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Some more q -Methods and their applications

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Abstract : This paper is a collection of q analogue of various problems. It also aims at focusing on work performed by various researchers and describes q analogues of various functions. We have also proposed q analogue of some integral transforms (viz. Wavelet Transforms, Gabor Transform etc.)
Keywords -q analogue, basic analogue, q method, classical method, basic hyper-geometric function

I. INTRODUCTION AND LITERATURE SURVEY

C.F.Gauss [1, 11] started the theory of q hyper-geometric series in 1812 and worked on it for more than five decades and he presented the series

$$1+\frac{ab}{c}z+\frac{a(a+1)b(b+1)}{1.2.c(c+1)}z^2+\cdots \quad \textbf{(1.1)}$$

, where a, b, c and z are complex numbers and c = 0, −1, −2, ...,at the Royal Society of Sciences, Gottingen. Thirty three years later E. Heine [1,11] converted a simple observation

$$\lim_{q\to 1}\frac{1-q^a}{1-q}=a \quad \textbf{(1.2)}$$

into a systematic theory of basic hyper-geometric series (q-hyper-geometric series or q-series)

$$1+\frac{(1-q^a)}{(1-q)}\frac{(1-q^b)}{(1-q^c)}z+ \quad \textbf{(1.3)}$$

In fact, the theory was started in 1748, when Euler [1, 11] considered the infinite product

$$\prod_{n=1}^{\infty}(1-q^n)^{-1} \quad \textbf{(1.4)}$$

as a generating function for $p(n)$, the number of partitions of a positive integer n, partition of a positive integer n is being a finite non-increasing sequence of positive integers whose sum is n. During 1860−1890, some more contributions to the theory of basic hyper-geometric series were made by J. Thomae and L. J. Rogers. In the beginning of twentieth century F. H. Jackson [1,11,17,18,19,60] started the program of developing the theory of basic hyper-geometric series in a systematic manner, studying q-differentiation, q-integration and deriving q-analogues of the hyper-geometric summation and transformation formulae that were discovered by A. C. Dixon, J. Dougall [1],L. Saalsch¨utz, F. J. W. Whipple[1] and others. During the same time Srinivasa Ramanujan has also made significant contributions to the theory of hyper-geometric and basic hyper-geometric series by recording many identities involving hyper-geometric and basic hyper-geometric series in his notebooks, which were later brought before the mathematical world by G. H. Hardy. During 1930's and 1940's many important results on hyper-geometric and basic hyper-geometric series were derived by W. N. Bailey[1]. Of these Bailey's transform is considered as Bailey's greatest work. The main contributors to the theory during 1950's are D. B. Sears, L. Carlitz, W. Hahn [1,11] and L. J. Slater [1,11]. In fact, Sears [1,11]derived several transformation formulae for $3\varphi2$-series, balanced $4\varphi3$-series and very-well-poised ${}_{r+1}\varphi_{r}$-series. After 1950, the theory of hyper-geometric and basic hyper-geometric series becomes an active field of research, kudos to R.P.Agrawal [53,54,55,56,57], G. E. Andrews [1,11,51,52] and R. Askey[11].

F.H.Jackson [1,11,17,18,19] proposed q-differentiation and q-integration and worked on transformation of q-series and generalized function of Legendre and Bessel. G.E.Andrews [11,51,52] contributed a lot on q theory and worked on q-mock theta function, problems and prospects on basic hyper-geometric series, q-analogue of Kummer's Theorem.

G.E.Andrew [11,51,52] with R.Askey [1] worked on q extension of Beta Function. J.Dougall [1] worked on Vondermonde's Theorem. H.Exton [1] worked a lot on basic hyper-geometric function and its applications.

T.M.MocRobert worked on integrals involving E Functions, Confluent Hyper-geometric Function, Gamma E Function,Fourier Series for E Function and basic multiplication formula. M.Rahman [11] with Nassarallah worked on q-Appells Function, q-Wilson polynomial, q-Projection Formulas. He also worked on reproducing Kample and bilinear sums for q-Racatanad and q-Wilson polynomial. I Gessel with D.Stanton worked on family of q-Lagrange inversion formulas. T.M. MacRobert worked on integrals involving E.Functions and confluent hyper-geometricseries. D.Stanton [1] worked on partition of q series. Studies in the nineteenth century included those of Ernst Kummer, and the fundamental characterization by Bernhard Riemann of the F-function by means of the differential equation it satisfies. Riemann showed that the second-order differential equation for F, examined in the complex plane, could be characterized by its three regular singularities: that effectively the entire algorithmic side of the theory was a result of vital facts and the use of Möbius transformations as a symmetry group.

Subsequently the hyper-geometric series [1, 11] were generalized to numerous variables, for example by Paul Emile Appell, but a comparable general theory took long to emerge. Many identities were found, some quite remarkable. A generalization, the q-series analogues, called the basic hyper-geometric series, was given by Eduard Heine [1, 11] in the late nineteenth century. Here, the ratio of successive terms, instead of being a rational function of n, is considered to be a rational function of qn. Another generalization, the elliptic hyper-geometric series, are those series where the ratio of terms is an elliptic function of n.
During the twentieth [68] century this was a prolific area of combinatorial mathematics, with many connections to other fields. q series can be developed on Riemannian symmetric spaces and semi-simple Lie groups. Their significance and role can be understood through a special case: the hyper-geometric series 2F1 is directly related to the Legendre Polynomial and when used in the form of spherical harmonics, it expresses, in a certain sense, the symmetry properties of the two-sphere or equivalently the rotations given by the Lie group SO(3) Concrete representations are analogous to the Clebsch-Gordan.

Among Indian researchers R.P.Agrawal [53,54 ,55,56,57] gave a lot to q function .He worked on fractional q derivative, q-integral,mock theta function, combinatorial analysis, extension of Meijer’s G Function, Pade approximants, continued fractions and generalized basic hyper-geometric function with unconnected bases. W.A.Al-Salam [2,3] and A.Verma [2,3] worked on quadratic transformations of basic series. N.A.Bhagirathi worked on generalized q hyper-geometric function and continued fractions.V.K.Jain and M.Verma worked on transformations of non terminating basic hyper-geometricseries, their contour integrals and applications to Rogers ramanujan’s identities. H.M.Srivastava with Karlsson worked on multiple Gaussians Hyper-geometric series, polynomial expansion for functions of several variables. S Ramanujan in his last working days worked on basic hyper-geometric series. G.E.Andrews [11,51,52] published an article on “The Lost Note Book of Ramanujan”.H.S.Shukla worked on certain transformation in the field of basic hyper-geometric function. A.Verma and V.K.Jain worked on summation formulas of q-hyper-geometric series, summation formulae for non terminating basic hyper-geometric series, q analogue of a transformation of Whipple and transformations between basic hyper-geometric series on different bases and identities of Rogers-Ramanujan Type. B.D.Sears worked on transformation theory of basic hyper-geometric functions. P.Rastogi worked on identities of Rogers Ramanujan type. A.Verma and M.Upadhyay worked on transformations of product of basic bilateral series and its transformations. Generally speaking, in particular in the areas of combitorics and special functions, a q-analogue of a theorem, identity or expression is a simplification involving a new parameter q that returns the novel theorem, identity or expression in the limit as $q \to 1$ (this limit is often formal, as q is often discrete-valued). Typically, mathematicians are [68] interested in q-analogues that occur naturally, rather than in randomly contriving q-analogues of recognized results. The primary q-analogue studied in detail is the basic hyper-geometric series, which was introduced in the nineteenth century.

q-analogues [11,61] find applications in a number of areas, including the study of fractals and multi-fractal measures, and expressions for the entropy of chaotic dynamical systems. The relationship [1,11,64] to fractals and dynamical systems results from the fact that many fractal patterns have the symmetries of Fuchsian groups in general and the modular group in particular. The connection passes through hyperbolic geometry and ergodic theory, where the elliptic integrals and modular forms play a prominent role; the q-series themselves are closely related to elliptic integrals.

q-analogues [68] also come into sight in the study of quantum groups and in q-deformed super algebras. The connection here is alike, in that much of string theory is set in the language of Riemann surfaces, ensuing in connections to elliptic curves, which in turn relate to q-series.

2. PREVIOUS WORK

2.1 q-Exponential Function

***q*-exponential** is a *q*-analogue [1,11,68] of the exponential function, namely the eigen function of a q-derivative. There are many q-derivatives, for example, the classical *q*-derivative, the Askey-Wilson [11,49] operator, etc. Therefore, unlike the classical exponentials, q-exponentials are not unique. Three variants [11] of exponential functions are given below. Third one is generalized formula.

$$E_{q^{-1}}(x) = \sum_{r=0}^{\infty} \frac{x^r q^{r(r-1)/2}}{[r;q]!} \quad (2.1)$$

$$E_q(x) = \sum_{r=0}^{\infty} \frac{x^r}{[r;q]!} \quad (2.2)$$

$$E(q, \alpha; x) = \sum_{r=0}^{\infty} \frac{x^r q^{\frac{r\alpha(r-1)}{2}}}{[r;q]!} \quad (2.3)$$

2.2 q-Integration

The inverse operation [11] to basic differentiation has also been discussed at some length by F.H.Jackson [18, 19].This is represented by the symbol $\int_b^a \emptyset(x)d\,(qx)$ and is referred to as q-integration or basic integration. When q tends to unity, the basic integral reduces to the ordinary integral. The operations of basic differentiation and integration correspond exactly in every way to ordinary differentiation and integration of which they are generalizations

$$\int_a^b f(x)d(qx) = (1-q)\{b\sum_{r=0}^{\infty} q^r f(q^r b) - a\sum_{r=0}^{\infty} q^r f(q^r a)\} \quad (2.4)$$

$$\int_0^c f(x)d(qx) = (1-q)\{c\sum_{r=0}^{\infty} q^r f(q^r c)\} \quad (2.5)$$

$$\int_{cq}^{\infty} f(x)d(qx) = (1-q)\{c\sum_{r=0}^{\infty} q^{r+1} f(q^{r+1} c)\} \quad (2.6)$$

$$\int_0^{\infty} f(x)d(q,x) = (1-q)\sum_{i=-\infty}^{\infty} q^i f(\,q^i) \quad (2.7)$$

2.3 Trigonometric Functions[11]

$$sin_q(x) = x\sum_{r=0}^{\infty} \frac{(-x^2)^r}{[2r+1;q]!} = x0F1(-;\tfrac{3}{2};q^2;-\left[\tfrac{1}{2};q^2\right]^2 x^2) \quad (2.8)$$

$$cos_q(x) = \sum_{r=0}^{\infty} \frac{(-x^2)^r}{[2r;q]!} = 0F1(-;\tfrac{1}{2};q^2;-\left[\tfrac{1}{2};q^2\right]^2 x^2) \quad (2.9)$$

$$sin_q(x) = \sum_{r=0}^{\infty} (-1)^r \frac{x^{2r+1}}{(q;2r+1)} \quad (2.10)$$

$$Sin_q(x) = \sum_{r=0}^{\infty} (-1)^r \frac{q^{r(2r+1)} x^{2r+1}}{(q;2r+1)} \quad (2.11)$$

$$cos_q(x) = \sum_{r=0}^{\infty} (-1)^r \frac{x^{2r}}{(q;2r)} \quad (2.12)$$

$$Cos_q(x) = \sum_{r=0}^{\infty} (-1)^r \frac{q^{r(2r-1)} x^{2r}}{(q;2r)} \quad (2.13)$$

2.4 Properties of Trigonometric Functions

$$\boldsymbol{sin_q(x)sin_{1/q}(x) + cos_q(x)cos_{1/q}(x) = 1} \quad \textbf{(2.14)}$$

$$\boldsymbol{cos_q(x)cos_{1/q}(x) - sin_q(x)sin_{1/q}(x) = cos_q(2x)} \quad \textbf{(2.15)}$$

2.5 Basic Differentiation operator

Jackson [18, 19] introduced the operative symbol for basic differentiation defined by the relation

$\Delta\{\emptyset$ (x)} = {(x)- (qx)}x^{-1} $(1\text{-}q)^{-1}$, The operation of basic differentiation is defined by the relations

$$\boldsymbol{B_{q,x}}\emptyset(\boldsymbol{x}) = \frac{\emptyset(x)-\emptyset(qx)}{x(1-q)} = \sum_{r=0}^{\infty}\frac{(q-1)^r x^r d^{r+1}\emptyset(x)}{(r+1)!\ dx^{r+1}}, \quad \textbf{(2.16)}$$

where x and q may be real or complex. This becomes the same as ordinary differentiation as the base q tends to unity. In order to avoid the possibility of confusion with the ordinary difference operator, we shall write B q,x instead of Δ. Furthermore, the subscripts q and x will be omitted provided that there is no chance of ambiguity. It will be seen [11] that the possibility now arises of the existence of certain types of difference equations based upon this operator.

$$\mathbf{D_{q,x}f(x)}=\frac{\mathbf{f(qx)-f(x)}}{x(q-1)} \quad \textbf{(2.17)}$$

2.6 Basic analogue of Taylor's Theorem

Jackson [18, 19] introduced q analogue of Taylor's Theorem

$$\boldsymbol{f(x) = f(a) + \frac{(x-a)^{(1)}}{[1;q]}D_q f(a) + \frac{(x-a)^{(2)}}{[2;q]!}D_q^2 f(a) + \ldots + \frac{(x-a)^{(n)}}{[n;q]!}D_q^n \mathbf{f(a)}}, \text{ where}$$

$$\mathbf{R_n}=\frac{(x-a)^{(n+1)}}{[n+1;q]!}\boldsymbol{D}^{(n+1)}\mathbf{f(\xi)}, \text{where } \xi \textit{ lies between } x \textit{ and } a. \quad \textbf{(2.18)}$$

2.7 Integration

$$\boldsymbol{\int_a^b f(x)d(qx) = (1-q)\{b\sum_{r=0}^{\infty} q^r f(q^r b) - a\sum_{r=0}^{\infty} q^r f(q^r a)\}} \quad \textbf{(2.19)}$$

$$\boldsymbol{\int_0^c f(x)d(qx) = (1-q)\{c\sum_{r=0}^{\infty} q^r f(q^r c)\}} \quad \textbf{(2.20)}$$

$$\boldsymbol{\int_{cq}^{\infty} f(x)d(qx) = (1-q)\{c\sum_{r=0}^{\infty} q^{r+1} f(q^{r+1} c)\}} \quad \textbf{(2.21)}$$

$$\boldsymbol{\int_0^{\infty} f(x)d(q,x) = (1-q)\sum_{i=-\infty}^{\infty} q^i f(\ q^i)} \quad \textbf{(2.22)}$$

$$\boldsymbol{\int_a^{\infty} f(x)d(q,x) = a(1-q)\sum_{i=0}^{\infty} q^{-i} f(\ q^{-i} a)} \quad \textbf{(2.23)}$$

2.8 Addition Theorem

$$\boldsymbol{(q;n) = (1-q)(1-q^2)\ldots(1-q^n) = (1-q)^n[n;q]}$$

$$\textit{where } 0 < q < 1.$$

$$\boldsymbol{(q,\infty) = \prod_{n=1}^{\infty}(1-q^n)} \quad \textbf{(2.24)}$$

2.9 Mellin Transform

$$\boldsymbol{f(s) = \int_0^{\infty} F(t)t^{s-1}d(qt)} \quad \textbf{(2.25)}$$

2.10 Hankel Transform

$$f(s) = \int_0^\infty F(t) t J_n(st) d(qt) \tag{2.26}$$

2.11 Variants of Laplace Transform

Hahn [48] defined two analogues of Laplace Transform by the help of the integral equations

$$L_{q,s} f(x) = \frac{1}{1-q}\int_0^{\frac{1}{s}} E_q\,(qsx) f(x) d(q,x) \tag{2.27}$$

$$\mathcal{L}_{q,s} f(x) = \frac{1}{1-q}\int_0^\infty e_q(-sx) f(x) d(q,x) \tag{2.28}$$

$$\mathrm{R1}(\mathbf{s}) \geq \mathbf{0}$$

$$L_{q,s} = \frac{1}{1-q}\int_0^{\frac{1}{s}} E_q\,(qsx) f(x) d(q,x) = \frac{(q,\infty)}{s}\sum_{j=0}^\infty \frac{q^j f(s^{-1}q^j)}{(q;j)} \tag{2.29}$$

$$\mathcal{L}_{q,s} f(x) = \frac{1}{1-q}\int_0^\infty E_q(-sx) f(x) d(q,x) = \frac{1}{\prod_{n=0}^\infty (1+sq^n)}\sum_{j=-\infty}^\infty q^j f(q^j)(1+s)_j \tag{2.30}$$

2.11.1 q-Laplace [48] of some of the elementary functions

$$L_q\{1\} = \frac{q}{s} \tag{2.31}$$

$$L_q\{x\} = \frac{q^2}{s^2} \tag{2.32}$$

$$L_q\{x^n\} = \frac{q^{n+1}}{s^{n+1}}[n;q]! \tag{2.33}$$

$$L_q\{E_q(ax)\} = \frac{q}{s-qa} \tag{2.34}$$

$$L_q sin_q ax = \frac{q^2 a}{s^2+q^2a^2} \tag{2.35}$$

$$L_q cos_q ax = \frac{qs}{s^2+q^2a^2} \tag{2.36}$$

2.11.2 q-Transform of derivatives

$$D_q f(x) = \frac{s}{q} F(s) - f(0) \tag{2.37}$$

$$D_q^2 f(x) = \frac{s^2}{q^2} F(s) - \frac{s}{q} f(0) - D_q f(0) \tag{2.38}$$

$$D_q^n f(x) = \frac{s^n}{q^n} F(s) - \sum_{j=0}^{j=n-1}\left(\frac{s}{q}\right)^{n-1-j} D_q^j f(0) \tag{2.39}$$

2.12 q-Transform of Integrals

$$\int_0^x f(x)\, d_q x = q\,\frac{F(s)}{s} \tag{2.40}$$

3.13 Heine's[11] Series

$$1+\frac{(1-q^a)(1-q^b)}{(1-q^c)(1-q)}x+\frac{(1-q^a)(1-q^{a+1})(1-q^b)(1-q^{b+1})}{(1-q^c)(1-q^{c+1})(1-q)(1-q^2)}x^2+\cdots \text{where} |q|<1 \text{ and } |x|<1 \qquad (2.41)$$

2.14 Euler's[11] Identity

$$1+\sum_{n=1}^{\infty}(-1)^n\left\{q^{n(3n-1)/2}+q^{n(3n+1)/2}\right\}=\prod_{n=1}^{\infty}(1-q^n) \qquad (2.42)$$

2.15 The Heine [11] Equation

The Gauss [11] hyper-geometric function 2F1 (a, b; c; x) is a particular solution of the equation

$$x(1-x)y''+\{c-(1+a+b)x\}y'+aby=0 \qquad (2.43)$$

which may be written in operational form as

$$x(\delta+a)(\delta+b)y-\delta(\delta+c-1)y=0. \qquad (2.44)$$

If we replace the symbolic operations by their basic analogues, we obtain the q-differential equation

$$x[\delta+a;q][\delta+b;bq]y-[\delta;q][\delta+c-1;q]y=0, \qquad (2.45)$$

which on expansion, takes the form

$$x\{q^c-q^{a+b+1}x\}\hat{B}^2y+\{[c;q]-\left(q^a[1+b;q]+q^b[a;q]\right)x\}-[a;q][b;q]y=0 \qquad (2.46)$$

This is one of an infinite number of possible q-analogues of the hyper-geometric equation.

2.16 q-Gauss [11] summation formula

$$\sum_{n=0}^{n=\infty}\frac{(a,b)_n}{(q,c)_n}\left(\frac{c}{ab}\right)^n=\frac{\left(\frac{c}{a},\frac{c}{b}\right)_\infty}{\left(c,\frac{c}{ab}\right)_\infty} \qquad (2.47)$$

2.17 q-Plaff-Saalschutz's [11] summation formula

$$\sum_{k=0}^{k=n}\frac{(q^{-n},A,B)_k}{(q,C,ABq^{1-n}/C)_k}q^k=\left(\frac{C}{A},\frac{C}{B}\right)_n\Big/\left(C,\frac{C}{AB}\right)_n \qquad (2.48)$$

2.18 Some identities of q –shifted factorials [1, 11] are

$$(a)_{-n}=\frac{1}{(aq^{-n})_n}=\frac{(^{-q}/_a)^n}{(^q/_a)_n}q\binom{n}{2} \qquad (2.49)$$

$$(a)_{n+k}=(a)_n(aq^n)_k \qquad (2.50)$$

$$(a)_{n-k}=\frac{(a)_n}{\left(\frac{q^{1-n}}{a}\right)_k}\left(\frac{-q}{a}\right)^n q^{\binom{k}{2}-nk} \qquad (2.51)$$

3. q-INTEGRAL TRANSFORMS

The impetus [68] behind integral transforms is simple to understand. There are many classes of problems that are hard to solve or at least quite unwieldy algebraically in their novel representations. An integral transform **maps** an equation from its original **domain** into another domain. Manipulating and solving the equation in the target domain can be much easier than manipulation and solution in the original domain. The solution is then mapped back to the original domain with the inverse of the integral transform. As an example of an application of integral transforms, consider the Laplace transform. This is a method that maps differential or integro-differential equations in the *time domain* into polynomial equations in what is termed as complex *frequency* domain. (Complex frequency is comparable to real, physical frequency but rather more general. Specifically, the imaginary component ω of the complex frequency $s = -\sigma + i\omega$ corresponds to the usual concept of frequency, *viz.*, the rate at which a sinusoid cycles, whereas the real component σ of the complex frequency corresponds to the degree of *damping*.)

The equation [68] cast in terms of complex frequency is readily solved in the complex frequency domain (roots of the polynomial equations in the complex frequency domain correspond to Eigen values in the time domain), leading to a *solution* formulated in the frequency domain. Employing the inverse transform, *i.e.*, the inverse procedure of the original Laplace transform, one obtains a time-domain solution. In this example, polynomials in the complex frequency domain (typically occurring in the denominator) correspond to power series in the time domain, while axial shifts in the complex frequency domain correspond to damping by decaying exponentials in the time domain. The Laplace transform finds broad application in physics and chiefly in electrical engineering, where the characteristic equations that explain the behaviour of an electric circuit in the complex frequency domain correspond to linear combinations of exponentially damped, scaled, and time-shifted sinusoids in the time domain. Other integral transforms find out unique applicability within other scientific and mathematical disciplines.

3.1 q analogue of Morlet Wavelet

It can be defined by

$$\Psi_q(t) = E_{\frac{1}{q}}\left(i\omega_0 t - \frac{t^2}{2}\right) \quad (3.1)$$

Fourier Transform of Morlet Wavelet is

$$\widehat{\Psi_q(t)} = \int_{-\infty}^{\infty} E_{\frac{1}{q}}\left(i\omega_0 t - \frac{t^2}{2}\right) E_q(-i\,\omega t)\, d(qt) = 2\int_0^{\infty} E_q\left(-\frac{t^2}{2}\right)\cos_q(\omega_0 - \omega)t\, d(qt) \quad (3.2)$$

Let $t = u^{1/2}$

$$d(qt) = \left[\frac{1}{2}; q\right] u^{-\frac{1}{2}} d(qu) \quad (3.3)$$

Integration will become

$$2\int_0^{\infty} \cos_q(\omega_0 - \omega)\sqrt{u}\, E_q\left(-\frac{u}{2}\right) d(qu) \quad (3.4)$$

It can be rewritten as a form of series

$$2L_q\left[1 - \frac{(\omega-\omega_0)^2}{[2;q]!} u + \frac{(\omega-\omega_0)^4}{[4;q]!} u^2 + \cdots.\right]$$

$$= 2\left[2q - \frac{(\omega-\omega_0)^2}{[2;q]!}[1;q](2q)^2 + \frac{(\omega-\omega_0)^4}{[4;q]!}(2q)^3[2;q]! + \cdots\right] \quad (3.5)$$

Parameter of Laplace Transform s=1/2.When q tends to one it will be equivalent to classical Fourier Transform.Fourier Transform of Morlet Wavelet is

$$\sqrt{2\pi} Exp\left[-\frac{(\omega-\omega_0)^2}{2}\right] \quad (3.6)$$

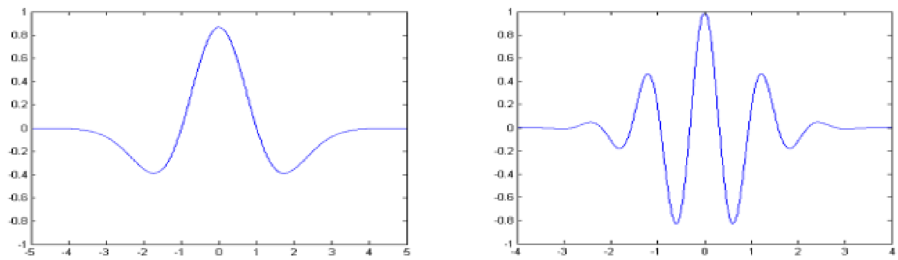

Figure 1: q Morlet Wavelet (q=1.0001)

3.2 q-transform of Mexican Hat Wavelet

$$\Psi_q(t) = (1 - t^2) E_{\frac{1}{q}}\left(-\frac{t^2}{2}\right) \quad (3.7)$$

We can calculate its Fourier Transform as

$$\widehat{\Psi_q(t)} = \int_{-\infty}^{\infty}(1-t^2)E_{\frac{1}{q}}\left(-\frac{t^2}{2}\right)E_q(-i\,\omega t)\,d(qt) = 2\int_0^{\infty}(1-t^2)\cos_q(\omega t)\,E_{\frac{1}{q}}\left(-\frac{t^2}{2}\right)d(qt)$$

(3.8)

Let $t=u^{\frac{1}{2}}$

$$d(qt) = \left[\frac{1}{2};q\right]u^{-\frac{1}{2}}d(qu) \quad (3.9)$$

$$\widehat{\Psi_q(t)} = 2\left[\frac{1}{2};q\right][A+B] \quad (3.10)$$

where A and B are two series

$$A = \left[-\frac{1}{2};q\right](2q)^{1/2} - (\omega^2/[2;q]!)[\frac{1}{2};q](2q)^{\frac{3}{2}} + (\omega^4[3/2;q]/[4;q]!)(2q)^{\frac{5}{2}} + (\omega^6[5/2;q]/[6;q]!)(2q)^{\frac{7}{2}}+\ldots.. \quad (3.11)$$

$$B = \left[\frac{1}{2};q\right](2q)^{3/2} - (\omega^2/[2;q]!)[\frac{3}{2};q](2q)^{\frac{5}{2}} + (\omega^4[5/2;q]/[4;q]!)(2q)^{\frac{7}{2}} + (\omega^6[7/2;q]/[6;q]!)(2q)^{\frac{9}{2}}+\ldots. \quad (3.12)$$

When we choose q very close to one from right or left we will get Fig 2.

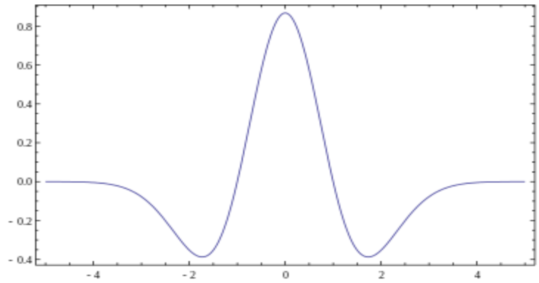

Figure 2:q Mexican Hat Wavelet(at q=0.999)

3.3 q-analogue of Haar Wavelet

$$\psi(t) = f(x) = \begin{cases} 1, & 0 \le t \le \frac{1}{2} \\ -1, & \frac{1}{2} \le t \le 1 \\ 0 & elsewhere \end{cases} \quad (3.13)$$

$$\widehat{\psi(t)} = (1-q)\left[\sum_{r=0}^{r=\infty} q^r E_{q^{-1}}\left(-{}^{i\omega q^r}/_2\right) - \sum_{r=0}^{r=\infty} q^r E_{q^{-1}}(-i\omega q^r)\right] \quad (3.14)$$

When q tends to one it will be equivalent to

$$\widehat{\psi(t)} = \frac{4i}{\omega}E_q\left(-\frac{i\omega}{2}\right)\sin_q{}^2(\frac{\omega}{4}) \quad (3.15)$$

When we choose q very close to one from right or left we will get Fig 3.

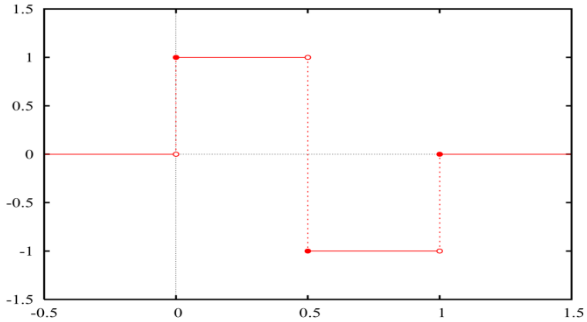

Figure 3: q Haar Wavelet (q=0.997)

3.4 q-Gabor Transform

The continuous Gabor Transform of a function $f\epsilon L^2(R)$ with respect to a window function $g\epsilon L^2(R)$ is denoted by $\boldsymbol{G[f](t,\omega) = \hat{f}_g(t,\omega) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)E_{\frac{1}{q}}(-i\omega\tau)d(q\tau)}$ **where**

$$\boldsymbol{g_{t,\omega}(\tau) = \bar{g}(\tau - t)E_{\frac{1}{q}}(-i\omega\tau)} \quad \textbf{(3.16)}$$

Example:

Gabor transform of $\boldsymbol{f(\tau) = E_q(-a^2\tau^2) with g(\tau) = 1}$

$$\boldsymbol{\widehat{f_{g,q}}(t,w) = \int_{-\infty}^{\infty} E_q(-a^2\tau^2)E_{\frac{1}{q}}(-i\omega\tau)g(\tau-t)d(q\tau)}$$

$$= \int_{-\infty}^{\infty} \boldsymbol{E_q(-a^2\tau^2)\, E_{\frac{1}{q}}(-i\omega\tau)}\mathrm{d}(\mathrm{q}\boldsymbol{\tau}) + \int_{-\infty}^{\infty} \boldsymbol{E_q(-a^2\tau^2)E_{\frac{1}{q}}(-i\omega\tau)\, d(q\tau)} \quad \textbf{(3.17)}$$

Putting $\boldsymbol{\tau = u^{1/2}} we get \boldsymbol{d(q\tau) = \frac{u^{\frac{1}{2}}q^{\frac{1}{2}} - u^{\frac{1}{2}}}{u(q-1)} d(qu)} = \boldsymbol{u^{\frac{-1}{2}}\left[\frac{1}{2};q\right] d(qu)}$, **(3.18)**

$$\boldsymbol{[\tfrac{1}{2};q]\left[\left[-\tfrac{1}{2};q\right]\left(\frac{q}{a^2}\right)^{\frac{1}{2}} + i\omega\left(\frac{q}{a^2}\right) - \frac{\omega^2[\frac{1}{2};q]}{[2;q]!}\left(\frac{q}{a^2}\right)^{\frac{3}{2}} - \frac{i\omega^3}{[3;q]!}[1;q]!\left(\frac{q}{a^2}\right)^2\right] + [\tfrac{1}{2};q]\left[\left[-\tfrac{1}{2};q\right]\left(\frac{q}{a^2}\right)^{\frac{1}{2}} - i\omega\left(\frac{q}{a^2}\right) - \frac{\omega^2[\frac{1}{2};q]}{[2;q]!}\left(\frac{q}{a^2}\right)^{\frac{3}{2}} + \frac{i\omega^3}{[3;q]!}[1;q]!\left(\frac{q}{a^2}\right)^2\right]}$$

$$\boldsymbol{= 2[\tfrac{1}{2};q]\left[\left[-\tfrac{1}{2};q\right]\left(\frac{q}{a^2}\right)^{\frac{1}{2}} - \frac{\omega^2}{[2;q]!}\left[\frac{1}{2};q\right]\left(\frac{q}{a^2}\right)^{\frac{3}{2}} + \cdots\right]} \quad \textbf{(3.19)}$$

When we choose q very close to one from right or left we will get Fig 4.with mean zero and standard deviation 1.

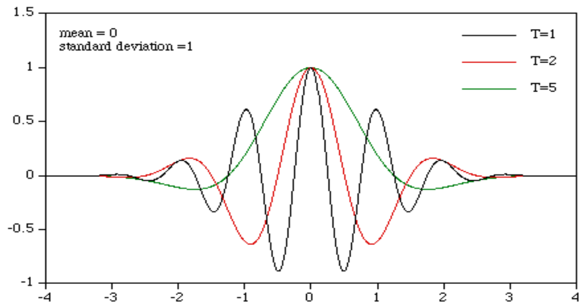

Figure 4:q Gabor Transform(q=0.99)

II. APPLICATIONS OF BASIC HYPER-GEOMETRIC FUNCTIONS

It has been used in number theory, combinatorial analysis and problems in Physics, Statistics and Numerical Analysis and can be used further in many areas. Some areas are as follows:

1. Numerical solutions of q-Differential Equations

2. The Operations Treatment of Difference Equations.

3. Statistical applications of q-Binomial Coefficient

4. q-Quantisations, q-Lommel Polynomials, q-Ultraspherical Polynomials

5. Basic Bessel, Basic Hermite Equation, Basic Legendre Equation

6. Combinatorial Interpretation of an identity of Ramanujan

7. Jacobi's Triple Product Identity

This is a list of basic analogues in mathematics and related areas

Field of Algebra

- Quantum affine algebra
- Quantum group
- Iwahori–Hecke algebra

Field of Analysis

- q-difference polynomial
- Quantum calculus
- Jackson integral
- q-derivative

Field of Combinatorics

- LLT polynomial
- q-binomial coefficient
- q-Pochhammer symbol
- q-Vandermonde identity

Field of Orthogonal Polynomials

- q-Bessel polynomials
- q-Charlier polynomials

- q-Hahn polynomials
- q-Jacobi polynomials:
- Big q-Jacobi polynomials
- Continuous q-Jacobi polynomials
- Little q-Jacobi polynomials
- q-Krawtchouk polynomials
- q-Laguerre polynomials
- q-Meixner polynomials
- q-Meixner–Pollaczek polynomials
- q-Racah polynomials

Field of Statistics

- Gaussian q-distribution
- q-exponential distribution
- q-Weibull distribution
- Tsallis q-Gaussian

Field of Special Functions

- Basic hyper-geometric series
- Elliptic gamma function

CONCLUSION

q method for solving problems of numerical method is an alternate method which finds applicability in most of the problems and this method is even better method for some of the problems where error terms are involved. It is better method for problems involving transcendental functions. q-**analogue** of a theorem, identity or expression is a simplification relating a new constraint q that returns the original theorem, identity or expression in the limit as $q \to 1$.

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