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AN APPLICATION OF CHOQUET AND SUGENO INTEGRALS

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Abstract: In real many problems, most of criteria have interactive characteristics, which cannot be evaluated by additive measures exactly. For the human subjective evaluation processes it will be more better to apply Choquet and Sugeno integrals model together with the definition of $\lambda -$ fuzzy measure, in which the property of additive is not necessary. This paper presents the application of fuzzy integrals as tool for criteria aggregation in the decision problems and evaluating medicine with illustrations of hierarchical structure of $\lambda -$ Fuzzy measure for Choquet and Sugeno integrals model.

Keywords: Choquet and Sugeno integrals, $\lambda -$ Fuzzy measure

Introduction:

Our tools for information of aggregation are the weighted average method for example, a linear Integral. These methods consider that the information sources involved and non-interactive (or) independent and, hence their weighted effects are viewed as additivity, but in real world in many problems ,this approach is not realistic. Sugeno gave $\lambda -$ fuzzy measures that satisfying the λ–additive axiom and it is the particular case of fuzzymeasure. $\lambda -$ fuzzy measures has great importance in practical applications in artificial intelligence.

The definition of Sugeno integral based on max (or) min ,the min− max integral calculation can only determine some interval at which the measure values are possibly located.

1.1 Preliminaries

1.1Definition of Fuzzy Measure

Suppose that $(\mathcal{X}, F, \mu)$ is a measureable space, a fuzzy measure is a function $\mu : F \longrightarrow [0,\infty]$ such that the following properties are hold

(i) $\mu(\emptyset) = 0$

(ii) If A, B $\in$F and A $\subseteq$B then $\mu(A) \leq \mu(B)$

(iii) If $A_n \in F$ and $A_1 \subseteq A_2 \subseteq$...... then $\lim_{n\to\infty} \mu(A_n) = \mu(\lim_{n\to\infty} A_n)$

1.2 Definition of the Additive Measure

Let us consider $(\mathcal{X}, F, m)$ to be a measure space .An additive measure m is a function m: $F \longrightarrow [0,\infty]$ i.e., defined on sigma-algebra F over a set X and taking values in the interval[0, ∞] such that the following properties are satisfied

(i) m $(\emptyset) = 0$

(ii) If E_1 , E_2, E_3 ,…… is a countable sequence of pair wise disjoint subsets of *F* then

$m(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} m(E_i)$

1.2 Definition of Sugeno Fuzzy Measure

Let $X = \{x_1, x_2, ...x_n\}$ be a finite set and consider $\lambda \in (-1,\infty)$, an λ measure is afunction $g_\lambda : 2^x \longrightarrow [0,1]$ such that it satisfied the following condition

(i) $g_\lambda(X) = 1$

(ii) If A,B $\in 2^x$ then

(iii) $g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + g_\lambda(A)\, g_\lambda(B)$ with $A \cap B = \phi$

2.usages of choquet integral and sugeno integral

Let us suppose that $g$be a fuzzy measure on X , then Choquet integral of a function $f : X \longrightarrow [0, \infty]$ w.r.to fuzzy measure g is defined

(c) $\int f\, dg = \sum_{i=1}^{n}(f(x_i) - f(x_{i-1}))\, g(A_i)$

where $A_i C_X$ for i = 1,2,3,.....,n

where { f(x_1), f(x_2) , ….., f(x_n)} are the ranges and they are defined as f(x)≤ f(x_1)≤ f(x_2) ….. f(x_n) and f(x_0) = 0

2.1Sugeno Integral

Suppose that μ is a fuzzy normalized measure on *X* , the Sugeno integral of a functiion f:X→[0, 1] w. r. to fuzzy measure μ is defined as ∫f(x)dμ=$\max_{1\leq i\leq n}$(min(𝑓(x_i)) where f(x_1)≤f(x_2)..........f(x_n)

2.1Example

Consider a set X = {x_1, x_2, x_3 } where the ranges are defined as f (x_1) = 0.6, (x_2) = 0.8, f (x_3) = 0.9, such that the fuzzy measure is defined as

Function	**Values**
$\mu(\phi)$	0
$\mu((\{x_1\})$	0.9
$\mu((\{x_2\})$	1
$\mu((\{x_3\})$	0.6
$\mu((\{x_1, x_2\})$	0.8
$\mu((\{x_2, x_3\})$	0.9
$\mu((\{x_1, x_3\})$	0.6
$\mu((\{x_1, x_2, x_3\})$	1.0

The Choquet Integral is

(c) $\int f\, d\mu = \sum_{i=1}^{3}[f(x_i) - f(x_{i-1})](A_i)$

$= f(x_1)[((\{x_1, x_2, x_3\}]+(f(x_2) - f(x_1))\mu(\{x_1, x_2\})+[f(x_3)- f(x_2)]\mu\{x_3\}$

= 0.6* 1.3 + (08-0.6)*0.6+(0.9-0.8)*0.5

$\int f\, d\mu$ = 0.5

2.2Example:

Consider a set X = {x_1, x_2, x_3 } where the ranges are defined as f (x_1) = 0.6, f(x_2) = 0.8, f (x_3) = 0.9, such that the fuzzy measure.

Function	**Values**
$\mu(\phi)$	0
$\mu((\{x_1\})$	0.9
$\mu((\{x_2\})$	1
$\mu((\{x_3\})$	0.6

$\mu((\{x_1, x_2\})$	0.8
$\mu((\{x_2, x_3\})$	0.9
$\mu((\{x_1, x_3\})$	0.6
$\mu((\{x_1, x_2, x_3\})$	1.2

we are dividing all of them by the largest value in measure.

Function	Values
$\mu(\phi)$	0
$\mu((\{x_1\})$	0.75
$\mu((\{x_2\})$	0.8333
$\mu((\{x_3\})$	0.5
$\mu((\{x_1, x_2\})$	0.6667
$\mu((\{x_2, x_3\})$	0.8333
$\mu((\{x_1, x_3\})$	0.5

The Sugeno integral

$$\int f(x)d\mu = (\min(((\min(f(x_i),\mu(A_i)))$$
$$= m \quad \max \quad .6,1),\min(0.8,0.8333),\min(0.9,0.5)$$
$$= m \; 1 \leq i \leq n \; 3,0.5)$$
$$\int f(x)d\mu = 0.8$$

3.Application of choquet integral

In this section we have to apply choquet integral in decision making and using medicine with examples

3.1 Mean

Consider the set of data X $\{x_1, x_2,..........\}$ then the arithmetic mean is defined as

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

3.2 Median

Median is defined the middle value of the given objects

med $(x_1, x_2,..........,x_n)= x_{(n+1)/_2}$ where n is odd

med $(x_1, x_2,..........,x_n)= \frac{1}{2}\,(x_{n/_2}+ x_{n/_2+1})$where n is even

3.3 Ordering Weighted Averaging Operator

The operator has been introduced by yager, which is defined as

$$\text{OWA} = F(a_1, a_2,\ldots, a_n) = \sum_{i=1}^{n} \omega_i a_{(i)}$$

where $a_{(1)}\leq a_{(2)}.........\, a_{(n)}$

and

$$\sum_{i=1}^{n} \omega_i = 1$$

3.4 Fuzzy Measure

A fuzzy measure of the set X is a function$g$: 2*→[0,1] such that the following condition are satisfied

(i) $g(\phi) = 0$

(ii) $g(X)$ =1 ((i) and (ii) is also called boundary conditions)

(iv) If $A \subseteq B \in X$ then g(A) ≤ g(B).

This property is called monotonicity where (A) indicates the weights of importance for a set A.A fuzzy measure is called additive if $(A \cup B) = g(A) + g(B)$ whenever $A \cap B$ = ,super additive if $g(A \cup B) \geq g(A) + g(B)$ whenever $A \cap B = \phi$ and sub additive if $g(A \cup B) \leq g(A) + g(B)$ whenever $A \cap B = \phi$.

Let X is a finite set X = {$x_1, x_2, \ldots, x_n$}and P(X) be the class of all subsets of X the fuzzy measures $g(X) = g_\lambda\{x_1, x_2, \ldots, x_n\}$ can be formulated as

$$(\{x_1, x_2, \ldots, x_n\}) = \sum_{i=1}^{n} g_i + \lambda \sum_{i_1=1}^{n-1} \sum_{i_2=i_1+1}^{n} g_{i_1} g_{i_2} + \ldots \lambda^{n-1} g_1 g_2 \ldots g_n$$

$$= {}^{1}/_{\lambda} \left[\prod_{i=1}^{n}(1+\lambda g_i) - 1\right]$$

Where $\lambda \in (-1,\infty)$

Example 3.1

The mathematics teacher has to evaluate her students according to their level in algebra , Real and differential equation she gives equal importance to algebra and Real Analysis and less importance to Differential Equation.

Students	Algebra	Real Analysis	Differential Equation
A_1	45	50	40
A_2	56	35	50
A_3	39	58	55
A_4	58	58	57

Graces of importance is given by

$g_\lambda(\{x_1\})$ = g({algebra}) = 0.45

$g_\lambda(\{x_2\})$ = ({Real Analysis}) = 0.45

g({x_3}) = ({Differental Equation}) = 0.3

$\lambda = -0.4492$

g({x_1, x_2}) = 0.8090

g({x_1, x_3}) = 0.6894

g({x_2, x_3}) = 0.6894

g({x}) = $g_\lambda(\{x_1, x_2, x_3\}) = 1$

$C_1 = (c)\int f\, dg_\lambda$

$= f(x_3)(\{x_1, x_2, x_3\}] + (f(x_1) - f(x_3))g_\lambda(\{x_1, x_2\}) + [f(x_2) - f(x_1)]g_\lambda(\{x_2\})$

$C_1 = (c)\int f\, dg_\lambda$ = 40* 1 +(45-40)*0.8090 + (50-45)*0.45

$C_1 = (c)\int f\, dg_\lambda$ = 46.295

$C_2 = (c)\int f\, dg$

$= f(x_2)(\{x_1, x_2, x_3\}] + (f(x_3) - f(x_2))g_\lambda(\{x_3, x_2\}) + [f(x_1) - f(x_2)]g_\lambda(\{x_1\})$

$C_2 = (c)\int f\, dg$= 35*1+ (50 – 35) *0.6894+ (56 – 50) *45

$C_2 = (c)\int f\, dg$ = 48.041

$C_3 = (c)\int f\, dg$

$= f(x_1)(\{x_1, x_2, x_3\}] + (f(x_3) - f(x_1))g_\lambda(\{x_2, x_3\}) + [f(x_2) - f(x_3)]g_\lambda(\{x_2\})$

= $(c)\int f\, dg$ = 39 *1 (55 - 39) *0.6894+ (58 - 55) *0.3

= 51.3804 3

$C_4 = (c)\int f\, dg$

$= f(x_2)(\{x_1, x_2, x_3\}] + (f(x_3) - f(x_2))g_\lambda(\{x_3, x_2\}) + [f(x_1) - f(x_2)]g_\lambda(\{x_1\})$

= 38 *1 + (57 - 38) * 0.6894 (58 - 57) * 0.45
= 51.5481 4
The ranking of students $A_4 > A_3 > A_2 > A_1$

Choquet integral in Medical Analysis:

Introduce the notions of a space of states $X = \{x_1, x_2, ..., x_n\}$and a decision space A = (a_1, a_2,....., a_n). Consider a decision model in which n alternativesa_1, a_2,....., $a_n \in$ A act as drugs used to treat. Patients who suffer from a diseases

The medicines should influences m states a_1, a_2,....., $a_n \in$ A, which are identified with m symptoms typical of the morbid.

Effectiveness	**Representing Z Values**	**μ(Z)**
None	0	0
Almost none	10	.01
Very little	20	0.2
Little	30	0.3
Rather little	40	0.4
Medium	50	0.5
Rather large	60	0.6
Large	70	0.7
Very Large	80	0.8
Almost complete	90	0.9
Complete	100	1

Example 2.2

The following clinical data concerns the diagnosis "heart disease".we consider the most substantial symptoms x_1 ="pain in chest", x_2 ="changes in ECG" andE_3 ="Statine LDL reductor".

a_i/x_j	x_1	x_2	x_3
a_1	Completeu_{11}=1	Very largeu_{12} = 0.8	Almostnone u_{13}=0.1
a_2	Mediumu_{21} = 0.5	Rather largeu_{22} = 0.6	**littleu_{23}=0.3**
a_3	Little u_{31}=0.3	littleu_{32} =0.3	**very largeu_{33}=0.8**

The physician status of a patient subjectively better if x_1="pain inchest" disappears. The next is assigned to x_2= "changes in ECG" and x_3 ="increased level of LDL cholesterol".

$$B = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} 1 & 3 & 5 \\ 1/3 & 1 & 3 \\ 1/5 & 1/3 & 1 \end{pmatrix} \end{array}$$

The largest eigen value of B_i is λ = 3.0385 and the eigen vector$V$ = (0.93295,0.303787,0.18659).

The Choquet integral
$a_1 = (c)\int f\, dg$

$$= f(x_3)g[\{x_1, x_2, x_3\}] + (f(x_2) - f(x_3))g_\lambda(\{x_1, x_2\}) + [f(x_1) - f(x_2)]g_\lambda(\{x_1\})$$

$a_2 = (c)\int f\, dg$
$$= f(x_3)g_\lambda\{x_1, x_2, x_3\}] + (f(x_1) - f(x_3))g_\lambda(\{x_1, x_2\}) + [f(x_2) - f(x_1)]g_\lambda(\{x_2\})$$

$a_3 = (c)\int f\, dg$

$=f(x_1)\boldsymbol{g_\lambda}(\{x_1, x_2, x_3\}]+(f(x_2) - f(x_1))g_\lambda(\{x_2, x_3\})+[f(x_3)- f(x_2)]g_\lambda(\{x_3\})$

$g_\lambda(\{x_1\}) = 0.93295$
$g_\lambda(\{x_2\}) = 0.303787$
$g_\lambda(\{x_3\}) = 0.08659$
Since $\lambda +1 = \prod_{i=1}^{n}(\lambda g_i + 1)$
$\lambda +1 = (0.93295\lambda +1)(0.303787\lambda +1)(0.18659\lambda +1)$
$0.05287 \lambda^3+ 0.5141\lambda^2 + 0.42329 \lambda= 0$
$\lambda = \{0,-8.81, - 0.9082\}$
But $\lambda= (-1,\infty)$
take λ = -0.9082 only, because λ = 0 is additively.

$g_\lambda(\{x_1, x_2\}) = (\{x_1\}) + g(\{x_2\}) + \lambda g_\lambda(\{x_1\})\ g_\lambda(\{x_2\})$
= 0.979386
$g_\lambda(\{x_2, x_3\}) = (\{x_3\}) + g(\{x_2\}) + \lambda g_\lambda(\{x_3\})\ g_\lambda(\{x_2\})$
= 0.438924
$g_\lambda(\{x_1, x_3\}) = (\{x_1\}) + g(\{x_3\}) + \lambda g_\lambda(\{x_1\}) + g_\lambda(\{x_3\})$
= 0.961496.

a_1 = (c)$\int f\ dg$
$=f(x_3)g_\lambda(\{x_1, x_2, x_3\}]+(f(x_2) - f(x_3))g_\lambda(\{x_1, x_2\})+[f(x_1)- f(x_2)]g_\lambda(\{x_1\})$
= 0.1*1+ (0.8 - 0.1)*0.9793+ (1- 0.8)*0.93295
= 0.97
a_2 = (c)$\int f\ dg$
$=f(x_3)g_\lambda(\{x_1, x_2, x_3\}]+(f(x_1) - f(x_3))g_\lambda(\{x_1, x_2\})+[f(x_2)- f(x_1)]g_\lambda(\{x_2\})$
= 0.3*1+ (0.5- 0.3)*0.96149+ (.6 - 0.5)*0.303
= 0.52
a_3 = (c)$\int f\ dg$
$=f(x_1)g_\lambda\{x_1, x_2, x_3\}]+(f(x_2) - f(x_1))g_\lambda(\{x_2, x_3\})+[f(x_3)- f(x_2)]g_\lambda(\{x_3\})$
= 0.3*1+ 0 + (.8- 0.3)*0.1866
= 0.39
The Sugeno integral in drug order
a_1 = (c)$\int f\ dg_\lambda$=max(min $(f(x_3)g_\lambda(\{x_1, x_2, x_3\}])$ min $(f(x_2), g_\lambda(\{x_1,x_2\})$,
min$(f(x_1)g_\lambda(\{ x_2\})))$,
= max(min(0.1,1),min(0.8,0.9793),min(1,0.93295))
= 0.93295
a_2 = max(min(0.3,1),min(0.5,0.979386),min(0.6,0.93295))
= 0.5
a_3 = max(min(0.3,1),min(0.3,0.438942),min(0.8,0.18659))
= max(0.3,0.3,0.18659)
= 0.3
Sugeno integral in the drug ranking
$a_1 > a_2 > a_3$.

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