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On g -derivative and g – Integral

Dr. N. Sarala[1], S. Jothi[2]

(1. Associate Professor, A.D.M. College for women, Nagapattinam, Tamilnadu, India.
2. Guest Lecturer, Thiru.Vi. Ka. Govt. Arts College, Thiruvarur Tamilnadi, India.)

Abstract: In this paper some theorems of g- calculus are proved. This is restricted to the strict pseudo addition $\oplus$ and the corresponding generator g. g calculus is developed in a smiliar way as done for the usual calculus.
Keywords: Pseudo addition, Pseudo multiplication, g –derivative, g – Integral.

Introduction:

The notation of g- calculus was introduced by E.Pap(3). It is based on the operations of pseudo addition and multiplication(1). The range interval [0,1] where t-conorms act to an arbitrary interval [a,b] contained in [-∞, ∞] (4) under the name pseudo addition. This enables us to develop a calculus (g –derivative and g – Integral) so called g – calculus in a similar way as for the usual calculus.

In the second section recall some necessary notion and notations for g –derivative(4). In section 4 prove some theorem and examples for g – Integral.

Preliminary:

Let [a,b] be a closed real interval. The operation $\oplus$ is a function $\oplus$: [a,b] x [a,b] → [a,b] which is commutative, non-decreasing associate and has a zero element denote by 0.

Pseudo addition $\oplus$ there exists a monotone function g.

$$g: \quad [a,b] \quad \rightarrow \quad [0,\infty] \quad g(a) = 0 \text{ or } g(b) = 0$$

$$x \oplus y = g^{-1}(g(x) + g(y))$$

Pseudo multiplication $\otimes$is a function $\otimes$: [a,b] x [a,b] → [a,b] which is commutative non-decreasing associative and has a unit element 1.

$$x \otimes y = g^{-1}(g(x) \,.\, g(y))$$

define the g derivative of f at the point $x \in$ (c,d) as

$$\frac{d^{\oplus} f(x)}{dx} := g^{-1}\left(\frac{d}{dx} g(f(x))\right)$$

here f is differentiable on (c,d).

for any measurable function f: [c,d] → [a,b]

$$\int_{[c,d]}^{\oplus} f\,dx := g^{-1}\left(\int_c^d g(f)\,dx\right)$$

2. g derivative

Let the function f be defined on the interval [c,d] and with values in [a.b] if f is differentiable on (c,d) and has some monotonicity as the function g then we define the g-derivative of f at the point $x \in$ (c,d) as

$$\frac{d^{\oplus} f(x)}{dx} = g^{-1}\left(\frac{d}{dx} g(f(x))\right)$$

Theorem

If there exist an $n - g$ derivative of f then we have

$$\frac{d^{(n)\oplus} f}{dx^n} = g^{-1}\left(\frac{d^n}{dx^n} g(f)\right)$$

Proof

By induction,

For n =1,

$$\frac{d^{(0)\oplus} f}{dx} = f$$

It is obvious. Suppose that the theorem is true to n-1

$$\frac{d^{n-1\oplus}f}{dx^{n-1}} = g^{-1}\left(\frac{d^{n-1}}{dx^{n-1}}g(f)\right)$$

$$\frac{d^{(n)\oplus}f}{dx^{n-1}} = \frac{d^{\oplus}}{dx}\left(\frac{d^{n-1\oplus}f}{dx^{n-1}}\right)$$

$$\frac{d^{\oplus}}{dx}\left(g^{-1}\left(\frac{d^{n-1}}{dx^{n-1}}g(f)\right)\right) = g^{-1}\left(\frac{d}{dx}gg^{-1}\left(\frac{d^{n-1}}{dx^{n-1}}g(f)\right)\right)$$

$$= {}^{-1}\frac{d^{(n)}}{dx^{n}}g(f)$$

$$\therefore \frac{d^{(n)\oplus}f}{dx^{n}} = g^{-1}\left(\frac{d^{(n)}}{dx^{n}}g(f)\right)$$

Theorem: 2

Let f_1 and f_2 be two functions defined on [c,d] and with values in [a,b] if both functions are differentiable then we have

$$\frac{d^{\oplus}(f_1 \otimes f_2)}{dx} = g^{-1}\left(\frac{d^{\oplus}f_1}{dx} \otimes f_2\right) \oplus \left(f_1 \otimes \frac{d^{\oplus}f_2}{dx}\right)$$

Proof:

L.H.S

$$\frac{d\oplus(f_1 \otimes f_2)}{dx} = g^{-1}\left(\frac{d}{dx}\left(g(f_1 \otimes f_2)\right)\right)$$

$$= g^{-1}\left(\frac{d}{dx}g\left(g^{-1}(g(f_1)\,g(f_2)\right)\right)$$

$$= g^{-1}(\ g^{1}(f_1)f_1^{1}g(f_2) + g(f_1)\ \ g^{1}(f_2)f_2^{1})$$

R.H.S.

$$\left(\frac{d^{\oplus}f_1}{dx} \otimes f_2\right) \oplus \left(f_1 \otimes \frac{d^{\oplus}f_2}{dx}\right) = g^{-1}\left(\left(\frac{d^{\oplus}f_1}{dx} \otimes f_2\right)\right)$$

$$+ g\left(\left(f_1 \otimes \frac{d^{\oplus}f_2}{dx}\right)\right)$$

$$= g^{-1}\left(g\left(g^{-1}\left(g\left(\frac{dg(f_1)}{dx}\right)g(f_2)\right)\right)\right) + g\left(g^{-1}\left(g\left(\frac{d^{\oplus}f_2}{dx}\right)\right)\right)$$

$$= g^{-1}\left(g\left(g^{-1}\left(\frac{d^{\oplus}f_1}{dx}\right)g(f_2)\right)\right) + g(f_2) + g(f_1)gg^{-1}\left(\frac{dg(f_2)}{dx}\right)$$

$$= g^{-1}g^{1}(f_1)f_1^{1}g(f_2) + g(f_1)\ \ g^{1}(f_2)f_2^{1}$$

The following examples the ordinary derivative and the corresponding g derivative for $g_1(x) = x^p$, g_1: [0,∞] → [0,∞] p > 0
$g_2(x) = e^{-x/c}$, g_2: [- ∞, ∞] → [0,∞] c > 0

for some elementary function

Let $g(x) = e^{-x/c}$, c > 0 , $x \in \mathcal{R}$

$$\frac{d^{\oplus} f}{dx} = f - \mathrm{CIn}\,(-f^{1}) + \mathrm{CInC}$$

Where f is strictly decreasing, $x \in$ [c,d]

$g(x) = x^{p}$, p > 0, $x \geq 0$

$$\frac{d^{\oplus} f}{dx} = p^{1/p} f^{p-1/p} (f^{1})^{1/p}$$

Where f is strictly increasing defined for $x \in$ [c,d]

Example:

i) $f(x)$ = c, $x \in \mathcal{R}$, C $\in \mathcal{R}$
$f^{1}(x) = 0$
$f_1^{1}(x) = 0$; c $\geq$ 0

ii) $f(x) = x^{n}$, $x > 0$, $n \in \mathcal{R}$
$f^{1}(x) = n\, x^{n-1}$
$f_1^{1}(x) = (np)^{1/p} x^{\frac{np-1}{p}}$, $x > 0$, $n \geq 0$

iii) $f(x) = \sin x$, $x \in \mathcal{R}$
$f^{1}(x) = \cos x$
$f_1^{1}(x) = (p\sin^{p-1} x\cos x)^{1/p}$, $x \in (2k\pi, \pi/2+2k\pi)$
$f_2^{1}(x) = \sin x + \mathrm{C\,In\,C} - \mathrm{C\,In}(-\cos x)$, $x \in \pi/2+2k\pi, \frac{3\pi}{2} + 2k\pi$, $k \in z$

iv) $f(x) = \tan x$, $k \in z$
$f^{1}(x) = \sec^{2} x = \frac{1}{\cos^{2} x}$
$f_1^{1}(x) = p^{\frac{1}{p}} \frac{\tan x}{(\sin x \cos x)^{1/p}}$, $x \in (k\pi, \pi/2+k, , k \in z)$

3. g-Integral

Let the function f be defined on the interval [c,d] and with values in [a,b]. if f is measurable on [c,d]

$$\int_{[c,d]}^{\oplus} f(x)\,dx := g^{-1}\left(\int_{c}^{d} g(f(x))dx\right)$$

Theorem 3.1:

Let f_1 and f_2 be continuous g-differentiable on the imterval (c,d) then for each $x \in$ (c,d)

$$\int_{[c,x]}^{\oplus} \left(\frac{d^{\oplus}}{dx} f_1(x) \otimes f_2(x)\right) dx \oplus \int_{[c,x]}^{\oplus} \left(f_1(x) \otimes \frac{d^{\oplus}}{dx} f_2(x)\right) dx \oplus f_1(c) \otimes f_2(c)$$

$$= f_1(x) \otimes f_2(x)$$

Proof

$$\int_{[c,x]}^{\oplus} \frac{d^{\oplus}}{dx} (f_1(x) \otimes f_2(x)) dx \oplus f_1(c) \otimes f_2(c)$$

$$= \int_{[c,x]}^{\oplus} \frac{d^{\oplus}}{dx} f_1(x) \otimes f_2(x) \oplus f_1(x) \otimes \frac{d^{\oplus}}{dx} f_2(x)\, dx \oplus f_1(c) \otimes f_2(c)$$

$$f_1(x) \otimes f_2(x) = \overset{\oplus}{\underset{[c,x]}{\int}} \left(\frac{d^{\oplus}}{dx} f_1(x) \otimes f_2(x)\right) dx \oplus \overset{\oplus}{\underset{[c,x]}{\int}} \left(f_1(x) \otimes \frac{d^{\oplus}}{dx} f_2(x)\right) dx$$

$$\oplus (f_1(c) \otimes f_2(c))$$

The following examples represents the ordinary integral and the corresponding g integral for $g_1(x) = x^{p}$, g_1 $[0,\infty] \to [0,\infty]$ p > 0 and
$g_2(x) = e^{-x/c}$, g_2: $[-\infty, \infty]$, c > 0

Examples:

i) $f(x) = a$, $x \in \mathcal{R}, a \in \mathcal{R}$
$\int f(x)\, dx = c$
$F_1(x) = (a^p x + c)^{1/p}, \; x \in \mathcal{R}$
$F_2(x) = -C\text{In}\left(x e^{-a/c} x + c\right), \; x \in \mathcal{R}$
ii) $f(x) = e^x$, $x \in \mathcal{R}$
$\int f(x)\, dx = e^x + c$
$F_1(x) = \left({}^1/_p\, e^{px} + c\right)^{1/p}$

iii) $f(x) = {}^1/_{cos^2 x}$; $x \neq (2K+1)\, {}^{\pi}/_2$, k $\in z$
$\int f(x)\, dx = \tan x + c$
$P = 2, F_1(x) = \left(\frac{Sinx}{3cos^3 n} + \frac{2\, Sinx}{3\, cosx} + c\right)^{1/2}$
$P = {}^1/_2, F_1(x) = \left(In\left|\frac{1+Sinx}{cosx}\right| + c\right)^2$

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