

## **Analysis and Performance of Linear Quadratic Regulator and PSO algorithm in optimal control of DC motor**

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**ABSTRACT :** DC motors are widely used in the mechanisms that require control of speed. PID controller are widely used for speed control. But they aren't suitable for high performance cases, because of the low robustness of PID controller. Many researchers have been studying various new control techniques in order to improve the system performance. The objective of the paper is control speed of DC motor using different control strategies for comparison purpose. A comparison is made between three control methods and the controllers are PID conventional controller by using Ziegler-Nichols method, Particle Swarm Optimization algorithm in tuning PID controller and Linear Quadratic Regulator. The DC Motor system drive is modeled in MATLAB/SIMULINK. LQR and PSO algorithm are implemented using MATLAB toolbox. Graphs obtained from the simulations demonstrate a very good performance of the proposed system using PSO algorithm then other controllers used in this paper.

**KEYWORDS** - Optimal control of speed, LQR, PSO

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### **I. INTRODUCTION**

DC motors are comprehensively used in various industrial applications such as electrical equipment, computer peripherals, robotic manipulators, actuators, steel rolling mills, electrical vehicles, and home appliances. Its applications spread from low horse power to the multi-mega watt due to its wide power, torque, speed ranges, high efficiency, fast response, and simple and continuous control characteristics. The speed of DC motor can be changed by controlling the armature and field voltages [1]. The most commonly used controller for the speed control of DC motors is conventional PID controller. Traditional PID controllers have been successfully used in control applications since 1940s and are the most often used industrial controller today [2]. Traditional PID controllers have very simple control structure and inexpensive cost. In spite of the major features of the fixed PID controller, it has some disadvantages such as the high starting overshoot in speed, the sensitivity to controller gains and the sluggish response due to sudden change in load torque disturbance. Tuning PID controller parameters is very difficult, poor robustness; therefore, it's difficult to achieve the optimal state under field conditions in the actual production. In order to overcome some problems that faced by conventional PID controller and achieve accurate control performance of speed control of a DC motor, the other type of control methods can be developed such as linear quadratic regulator[3]. Linear quadratic regulator design technique is well known in modern optimal control theory and has been widely used in many applications. It has a very nice robustness property. Thus, the linear quadratic regulator theory has received considerable attention since 1950s. The linear quadratic regulator technique seeks to find the optimal controller that minimizes a given performance index. This performance index is parameterized by two matrices, Q and R, that weight the state vector and the system input respectively. The value of the elements in Q and R is related to its contribution to the performance index. In this paper, another technique that is used for tuning PID parameters which is based on Swarm Optimization is the Particle Swarm Optimization[4] (PSO) technique. A swarm consists of individuals called particles, each of which represents a different possible set of the unknown parameters to be optimized. The "swarm" is initialized with a population of random solutions. In a PSO particles fly about in a multi-dimensional search space adjusting its position according to its own experience and the experience of its neighboring particle. The goal is to efficiently search the solution space by swarming the particles towards the best fitting solution encountered in previous iterations with the intention of encountering better solutions through the course of the process and eventually converging on a single minimum or maximum solution. In this paper the optimal parameters of PID controller are proposed by using PSO algorithm method. The remainder of the paper is organized as follows: at first the dynamic model of the separately excited DC motor is briefly reviewed for the purpose of speed control. The next section the basic concept and design of linear quadratic regulator controller and Particle Swarm Optimization (PSO) technique are briefly reviewed. Then the simulation results are presented. Finally, the last section states the main conclusion. Iterative simulation results shows that the effectiveness of Particle Swarm Optimization approach PSO since it allows the operator to find a near optimal good compromise among the proposed goals, which is best trade-off lowest cost PID controller design.

## II. MODEL OF DC MOTOR

DC motor is a power actuator which transforms electrical energy in to mechanical energy. DC motor is widely used in many industrial applications where wide range of speed –torque characteristics required. In this paper, the separated excited DC motor model is chosen according to his good electrical and mechanical performances more than the other DC motor models. Fig.1 show the equivalent circuit of DC motor with separate excitation.

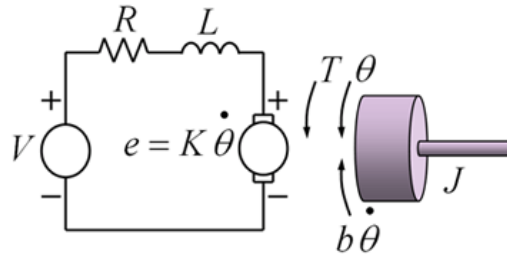


Fig.1 Equivalent circuit of DC motor

The dynamic equations of DC motor are given as follow:

$$\begin{bmatrix} \frac{d\omega}{dt} \\ \frac{di}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-b}{J} & \frac{k}{J} \\ \frac{-k}{L} & \frac{-R}{L} \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u \quad (1)$$

The transfer function of DC motor is:

$$G(s) = \frac{\omega(s)}{U(s)} = \frac{k}{JLs^2 + bLs + Rb + kk_b} \quad (2)$$

## III. PID CONTROLLER

The PID controllers have a wide range of applications in industrial control because of their simple control structure. The PID controllers need of less plant information than a complete mathematical model. The PID controller calculation (algorithm) involves three constant parameters called the proportional (P), integral (I), derivative (D) values, these value can be interpreted in terms of time. The transfer function of PID controller is:

$$G_c(s) = k_p + \frac{k_i}{s} + k_d s = k_p \left( 1 + \frac{1}{T_i s} + T_d s \right) \quad (3)$$

The Ziegler-Nichols design methods are the most popular methods used in process control to determine the parameters of a PID controller. The step response method is based on an open-loop step response test of the process.

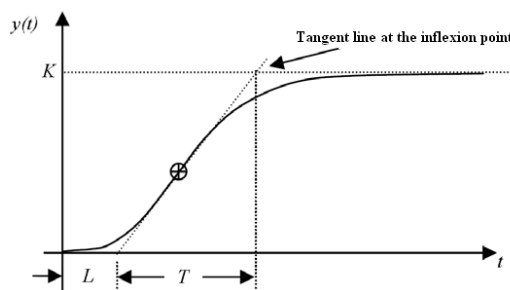


Fig.2 Open loop step response for Ziegler –Nichols method

choosing L and T parameter using

The step response of the process is characterized by two parameters L and T that are determined by drawing a tangent line at the inflexion point. The intersections of the tangent and the coordinate axes give the process parameters as shown in Fig.2, and these are used in calculating the controller parameters. The parameters for PID controllers obtained from the Ziegler-Nichols step response method are shown in Table 1.

**Table 1** PID parameters.

Type of controller	$K_p$	$T_i$	$T_d$
<b>P</b>	T/L	$\infty$	0
<b>PI</b>	0.9T/L	L/0.3	0
<b>PID</b>	1.2T/L	2L	0.5L

#### IV. LINEAR QUADRATIC REGULATOR

Linear quadratic regulator[4] design technique is well known in modern optimal control theory and has been widely used in many applications. Thus, the linear quadratic regulator theory has received considerable attention since 1950s. The liner quadratic regulator technique seeks to find the optimal controller that minimizes a given cost function. This cost function is parameterized by two matrices, Q and R, that weight the state vector and the system input respectively. These weighting matrices regulate the penalties on the excursion of state variables and control signal. One practical method is to Q and R to be diagonal matrix. The value of the elements in Q and R is related to its contribution to the cost function. Consider the single-input, single-output (SISO) system with

$$\dot{x} = Ax + Bu \quad (4)$$

and feedback

$$u = -Kx = -[k_1 \quad k_2 \dots \dots \quad k_n]x \quad (5)$$

The cost function is

$$J = \int_0^{\infty} (x^T Q x + R u^2) dt \quad (6)$$

where R is a scalar weighting factor. The index minimized when  $K = R^{-1} B^T P$  the n x n matrice P is determined from the solution of equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (7)$$

The two matrices Q and R are selected by tray and error. Generally, selecting Q large mean that to keep J small. On the other hand selecting R large means that the control input u must be smaller to keep J small. One should select Q to be positive semidefinite and R to be positive definite. The structure of LQR is shown in Fig.3.

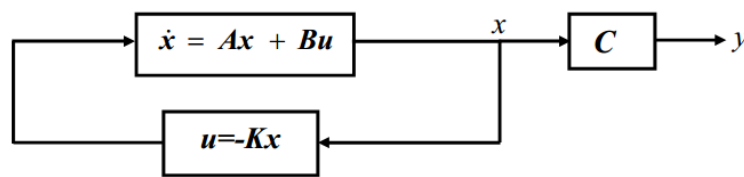


Fig.3 Structure of LQR (Linear Quadratic Regulator)

#### V. PARTICLE SWARM OPTIMIZATION (PSO) TECHNIQUE

Particle Swarm Optimization (PSO) technique, proposed by Kennedy and Eberhart is an evolutionary-type global optimization technique developed due to the inspiration of social activities in flock of birds and school of fish and is widely applied in various engineering problems due to its high computational efficiency. This techniques is used to minimize the maximum overshoot, minimize the rise time, minimize speed tracking error, minimize the steady state error, and minimize the settling time, optimization solution results are set of near optimal trade-off value which are called the Pareto front or optimally surfaces. PSO is a robust stochastic optimization technique based on the movement and intelligence of swarms. The components of PSO are Swarm Size, Velocity, position components and maximum no of iteration. The structure of the PID controller with PSO algorithm is shown in Fig.4.

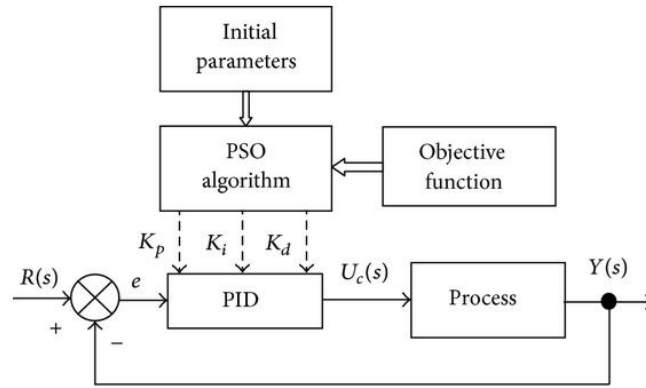


Fig.4. The structure of the PID controller with PSO algorithm

The Particle Swarm Optimization (PSO) model consists of a swarm of particles, which are initialized with a population of random candidate solutions. They move iteratively through the d-dimension problem space to search the new solutions. Each particle has a position represented by a position vector  $X_k^i$  where (i is the index of the particle), and a velocity represented by a velocity vector  $V_k^i$ . Each particle remembers its own best position  $P_{best}^i$ . The best position vector among the swarm then stored in a vector  $P_{global}^i$ . During the iteration time k, the update of the velocity from the previous velocity to the new velocity is determined by:

$$V_{k+1}^i = wV_k^i + c_1R_1(P_{best}^i - X_k^i) + c_2R_2(P_{global}^i - X_k^i) \quad (8)$$

The new position is then determined by the sum of the previous position and the new velocity:

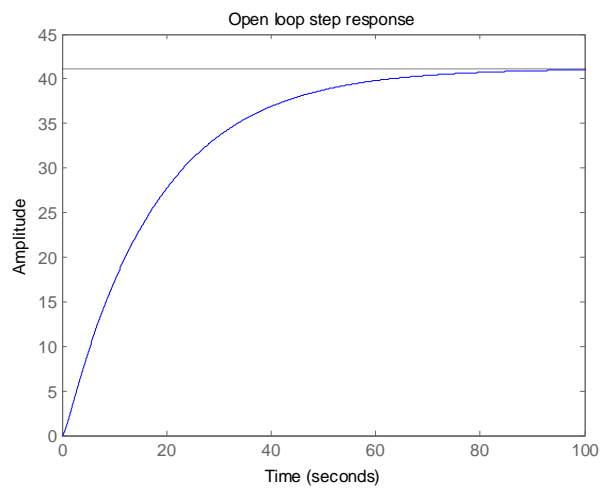
$$X_{k+1}^i = X_k^i + V_{k+1}^i \quad (9)$$

where  $R_1$  and  $R_2$  are random numbers. A particle decides where to move next, considering its own experience, which is the memory of its best past position, and the experience of the most successful particle in the swarm.

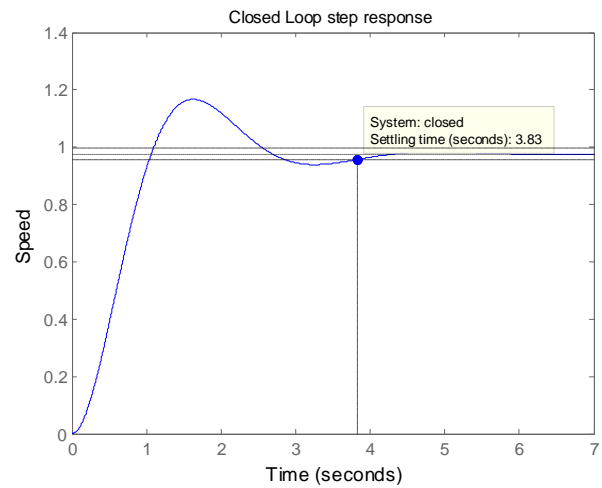
## VI. CONCLUSION AND RESULT

The model of DC motor and the optimal control of speed were numerically simulated using a state space model and Matlab/Simulink software for a separated excited DC motor with the following parameters: DC motor 30W, 12 V, 300 rad/s, total inertia  $0.01\text{kgm}^2$ . The simulation procedure may be summarized as follows:

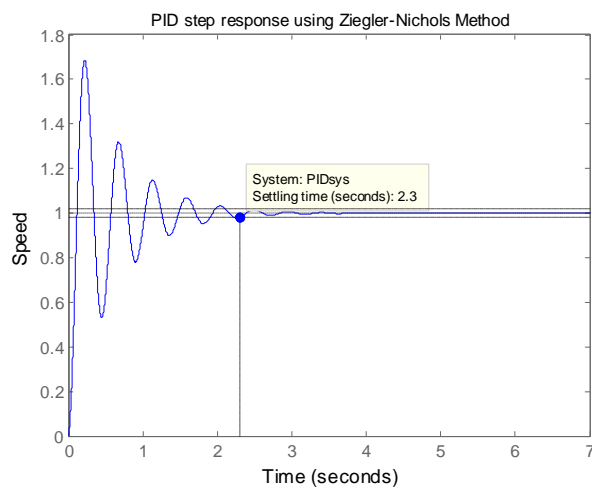
- First input the DC motor data,
- Write the differential equations for the model then get the state space representation
- Get the open loop transfer function and the closed loop step response
- Finally performing the performance of PID controller by Ziegler Nichols method, LQR and PID controller by using PSO algorithm and compare the results.



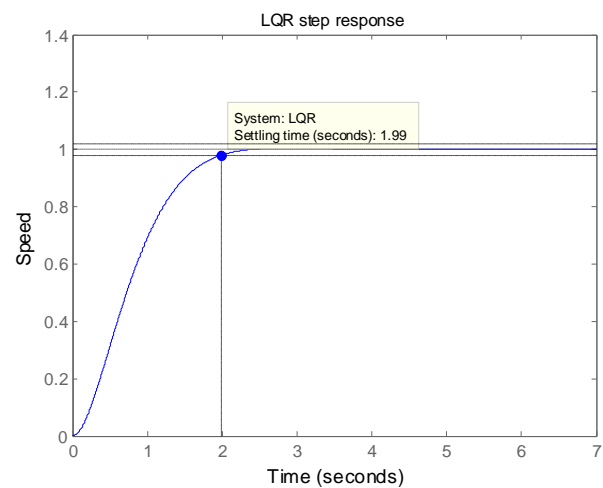
**Fig.5** Step response for open loop system



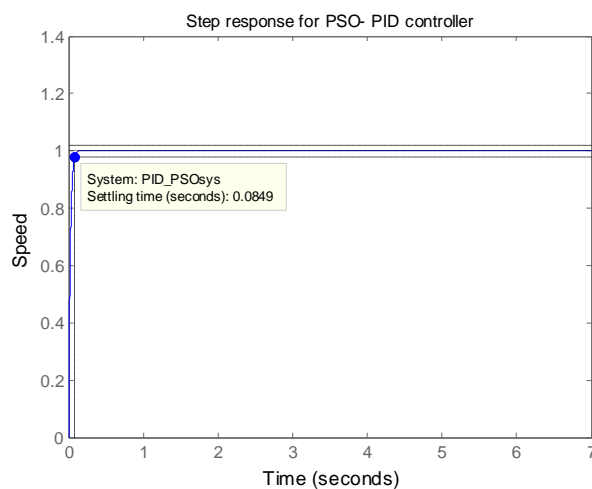
**Fig.6** Step response for closed loop system



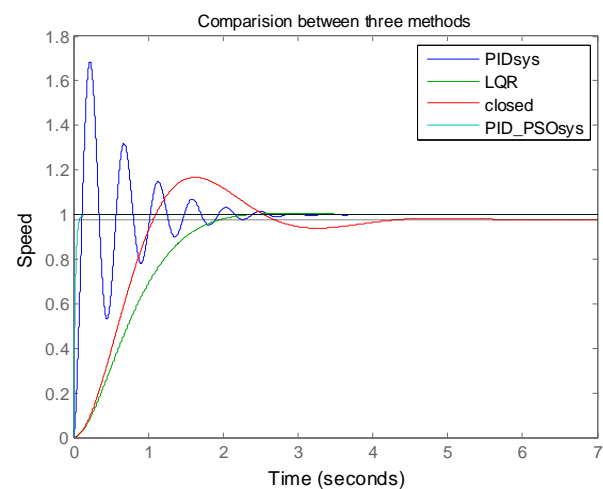
**Fig.7** Step response with PID control with Ziegler-Nichols method



**Fig.8** Step response with linear quadratic regulator



**Fig.9** Step response with PSO-PID



**Fig.10** Step response for all system

To show the effectiveness of the proposed approach, is made a comparison with designed of PID controller using Ziegler-Nichols method, Linear Quadratic Regulator and PSO algorithm method in Table 2.

Table.2 Comparison of the system

<i>Method</i>	<i>Settling time (sec)</i>	<i>Overshoot (%)</i>	<i>Rise time (sec)</i>
<b><i>Closed System</i></b>	3.83	19.5	0.721
<b><i>Z-N PID</i></b>	2.3	68.6	0.0815
<b><i>PSO-PID</i></b>	0.0849	0	0.0451
<b><i>LQR</i></b>	1.99	0	1.26

PID controllers are a widespread control solution due to their simple architecture, generally acceptable control performance and ease of use. In this work PID controller has been tuned using Ziegler-Nichols method, Linear Quadratic Regulator and Particle Swarm Optimization (PSO) through simulation of DC motor speed control system. The performance of the PSO algorithm method of tuning a PID controller has been proved to be better than traditional method Ziegler-Nichols method and LQR, in terms of the system overshoot, settling time and rise time.

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