint : $0 < a < b < c < 1$. For a triangular distribution, it is well known that the expected value for R is given as $E[R] = (a + b + c)/3$. For this case from Fig.(1), the maximum inventory I_{max} is also computed as the sum of $I_1 + I_2$. From to triangle (146), we know that,

$$\tan \theta_1 = P(1 - E(R)) - D = \frac{I_1 + B}{(T_1 + T_2)}$$

The production time of producing Q units is $T_p = T_1 + T_2$. Therefore, $T_1 + T_2 = Q/P$, substituting the expected value E[R] and $T_1 + T_2$. We obtain,

We obtain

$$P\left(1 - \frac{a + b + c}{2}\right) - D = \frac{I_1 + B}{T_p}$$

(or)

$$I_1 = Q\left[1 - \frac{a + b + c}{2} - \frac{D}{P}\right] - B$$

According to triangle (689),

$$T_3 = \frac{Q(a + b + c)}{3P}$$

where T_3 is the production time of producing the defective products. Thus T3 is equal to E[R]Q/P. Hence,

$$I_2 = Q\left(\frac{a + b + c}{3}\right)\left[1 - \frac{D}{P}\right]$$

Therefore, the maximum inventory I_{max} can be found as,

$$I_{max} = I_1 + I_2$$

$$= Q\left[1 - \frac{D}{P}\left(1 - \frac{a + b + c}{3}\right)\right] - B$$

As before from triangle (356), we obtain T_2 as

$$T_2 = \frac{Q\left[\left(1-\frac{a+b+c}{3}\right)-\frac{D}{P}\right]-B}{P\left(1-\frac{a+b+c}{3}\right)-D}$$

where T_2 is the time needed to build up I_1 units in inventory, as in the previous case, T_3 is equal to E[R]Q/P = Q(a + b + c)/3P.

Thus, the area of triangles : (356), (567) and (679) are given by

Area of triangle (356) $= \frac{1}{2} T_2 \, I_1$

$$= \frac{\left[Q\left[\left(1-\frac{a+b+c}{3}\right)-\frac{D}{P}\right]-B\right]^2}{2\left[P\left(1-\frac{a+b+c}{3}\right)-D\right]}$$

Area of triangle (567) $$= \frac{\frac{Q(a+b+c)}{3}\left[Q\left[\left(1-\frac{a+b+c}{3}\right)-\frac{D}{P}\right]-B\right]}{2P}$$

Area of triangle (679) $$= \frac{\frac{Q(a+b+c)}{3}\left[Q\left[1-\frac{D}{P}\left(1+\frac{a+b+c}{3}\right)\right]-B\right]}{2P}$$

According to the 7910

$$T_4 = \frac{Q\left[1-\frac{D}{P}\left(1+\frac{a+b+c}{3}\right)-B\right]}{D}$$

where T_4 is the time needed for consumption at hand maximum inventory level I_{max}, then

Area of triangle (7910) $= \frac{T_4 \, I_{max}}{2}$

$$= \frac{\left[Q\left[\left(1-\frac{D}{P}\left(1+\frac{a+b+c}{3}\right)\right)\right]-B\right]^2}{2D}$$

Finally the inventory average $\bar{I}$ can be calculated summing the area of triangles (356), (567), (679) and (7910) : and divided by T. Hence, we obtain $\bar{I}_{Tri}$ as

$$\bar{I}_{Tri} = \frac{D}{Q}\left[\frac{\left[Q\left(1-\frac{a+b+c}{3}\right)-B\right]^2}{2\left[P\left(1-\frac{a+b+c}{3}-D\right)\right]} + \frac{\frac{Q(a+b+c)}{3}\left[Q\left(1-\frac{a+b+c}{3}-\frac{D}{P}\right)-B\right]}{2P} + \frac{\frac{Q(a+b+c)}{3}\left[Q\left[1-\frac{D}{P}\left(1+\frac{a+b+c}{3}\right)\right]-B\right]}{2P} + \frac{\left[Q\left[1-\frac{D}{P}\left(1+\frac{a+b+c}{3}\right)\right]-B\right]^2}{2d}\right]$$

In order to express the above mathematical equation in a more compact expression, let us define the following symbols :

$$A_{Tri} = 1-\frac{a+b+c}{3}$$

$$E_{Tri} = 1-\frac{a+b+c}{3}-\frac{D}{P}$$

$$I_{Tri} = \left[1+\frac{a+b+c}{3}+\left(\frac{a+b+c}{3}\right)^2\right]\left(\frac{D^2}{P^2}\right)$$

$$O_{Tri} = \left[\left(\frac{a+b+c}{3}\right)^3-2\right]\left(\frac{D}{P}\right)$$

$$U_{Tri} = \frac{D}{P}+\frac{a+b+c}{3}-1 = -E_{Tri}$$

then

$$\bar{I}_{Tri} = \frac{Q}{2}\left[1-\left(1+\frac{a+b+c}{3}+\left(\frac{a+b+c}{3}\right)^2\right)\left(\frac{D}{P}\right)\right]+\frac{B^2\left(1-\frac{a+b+c}{3}\right)}{2Q\left(1-\frac{a+b+c}{3}-\frac{D}{P}\right)} - B \qquad \text{. . . (11)}$$

L_{Tri} defined as,

$$L_{Tri} = 1-\left(1+\frac{a+b+c}{3}+\left(\frac{a+b+c}{3}\right)^2\right)\left(\frac{D}{P}\right)$$

Finally, the inventory average is given by

$$\bar{I}_{Tri} = \frac{Q}{2}L_{Tri} + \frac{B^2A_{Tri}}{2QE_{Tri}} - B \qquad \text{. . . (12)}$$

From the triangle 123

$$T_1 = \frac{B}{P\left(1-\frac{a+b+c}{3}\right)-D}$$

where T_1 is the time needed to satisfy the backorders level once production process. Hence

$$\text{Area of triangle 123} = \frac{T_1 B}{2} = \frac{B^2}{2\left[P\left(1-\frac{a+b+c}{3}\right)-D\right]}$$

From triangle (101112), T_5 is given by

$$T_5 = \frac{B}{D}$$

where T_5 is the time needed to build up the backorders level of B units. Hence

$$\text{Area of triangle 101112} = \frac{T_5 B}{2} = \frac{B^2}{2D}$$

Thus, the inventory average of backorders J_{Tri} can be calculated adding the area of triangles : (123) and (101112), and divided by T.

J_{Tri} obtains as,

$$J_{Tri} = \frac{1}{T}\left[\frac{B^2}{2\left[P\left(1-\frac{a+b+c}{3}\right)-D\right]} + \frac{B^2}{2D}\right]$$

$$J_{Tri} = \frac{B^2 A_{Tri}}{2QE_{Tri}}$$

Therefore, the total system cost of this distribution is defined as setup cost, inventory cost, backorder cost, production cost, transportation cost, overage cost and inspection cost.

$$TC = \frac{KD}{Q} + HG_{Tri} + \frac{FBD}{Q} + WJ_{Tri} + CD(1 + E[R]) + \frac{F'D}{Q} + \frac{(C'-V)D}{Q} + CI.D$$

After substituting the value of above expression, we obtain,

$$TC(Q, B) = \frac{KD}{Q} + \frac{HQL_{Tri}}{2} + \frac{HB^2A_{Tri}}{2QE_{Tri}} - HB + \frac{FBD}{Q} + \frac{WB^2A_{Tri}}{2QE_{Tri}} + CD(2 - A_{Tri}) + \frac{F'D}{Q} + \frac{(C' - V).D}{Q} + CI\,.\,D \qquad ...(13)$$

Differentiate the above equation with respect to 'B', we get the optimal backorder quantity, as

$$B^* = \frac{(HQ - FD)E_{Tri}}{(W + H)A_{Tri}} \qquad ...(14)$$

After substituting the values of B in equation (13) and differentiate with respect to Q, we get the optimal order quantity, Q as

$$Q^* = \sqrt{\frac{2D\left[K + F + (C' - V)\right](W + H)A_{Tri} \; - \; F^2D^2E_{Tri}}{H\left[A_{Tri}L_{Tri}(W + H) - E_{Tri}H\right]}} \qquad ...(15)$$

3.3. Case C : The proportion of defective products follows a triangular distribution

In this case, it is assumed that R follows a beta distribution with range [α, β] where parameters α and β are the inferior and superior limits respectively of the beta distribution. As both limits represent a proportion of defective products. Obviously, they must satisfy the following constraint : $0 < \alpha < \beta < 1$. For a beta distribution, it is well known that the expected value for R is given as $E[R] = \alpha/(\alpha + \beta)$.

We follow the same procedure to obtain the decision variable Q and B. From fig.(1), the maximum inventory I_{max} is calculated as the sum of $I_1 + I_2$. According to triangle 146, we obtain,

$$P(1 - E(R)) - D = \frac{I_1 + B}{(I_1 + I_2)}$$

As the production time of producing Q units is $T_p = T_1 + T_2$. Hence, $T_1 + T_2 = Q/P$, substituting the expected value E[R] and $T_1 + T_2$. We obtain,

$$P\left(1 - \frac{\alpha}{\alpha + \beta}\right) - D = \frac{I_1 + B}{T_p}$$

(or)

$$I_1 = Q\left[1 - \frac{\alpha}{\alpha + \beta} - \frac{D}{P}\right] - B$$

From the triangle (689),

$$T_3 = \frac{Q\alpha}{P(\alpha + \beta)}$$

where T_3 is the production time of producing the defective products. Thus T_3 is equal to E[R]Q/P. Hence,

$$I_2 = Q\left(\frac{\alpha}{\alpha + \beta}\right)\left[1 - \frac{D}{P}\right]$$

Therefore, the maximum inventory I_{max} can be found as,

$$I_{max} = I_1 + I_2$$

$$= Q\left[1 - \frac{D}{P}\left(1 + \frac{\alpha}{\alpha + \beta}\right)\right] - B$$

From the triangle (356), T_2 can be obtained as

$$T_2 = \frac{Q\left[1 - \frac{\alpha}{\alpha + \beta} - \frac{D}{P}\right] - B}{P\left(1 - \frac{\alpha}{\alpha + \beta}\right) - D}$$

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where T_2 is the time needed to build up I_1 units in inventory, as in the previous case, T_3 is equal to E[R]Q/P = αR/(P(α + β)).

Thus, the area of triangles : (356), (567) and (679) are given by

$$\text{Area of triangle (356)} = \frac{Q\left[1-\frac{\alpha}{\alpha+\beta}-\frac{D}{P}-B\right]^2}{2\left[P\left(1-\frac{\alpha}{\alpha+\beta}\right)-D\right]}$$

$$\text{Area of triangle (567)} = \frac{Q\frac{\alpha}{\alpha+\beta}\left[Q\left(1-\frac{\alpha}{\alpha+\beta}-\frac{D}{P}\right)-B\right]}{2P}$$

$$\text{Area of triangle (679)} = \frac{I_{max}T_3}{2}$$

$$= \frac{Q\frac{\alpha}{\alpha+\beta}\left[Q\left(1-\frac{D}{P}\left(1+\frac{\alpha}{\alpha+\beta}\right)\right)-B\right]}{2P}$$

According to the triangle 7910

$$T_4 = \frac{Q\left[1-\frac{D}{P}\left(1+\frac{\alpha}{\alpha+\beta}\right)-B\right]}{D}$$

where T_4 is the time needed for consumption at hand maximum inventory level I_{max}, then

$$\text{Area of the triangle (7910)} = \frac{\left[Q\left[1-\frac{D}{P}\left(1+\frac{\alpha}{\alpha+\beta}\right)\right]-B\right]^2}{2D}$$

Finally as before, the inventory average $\overline{I}_{Beta}$ can be calculated summing the area of triangles (356), (567), (679) and (7910) and divided by T. Hence, we obtain $\overline{I}_{Beta}$ as

$$\overline{I}_{Beta} = \frac{D}{Q}\left[\frac{\left[Q\left(1-\frac{\alpha}{\alpha+\beta}-D\right)-B\right]^2}{2\left[P\left(1-\frac{\alpha}{\alpha+\beta}-D\right)\right]}+\frac{Q\frac{\alpha}{\alpha+\beta}\left[Q\left(1-\frac{\alpha}{\alpha+\beta}-\frac{D}{P}\right)-B\right]}{2P}\right.$$
$$\left.+\frac{Q\frac{\alpha}{\alpha+\beta}\left[Q\left[1-\frac{D}{P}\left(1+\frac{\alpha}{\alpha+\beta}\right)\right]-B\right]}{2P}+\frac{\left[Q\left[1-\frac{D}{P}\left(1+\frac{\alpha}{\alpha+\beta}\right)\right]-B\right]^2}{2D}\right]$$

In order to express the above mathematical equation in a more compact expression, let us define the following symbols :

$$A_{Beta} = 1-\frac{\alpha}{\alpha+\beta}$$

$$E_{Beta} = 1 - \frac{\alpha}{\alpha+\beta} - \frac{D}{P}$$

$$I_{Beta} = \left[1 + \frac{\alpha}{\alpha+\beta} + \left(\frac{\alpha}{\alpha+\beta}\right)^2\right]\left(\frac{D^2}{P^2}\right)$$

$$O_{Beta} = \left[\left(\frac{\alpha}{\alpha+\beta}\right)^3 - 2\right]\left(\frac{D}{P}\right)$$

$$U_{Beta} = \frac{D}{P} + \frac{\alpha}{\alpha+\beta} - 1 = -E_{Beta}$$

then

$$\bar{I}_{Beta} = \frac{Q}{2}\left(\frac{A_{Beta} + I_{Beta} + O_{Beta}}{E_{Beta}}\right) + \frac{B^2 A_{Beta}}{2QE_{Beta}} - B$$

Simplifying this we obtain

$$\bar{I}_{Beta} = \frac{Q}{2}\left[1-\left(1+\frac{\alpha}{\alpha+\beta}+\left(\frac{\alpha}{\alpha+\beta}\right)^2\right)\left(\frac{D}{P}\right)\right]+\frac{B^2\left(1-\frac{\alpha}{\alpha+\beta}\right)}{2Q\left(1-\frac{\alpha}{\alpha+\beta}-\frac{D}{P}\right)} - B \qquad ...(16)$$

If we define L_{Beta} as

$$L_{Beta} = 1-\left(1+\frac{\alpha}{\alpha+\beta}+\left(\frac{\alpha}{\alpha+\beta}\right)^2\right)\left(\frac{D}{P}\right)$$

The inventory average is given by

$$\bar{I}_{Beta} = \frac{Q}{2}L_{Beta} + \frac{B^2 A_{Beta}}{2QE_{Beta}} - B \qquad ...(17)$$

From the triangle 123

$$T_1 = \frac{B}{P\left(1-\frac{\alpha}{\alpha+\beta}\right)-D}$$

where T_1 is the time needed to satisfy the backorders level once production process. Hence

$$\text{Area of triangle 123} = T_1\ B = \frac{B^2}{2\left[P\left(1-\frac{\alpha}{\alpha+\beta}\right)-D\right]}$$

From triangle 101112, T_5 is given by

$$T_5 = \frac{B}{D}$$

where T_5 is the time needed to build up the backorders level of B units. Hence

$$\text{Area of triangle 101112} = \frac{T_5\ B}{2} = \frac{B^2}{2D}$$

Thus, the inventory average of backorders J_{Beta} can be calculated adding the area of triangles : (123) and (10112) and divided by T.
So the inventory average of backorders J_{Beta} can be obtained as

$$J_{Beta} = \frac{B^2 A_{Beta}}{2QE_{Beta}}$$

Therefore, the total cost of the system by considering setup cost, inventory cost, backorder cost, production cost, transportation cost, overage cost and inspection cost.

$$TC = \frac{KD}{Q}+HG_{Beta}+\frac{FBD}{Q}+WJ_{Beta} + CD(1 + E[R]) + \frac{F'D}{Q}+\frac{(C'-V)D}{Q}+CI.D \qquad ...(18)$$

After substituting the value of above expressions we have

$$TC(Q, B) = \frac{KD}{Q}+\frac{HQL_{Beta}}{2}+\frac{HB^2A_{Beta}}{2QE_{Beta}}-HB+\frac{FBD}{Q}+\frac{WB^2A_{Beta}}{2QE_{Beta}}+CD(2-A_{Beta})$$
$$+\frac{F'D}{Q}+\frac{(C'-V).D}{Q}+CI\ .\ D \qquad ...(19)$$

Differentiate the above equation with respect to ‘B’, we get the optimal backorder quantity, as

$$B^* = \frac{(HQ - FD)E_{Beta}}{(W + H)A_{Beta}} \quad ...(20)$$

After substituting the values of B in the above equation and differentiate the equation with respect to Q, we get the optimal order quantity, Q as

$$Q^* = \sqrt{\frac{2D\left[K + F + (C' - V)\right](W + H)A_{Beta} - F^2D^2E_{Beta}}{H\left[A_{Beta}L_{Beta}(W + H) - E_{Beta}H\right]}} \quad ...(21)$$

4. NUMERICAL EXAMPLES

Example 1 : The values of the following parameters are to be taken in appropriate units : D = 300 units/year, a = 0.03, b = 0.07, P = 550 units/year, W = \$10/unit/year, H = \$50/unit/year, F = \$1/unit short, K = \$50/lotsize, C = \$7/unit, F′ = \$100/order, CI = \$0.1/unit, C′ = \$22/units, V = \$20/units. Then the optimal solution is Q^* = 160 units, B^* = 55 units, TC = \$2980/year.

Example 2 : The values of the following parameters are to be taken in appropriate units : D = 300 units/year, a = 0.03, b = 0.04, c = 0.07, P = 550 units/year, W = \$10/unit/year, H = \$50/unit/year, F = \$1/unit short, K = \$50/lotsize, C = \$7/unit, F′ = \$100/order, CI = \$0.1/unit, C′ = \$22/units, V = \$20/units. Then the optimal solution is Q^* = 160 units, B^* = 55 units, TC = \$2900/year.

Example 3 : The values of the following parameters are to be taken in appropriate units : D = 300 units/year, α = 0.03, β = 0.07, P = 550 units/year, W = \$10/unit/year, H = \$50/unit/year, F = \$1/unit short, K = \$50/lotsize, C = \$7/unit, F′ = \$100/order, CI = \$0.1/unit, C′ = \$22/units, V = \$20/units. Then the optimal solution is Q^* = 176 units, B^* = 31.26 units, TC = \$3331/year.

5. CONCLUSION

In this paper we present an inventory model with imperfect quality items under the three probabilistic distribution function for the proportion of the defective items. According to the numerical results, we can conclude that the minimum cost is obtained for the case of triangular distribution after adding the inspection cost in the total system cost.

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