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Generalization of the Great Ricci Curvature Vector Formulae Based Upon the Great Metric Tensor in Cartesian Coordinates

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Abstract: In this paper, we expressed the Riemannian curvature tensor in terms of the Great Metric Tensor for all gravitational fields in nature in the Cartesian coordinates. Hence the results are used to derive the generalized Riemannian curvature scalar and Ricci Curvature tensor formulae in the Cartesian coordinates which are mathematically most elegant, physically most natural and satisfactory.

Keywords: Riemannian, Great Metric Tensor, Cartesian Coordinates and Ricci curvature Vector.

1:0 INTRODUCTIONS

The Cartesian coordinates (x, y, z, x^0) are defined in terms of the spherical polar coordinates $(r, \theta, \varphi, x^0)$ by [1-2]

$$x = rsin\theta cos\varphi \quad (1)$$

$$y = rsin\theta sin\varphi \quad (2)$$

$$z = rcos\theta \quad (3)$$

Here:

$$r = [x^2 + y^2 + z^2]^{\frac{1}{2}} \quad (4)$$

and

$$\theta = \cos^{-1}\left[\frac{z}{(x^2+y^2+z^2)^{\frac{1}{2}}}\right] \quad (5)$$

and

$$\phi = \tan^{-1}\left[\frac{y}{x}\right] \quad (6)$$

The great metric tensor for all gravitational fields in nature in spherical polar coordinates (r, θ, ϕ, x^0) is given as [3].

$$g_{00} = -\left(1 + \frac{2}{c^2} f\right) \quad (7)$$

$$g_{11} = \left(1 + \frac{2}{c^2} f\right)^{-1} \quad (8)$$

$$g_{22} = r^2 \quad (9)$$

$$g_{33} = r^2 \sin^2\theta \quad (10)$$

$$g_{uv} = 0\,; Otherwise \quad (11)$$

From the well know transformation equation given by the covariant tensor [6,7] and consequently, upon transformation by using (2.1)-(2.8) we obtained the Riemannian metric tensor for all gravitational fields in the Cartesian coordinates explicitly as [4]:

$$g_{uv} = h_{uv} + f_{uv} \quad (12)$$

where

$$h_{11} = h_{22} = h_{33} = 1 \quad (13)$$

$$f_{11} = \frac{x^2}{[x^2 + y^2 + z^2]} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \quad (14)$$

$$f_{12} = f_{21} = \frac{xy}{[x^2 + y^2 + z^2]} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n = g_{21} \quad (15)$$

$$f_{13} = f_{31} = \frac{xz}{[x^2 + y^2 + z^2]} \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \quad (16)$$

$$f_{22} = \frac{y^2}{[x^2+y^2+z^2]}\sum_{n=1}^{\infty}\binom{-1}{n}\left(\frac{2}{c^2}\right)^n f^n \quad (17)$$

$$f_{23} = f_{32} = \frac{yz}{[x^2+y^2+z^2]}\sum_{n=1}^{\infty}\binom{-1}{n}\left(\frac{2}{c^2}\right)^n f^n \quad (18)$$

$$f_{33} = \frac{z^2}{[x^2+y^2+z^2]}\sum_{n=1}^{\infty}\binom{-1}{n}\left(\frac{2}{c^2}\right)^n f^n \quad (19)$$

$$h_{oo} = -1 \quad (20)$$

$$f_{oo} = -\frac{2}{c^2}f^n \quad (21)$$

and

$$h_{uv} = 0 = f_{uv}\ ; Otherwise \quad (22)$$

and the determinant of this metric tensor, denoted as g is given by

$$g = -1 \quad (23)$$

if g_{uv} is a covariant metric tensor, then according to tensor analysis the contravariant metric tensor for this Riemannian metric tensor denote as g^{uv} is given as:

$$g^{00} = -\left(1+\frac{2}{c^2}f\right)^{-1} \quad (24)$$

$$g^{11} = \left[1+\frac{(x^2+y^2)}{[x^2+y^2+z^2]}\sum_{n=1}^{\infty}\binom{-1}{n}\left(\frac{2}{c^2}\right)^n f^n\right]\left(1+\frac{2}{c^2}f\right) \quad (25)$$

$$g^{12} = \left[\frac{xy}{[x^2+y^2+z^2]}\sum_{n=1}^{\infty}\binom{-1}{n}\left(\frac{2}{c^2}\right)^n f^n\right]\left(1+\frac{2}{c^2}f\right) = g^{21} \quad (26)$$

$$g^{13} = \left[\frac{xz}{[x^2+y^2+z^2]}\sum_{n=1}^{\infty}\binom{-1}{n}\left(\frac{2}{c^2}\right)^n f^n\right]\left(1+\frac{2}{c^2}f\right) = g^{31} \quad (27)$$

$$g^{22} = \left[1+\frac{(x^2+z^2)}{[x^2+y^2+z^2]}\sum_{n=1}^{\infty}\binom{-1}{n}\left(\frac{2}{c^2}\right)^n f^n\right]\left(1+\frac{2}{c^2}f\right) \quad (28)$$

and

$$g^{uv} = 0, otherwise \quad (29)$$

2.0 THEORY

According to the theory of tensor analysis the Riemann curvature tensor, $R^{\delta}_{\alpha\beta\gamma}$ is defined [3] as:

$$R^{\delta}_{\alpha\beta\gamma} = \Gamma^{\delta}_{\alpha\gamma,\beta} - \Gamma^{\delta}_{\alpha\beta,\gamma} + \Gamma^{\epsilon}_{\alpha\gamma}\Gamma^{\delta}_{\epsilon\beta} - \Gamma^{\epsilon}_{\alpha\beta}\Gamma^{\delta}_{\epsilon\gamma} \quad (30)$$

where $\Gamma^{\delta}_{\mu\nu}$ is the Christoffel symbol of the second kind Pseudo tensor and comma denotes one partial differentiation with respect to the corresponding space-times co-ordinates. Therefore the Riemann curvature tensor based upon the Great metric tensor in Cartesian coordinates are given in terms of the Christoffel symbols of the second kind of pseudo tensor as follows

$$R^{0}_{000} = 0 \quad (31)$$

$$R^{1}_{001} = \Gamma^{1}_{01,0} - \Gamma^{1}_{00,1} + \Gamma^{0}_{01}\Gamma^{1}_{00} - \Gamma^{0}_{00}\Gamma^{1}_{01} + \Gamma^{1}_{01}\Gamma^{1}_{10} - \Gamma^{1}_{00}\Gamma^{1}_{11} + \Gamma^{2}_{01}\Gamma^{1}_{20} - \Gamma^{2}_{00}\Gamma^{1}_{21} + \Gamma^{3}_{01}\Gamma^{1}_{30} - \Gamma^{3}_{00}\Gamma^{1}_{31} \quad (33)$$

$$R^{2}_{002} = \Gamma^{2}_{02,0} - \Gamma^{2}_{00,2} + \Gamma^{0}_{02}\Gamma^{2}_{00} - \Gamma^{0}_{00}\Gamma^{2}_{02} + \Gamma^{1}_{02}\Gamma^{2}_{10} - \Gamma^{1}_{00}\Gamma^{2}_{12} + \Gamma^{2}_{02}\Gamma^{2}_{20} - \Gamma^{2}_{00}\Gamma^{2}_{22} + \Gamma^{3}_{02}\Gamma^{2}_{30} - \Gamma^{3}_{00}\Gamma^{2}_{32} \quad (34)$$

$$R^{3}_{003} = \Gamma^{3}_{03,0} - \Gamma^{3}_{00,3} + \Gamma^{0}_{03}\Gamma^{3}_{00} - \Gamma^{0}_{00}\Gamma^{3}_{03} + \Gamma^{1}_{03}\Gamma^{3}_{10} - \Gamma^{1}_{00}\Gamma^{3}_{13} + \Gamma^{2}_{03}\Gamma^{3}_{20} - \Gamma^{2}_{00}\Gamma^{3}_{23} + \Gamma^{3}_{03}\Gamma^{3}_{30} - \Gamma^{3}_{00}\Gamma^{3}_{33} \quad (35)$$

$$R^{0}_{010} = \Gamma^{0}_{00,1} - \Gamma^{0}_{01,0} + \Gamma^{1}_{00}\Gamma^{0}_{11} - \Gamma^{1}_{01}\Gamma^{0}_{10} + \Gamma^{2}_{00}\Gamma^{0}_{21} - \Gamma^{2}_{01}\Gamma^{0}_{20} + \Gamma^{3}_{00}\Gamma^{0}_{31} - \Gamma^{3}_{01}\Gamma^{0}_{30} \quad (36)$$

$$R^{1}_{011} = 0 \quad (37)$$

$$R^{2}_{012} = \Gamma^{2}_{02,1} - \Gamma^{2}_{01,2} + \Gamma^{0}_{02}\Gamma^{2}_{01} - \Gamma^{0}_{02}\Gamma^{2}_{01} + \Gamma^{1}_{02}\Gamma^{2}_{11} - \Gamma^{1}_{01}\Gamma^{2}_{12} + \Gamma^{2}_{02}\Gamma^{2}_{21} - \Gamma^{2}_{01}\Gamma^{2}_{22} + \Gamma^{3}_{02}\Gamma^{2}_{31} - \Gamma^{3}_{01}\Gamma^{2}_{32} \quad (38)$$

$$R^{3}_{013} = \Gamma^{3}_{03,1} - \Gamma^{3}_{01,3} + \Gamma^{0}_{03}\Gamma^{3}_{01} - \Gamma^{0}_{01}\Gamma^{3}_{03} + \Gamma^{1}_{03}\Gamma^{3}_{11} - \Gamma^{1}_{01}\Gamma^{3}_{13} - \Gamma^{2}_{03}\Gamma^{3}_{21} - \Gamma^{2}_{01}\Gamma^{3}_{23} + \Gamma^{3}_{03}\Gamma^{3}_{31} - \Gamma^{3}_{01}\Gamma^{3}_{33} \quad (39)$$

$$R^{0}_{020} = \Gamma^{0}_{00,2} - \Gamma^{0}_{02,0} + \Gamma^{1}_{00}\Gamma^{0}_{12} - \Gamma^{1}_{02}\Gamma^{0}_{10} + \Gamma^{2}_{00}\Gamma^{0}_{02} - \Gamma^{2}_{02}\Gamma^{0}_{20} + \Gamma^{3}_{00}\Gamma^{0}_{32} - \Gamma^{3}_{02}\Gamma^{0}_{30} \quad (40)$$

$$R^1_{021} = \Gamma^1_{01,2} - \Gamma^1_{02,1} + \Gamma^0_{01}\Gamma^1_{02} - \Gamma^0_{02}\Gamma^1_{01} + \Gamma^1_{01}\Gamma^1_{12} - \Gamma^1_{02}\Gamma^1_{01} + \Gamma^2_{01}\Gamma^1_{22} - \Gamma^2_{02}\Gamma^1_{21} + \Gamma^3_{01}\Gamma^1_{32} - \Gamma^3_{02}\Gamma^1_{31} \quad (41)$$

$$R^2_{022} = 0 \quad (42)$$

$$R^3_{023} = \Gamma^3_{03,2} - \Gamma^3_{02,3} + \Gamma^0_{03}\Gamma^3_{02} - \Gamma^0_{02}\Gamma^3_{03} + \Gamma^1_{03}\Gamma^3_{12} - \Gamma^1_{02}\Gamma^3_{13} + \Gamma^2_{03}\Gamma^3_{22} - \Gamma^2_{02}\Gamma^2_{23} + \Gamma^3_{03}\Gamma^3_{32} - \Gamma^3_{02}\Gamma^3_{33} \quad (43)$$

$$R^0_{030} = \Gamma^0_{00,3} - \Gamma^0_{03,0} + \Gamma^1_{00}\Gamma^0_{113} - \Gamma^1_{03}\Gamma^0_{10} + \Gamma^2_{00}\Gamma^0_{23} - \Gamma^2_{03}\Gamma^0_{20} + \Gamma^3_{00}\Gamma^0_{33} - \Gamma^3_{03}\Gamma^0_{30} \quad (44)$$

$$R^1_{031} = \Gamma^1_{01,3} - \Gamma^1_{03,1} + \Gamma^0_{01}\Gamma^1_{03} - \Gamma^0_{03}\Gamma^1_{03} + \Gamma^1_{01}\Gamma^1_{13} - \Gamma^1_{03}\Gamma^1_{11} + \Gamma^2_{02}\Gamma^1_{23} - \Gamma^2_{03}\Gamma^1_{21} + \Gamma^3_{01}\Gamma^1_{33} - \Gamma^3_{03}\Gamma^1_{31} \quad (45)$$

$$R^2_{032} = \Gamma^2_{02,3} - \Gamma^2_{03,2} + \Gamma^0_{02}\Gamma^2_{03} - \Gamma^0_{03}\Gamma^2_{02} + \Gamma^1_{02}\Gamma^2_{13} - \Gamma^1_{03}\Gamma^2_{12} + \Gamma^2_{02}\Gamma^2_{23} - \Gamma^2_{03}\Gamma^2_{22} - \Gamma^3_{02}\Gamma^2_{33} - \Gamma^3_{03}\Gamma^2_{32} \quad (46)$$

$$R^3_{033} = 0 \quad (47)$$

$$R^0_{110} = \Gamma^0_{10,1} - \Gamma^0_{11,0} + \Gamma^0_{10}\Gamma^0_{01} - \Gamma^0_{11}\Gamma^0_{00} + \Gamma^0_{10}\Gamma^0_{11} - \Gamma^1_{11}\Gamma^0_{10} + \Gamma^2_{10}\Gamma^0_{21} - \Gamma^2_{11}\Gamma^0_{20} + \Gamma^3_{10}\Gamma^0_{31} - \Gamma^3_{11}\Gamma^0_{30} \quad (48)$$

$$R^1_{111} = 0 \quad (49)$$

$$R^2_{112} = \Gamma^2_{12,1} - \Gamma^2_{11,2} + \Gamma^0_{12}\Gamma^2_{01} - \Gamma^0_{11}\Gamma^2_{02} + \Gamma^1_{12}\Gamma^2_{11} - \Gamma^1_{11}\Gamma^2_{12} + \Gamma^2_{12}\Gamma^2_{21} - \Gamma^2_{11}\Gamma^2_{22} + \Gamma^3_{12}\Gamma^2_{31} - \Gamma^3_{11}\Gamma^2_{32} \quad (50)$$

$$R^3_{113} = \Gamma^3_{13,1} - \Gamma^3_{11,3} + \Gamma^0_{13}\Gamma^3_{01} - \Gamma^0_{11}\Gamma^3_{03} + \Gamma^1_{13}\Gamma^3_{11} - \Gamma^1_{11}\Gamma^3_{13} + \Gamma^2_{13}\Gamma^3_{21} - \Gamma^2_{11}\Gamma^3_{23} + \Gamma^3_{13}\Gamma^3_{31} - \Gamma^3_{11}\Gamma^3_{33} \quad (51)$$

$$R^0_{120} = \Gamma^0_{10,2} - \Gamma^0_{12,0} + \Gamma^0_{10}\Gamma^0_{02} - \Gamma^0_{12}\Gamma^0_{00} + \Gamma^1_{10}\Gamma^0_{12} - \Gamma^1_{12}\Gamma^0_{10} + \Gamma^2_{10}\Gamma^0_{22} - \Gamma^2_{12}\Gamma^0_{20} + \Gamma^3_{10}\Gamma^0_{32} - \Gamma^3_{12}\Gamma^0_{30} \quad (52)$$

$$R^1_{121} = \Gamma^1_{11,2} - \Gamma^1_{12,1} + \Gamma^0_{11}\Gamma^1_{02} - \Gamma^0_{12}\Gamma^1_{01} + \Gamma^2_{11}\Gamma^1_{22} - \Gamma^2_{12}\Gamma^1_{21} + \Gamma^3_{11}\Gamma^1_{32} - \Gamma^3_{12}\Gamma^1_{31} \quad (53)$$

$$R^2_{122} = 0 \quad (54)$$

$$R^3_{123} = \Gamma^3_{13,2} - \Gamma^3_{12,3} + \Gamma^0_{13}\Gamma^3_{02} - \Gamma^0_{12}\Gamma^3_{03} + \Gamma^1_{13}\Gamma^3_{12} - \Gamma^1_{12}\Gamma^3_{13} + \Gamma^2_{13}\Gamma^3_{22} - \Gamma^2_{12}\Gamma^3_{23} + \Gamma^3_{13}\Gamma^3_{32} - \Gamma^3_{12}\Gamma^3_{33} \quad (55)$$

$$R^0_{130} = \Gamma^0_{10,3} - \Gamma^0_{13,0} + \Gamma^0_{10}\Gamma^0_{03} - \Gamma^0_{13}\Gamma^0_{00} + \Gamma^1_{10}\Gamma^0_{13} - \Gamma^1_{13}\Gamma^0_{10} + \Gamma^2_{10}\Gamma^0_{23} - \Gamma^2_{13}\Gamma^0_{20} + \Gamma^3_{10}\Gamma^0_{33} - \Gamma^3_{13}\Gamma^0_{30} \quad (56)$$

$$R^1_{131} = \Gamma^1_{11,3} - \Gamma^1_{13,1} + \Gamma^0_{11}\Gamma^1_{03} - \Gamma^0_{13}\Gamma^1_{01} + \Gamma^2_{11}\Gamma^1_{23} - \Gamma^2_{13}\Gamma^1_{21} + \Gamma^3_{11}\Gamma^1_{33} - \Gamma^3_{13}\Gamma^1_{31} \quad (57)$$

$$R^2_{132} = \Gamma^2_{12,3} - \Gamma^2_{13,2} + \Gamma^0_{12}\Gamma^2_{03} - \Gamma^0_{13}\Gamma^2_{03} + \Gamma^1_{12}\Gamma^2_{13} - \Gamma^1_{13}\Gamma^2_{12} + \Gamma^2_{12}\Gamma^2_{23} - \Gamma^2_{13}\Gamma^2_{22} + \Gamma^3_{12}\Gamma^2_{33} - \Gamma^3_{13}\Gamma^2_{32} \quad (58)$$

$$R^3_{133} = 0 \quad (59)$$

$$R^0_{220} = \Gamma^0_{20,2} - \Gamma^0_{22,0} + \Gamma^0_{20}\Gamma^0_{02} - \Gamma^0_{22}\Gamma^0_{00} + \Gamma^1_{20}\Gamma^0_{12} - \Gamma^1_{22}\Gamma^0_{10} + \Gamma^2_{20}\Gamma^0_{22} - \Gamma^2_{22}\Gamma^0_{20} + \Gamma^3_{20}\Gamma^0_{32} - \Gamma^3_{22}\Gamma^0_{30} \quad (60)$$

$$R^1_{221} = \Gamma^1_{21,2} - \Gamma^1_{22,1} + \Gamma^0_{21}\Gamma^1_{02} - \Gamma^0_{22}\Gamma^1_{01} + \Gamma^1_{21}\Gamma^1_{12} - \Gamma^1_{22}\Gamma^1_{11} + \Gamma^2_{21}\Gamma^1_{22} - \Gamma^2_{22}\Gamma^1_{21} + \Gamma^3_{21}\Gamma^1_{32} - \Gamma^3_{22}\Gamma^1_{31} \quad (61)$$

$$R^2_{222} = 0 \quad (62)$$

$$R^3_{223} = \Gamma^3_{23,2} - \Gamma^3_{22,3} + \Gamma^0_{23}\Gamma^3_{02} - \Gamma^0_{22}\Gamma^3_{03} + \Gamma^1_{23}\Gamma^3_{12} - \Gamma^1_{22}\Gamma^3_{13} + \Gamma^2_{23}\Gamma^3_{22} - \Gamma^2_{22}\Gamma^3_{23} + \Gamma^3_{23}\Gamma^3_{32} - \Gamma^3_{22}\Gamma^3_{33} \quad (63)$$

$$R^0_{230} = \Gamma^0_{20,3} - \Gamma^0_{23,0} + \Gamma^0_{20}\Gamma^0_{03} - \Gamma^0_{23}\Gamma^0_{00} + \Gamma^1_{20}\Gamma^0_{13} - \Gamma^1_{23}\Gamma^0_{10} + \Gamma^2_{20}\Gamma^0_{23} - \Gamma^2_{23}\Gamma^0_{20} + \Gamma^3_{20}\Gamma^0_{23} - \Gamma^3_{23}\Gamma^0_{30} \quad (64)$$

$$R^1_{231} = \Gamma^1_{21,3} - \Gamma^1_{23,1} + \Gamma^0_{21}\Gamma^1_{03} - \Gamma^0_{23}\Gamma^1_{01} + \Gamma^1_{21}\Gamma^1_{23} - \Gamma^1_{23}\Gamma^1_{21} + \Gamma^2_{21}\Gamma^1_{23} - \Gamma^2_{23}\Gamma^1_{21} + \Gamma^3_{21}\Gamma^1_{33} - \Gamma^3_{23}\Gamma^1_{31} \quad (65)$$

$$R^2_{232} = \Gamma^2_{22,3} - \Gamma^2_{23,2} + \Gamma^0_{22}\Gamma^2_{03} - \Gamma^0_{23}\Gamma^2_{02} + \Gamma^1_{22}\Gamma^2_{13} - \Gamma^1_{23}\Gamma^2_{12} + \Gamma^3_{22}\Gamma^2_{33} - \Gamma^3_{23}\Gamma^2_{32} \quad (66)$$

$$R^3_{233} = 0 \quad (67)$$

$$R^0_{330} = \Gamma^0_{30,3} - \Gamma^0_{33,0} + \Gamma^0_{30}\Gamma^0_{03} - \Gamma^0_{33}\Gamma^0_{00} + \Gamma^1_{30}\Gamma^0_{13} - \Gamma^1_{33}\Gamma^0_{10} + \Gamma^2_{30}\Gamma^0_{23} - \Gamma^2_{33}\Gamma^0_{20} + \Gamma^3_{30}\Gamma^0_{33} - \Gamma^3_{33}\Gamma^0_{30} \quad (68)$$

$$R^1_{331} = \Gamma^1_{31,3} - \Gamma^1_{33,1} + \Gamma^0_{31}\Gamma^1_{03} - \Gamma^0_{33}\Gamma^1_{01} + \Gamma^1_{31}\Gamma^1_{13} - \Gamma^1_{33}\Gamma^1_{11} + \Gamma^2_{31}\Gamma^1_{23} - \Gamma^2_{33}\Gamma^1_{21} + \Gamma^3_{31}\Gamma^1_{33} - \Gamma^3_{33}\Gamma^1_{31} \quad (69)$$

$$R^2_{332} = \Gamma^2_{33,3} - \Gamma^2_{33,2} + \Gamma^0_{32}\Gamma^2_{03} - \Gamma^0_{33}\Gamma^2_{02} + \Gamma^1_{31}\Gamma^2_{13} - \Gamma^1_{33}\Gamma^2_{12} + \Gamma^2_{32}\Gamma^2_{23} - \Gamma^2_{33}\Gamma^2_{22} + \Gamma^3_{32}\Gamma^2_{33} - \Gamma^3_{33}\Gamma^2_{32} \quad (70)$$

$$R^3_{333} = 0 \quad (71)$$

These are the formulae for the Riemann Curvature Tensor in all gravitational fields based upon great metric tensors in Cartesian Coordinates. In this paper, we demonstrate how to use the Riemann curvature tensor in Cartesian coordinates to formulate hitherto unknown but mathematically most elegant, physically most natural generalize Great Riemannian Curvature Scalar and Great Ricci Curvature Tensor formulae in Cartesian coordinate. It is imperative to say that a comma indicate a partial differentiation with respect to the unit vector. Thus (1,2,3,0) denotes partial differentiation with respect to (x, y, z, ct).

2.1 The Generalization of the Great Ricci Curvature Tensor in Cartesian Coordinates

It may be noted that the Ricci Curvature Tensor in 4 dimensional space-times denotes as, $R_{\mu\nu}$, is given in all gravitational fields and all Orthogonal Curvilinear Coordinates x^{α} by [5]:

$$R_{\mu\nu} = R^{\epsilon}_{\mu\nu\epsilon}, \ \epsilon = 0,1,2,3 \qquad (72)$$

Based upon the great metric tensors in Cartesian Coordinates, the Great Ricci Curvature tensors formulae are given as follows:

$$\begin{aligned} R_{00} = {} & \Gamma^{1}_{01,0} - \Gamma^{1}_{00,1} + \Gamma^{2}_{02,0} - \Gamma^{2}_{00,2} + \Gamma^{3}_{03,0} - \Gamma^{3}_{00,3} + \Gamma^{0}_{01}\Gamma^{1}_{00} - \Gamma^{0}_{00}\Gamma^{1}_{01} + \Gamma^{1}_{01}\Gamma^{1}_{10} - \Gamma^{1}_{00}\Gamma^{1}_{11} + 2\Gamma^{2}_{01}\Gamma^{1}_{20} \\ & - \Gamma^{2}_{00}\Gamma^{1}_{21} + 2\Gamma^{3}_{01}\Gamma^{1}_{30} - \Gamma^{3}_{00}\Gamma^{1}_{31} + \Gamma^{0}_{02}\Gamma^{2}_{00} - \Gamma^{0}_{00}\Gamma^{2}_{02} - \Gamma^{1}_{00}\Gamma^{2}_{12} + \Gamma^{2}_{02}\Gamma^{2}_{20} - \Gamma^{2}_{00}\Gamma^{2}_{22} + 2\Gamma^{3}_{02}\Gamma^{2}_{30} \\ & - \Gamma^{3}_{00}\Gamma^{2}_{32} + \Gamma^{0}_{03}\Gamma^{3}_{00} - \Gamma^{0}_{00}\Gamma^{3}_{03} - \Gamma^{1}_{00}\Gamma^{1}_{13} - \Gamma^{2}_{00}\Gamma^{3}_{23} + \Gamma^{3}_{03}\Gamma^{3}_{30} \\ & - \Gamma^{3}_{00}\Gamma^{3}_{33} \qquad (73) \end{aligned}$$

$$\begin{aligned} R_{01} = R_{10} = {} & \Gamma^{0}_{00,1} - \Gamma^{0}_{01,0} + \Gamma^{2}_{02,1} - \Gamma^{2}_{01,2} + \Gamma^{3}_{03,1} - \Gamma^{3}_{01,3} + \Gamma^{1}_{00}\Gamma^{0}_{11} - \Gamma^{1}_{01}\Gamma^{0}_{10} + \Gamma^{2}_{00}\Gamma^{0}_{21} + \Gamma^{3}_{00}\Gamma^{0}_{31} - \Gamma^{0}_{01}\Gamma^{2}_{02} \\ & + \Gamma^{1}_{02}\Gamma^{2}_{11} - \Gamma^{1}_{01}\Gamma^{2}_{12} + \Gamma^{2}_{02}\Gamma^{2}_{21} - \Gamma^{2}_{01}\Gamma^{2}_{22} + \Gamma^{3}_{02}\Gamma^{2}_{31} - \Gamma^{3}_{01}\Gamma^{2}_{32} - \Gamma^{0}_{01}\Gamma^{3}_{03} + \Gamma^{1}_{03}\Gamma^{3}_{11} - \Gamma^{1}_{01}\Gamma^{3}_{13} \\ & + \Gamma^{2}_{03}\Gamma^{3}_{21} - \Gamma^{2}_{01}\Gamma^{3}_{23} + \Gamma^{3}_{03}\Gamma^{3}_{31} - \Gamma^{3}_{01}\Gamma^{3}_{33} \qquad (74) \end{aligned}$$

$$\begin{aligned} R_{02} = R_{20} = {} & \Gamma^{0}_{00,2} - \Gamma^{0}_{02,0} + \Gamma^{1}_{01,2} - \Gamma^{1}_{02,1} + \Gamma^{3}_{03,2} - \Gamma^{3}_{02,3} + \Gamma^{1}_{00}\Gamma^{0}_{12} + \Gamma^{2}_{00}\Gamma^{0}_{22} - \Gamma^{2}_{02}\Gamma^{0}_{20} + \Gamma^{3}_{00}\Gamma^{0}_{32} - \Gamma^{0}_{02}\Gamma^{1}_{01} \\ & + \Gamma^{1}_{01}\Gamma^{1}_{12} - \Gamma^{1}_{02}\Gamma^{1}_{11} + \Gamma^{2}_{01}\Gamma^{1}_{22} - \Gamma^{2}_{02}\Gamma^{1}_{21} + \Gamma^{3}_{01}\Gamma^{1}_{32} - \Gamma^{3}_{02}\Gamma^{1}_{31} - \Gamma^{0}_{02}\Gamma^{3}_{03} + \Gamma^{1}_{03}\Gamma^{3}_{12} - \Gamma^{1}_{02}\Gamma^{3}_{13} \\ & + \Gamma^{2}_{03}\Gamma^{3}_{22} - \Gamma^{2}_{02}\Gamma^{3}_{23} + \Gamma^{3}_{03}\Gamma^{3}_{32} - \Gamma^{3}_{02}\Gamma^{3}_{33} \qquad (75) \end{aligned}$$

$$\begin{aligned} R_{03} = R_{30} = {} & \Gamma^{0}_{00,3} - \Gamma^{0}_{03,0} + \Gamma^{1}_{01,1} - \Gamma^{1}_{03,2} + \Gamma^{2}_{02,3} - \Gamma^{2}_{03,2} + \Gamma^{1}_{00}\Gamma^{0}_{13} + \Gamma^{2}_{00}\Gamma^{0}_{23} + \Gamma^{3}_{00}\Gamma^{0}_{33} - \Gamma^{3}_{03}\Gamma^{0}_{30} - \Gamma^{0}_{03}\Gamma^{1}_{01} \\ & + \Gamma^{1}_{01}\Gamma^{1}_{13} - \Gamma^{1}_{03}\Gamma^{1}_{11} + \Gamma^{2}_{01}\Gamma^{1}_{23} - \Gamma^{2}_{03}\Gamma^{1}_{21} + \Gamma^{3}_{01}\Gamma^{1}_{33} - \Gamma^{3}_{03}\Gamma^{1}_{31} - \Gamma^{0}_{03}\Gamma^{2}_{02} + \Gamma^{1}_{02}\Gamma^{2}_{13} - \Gamma^{1}_{03}\Gamma^{2}_{12} \\ & + \Gamma^{2}_{02}\Gamma^{2}_{23} - \Gamma^{2}_{03}\Gamma^{2}_{22} + \Gamma^{3}_{02}\Gamma^{2}_{33} - \Gamma^{3}_{03}\Gamma^{2}_{32} \qquad (76) \end{aligned}$$

$$\begin{aligned} R_{11} = {} & \Gamma^{0}_{10,1} - \Gamma^{0}_{11,0} + \Gamma^{2}_{12,1} - \Gamma^{2}_{11,2} + \Gamma^{3}_{13,1} - \Gamma^{3}_{11,3} + \Gamma^{0}_{10}\Gamma^{0}_{01} - \Gamma^{0}_{11}\Gamma^{0}_{00} + \Gamma^{1}_{10}\Gamma^{1}_{11} - \Gamma^{1}_{11}\Gamma^{0}_{10} + 2\Gamma^{2}_{10}\Gamma^{0}_{21} - \Gamma^{2}_{11}\Gamma^{0}_{20} \\ & + 2\Gamma^{3}_{10}\Gamma^{0}_{31} - \Gamma^{3}_{11}\Gamma^{0}_{30} - \Gamma^{0}_{11}\Gamma^{2}_{02} + \Gamma^{1}_{12}\Gamma^{2}_{11} - \Gamma^{1}_{11}\Gamma^{2}_{12} + \Gamma^{2}_{12}\Gamma^{2}_{21} - \Gamma^{2}_{11}\Gamma^{2}_{22} + 2\Gamma^{3}_{12}\Gamma^{2}_{31} - \Gamma^{3}_{11}\Gamma^{2}_{32} \\ & + \Gamma^{0}_{11}\Gamma^{3}_{03} + \Gamma^{1}_{13}\Gamma^{3}_{11} - \Gamma^{1}_{11}\Gamma^{3}_{13} - \Gamma^{2}_{11}\Gamma^{3}_{23} + \Gamma^{3}_{13}\Gamma^{3}_{31} \\ & - \Gamma^{3}_{11}\Gamma^{3}_{33} \qquad (77) \end{aligned}$$

$$\begin{aligned} R_{12} = R_{21} = {} & \Gamma^{0}_{10,2} - \Gamma^{0}_{12,0} + \Gamma^{1}_{11,2} - \Gamma^{1}_{12,1} + \Gamma^{3}_{13,2} - \Gamma^{3}_{12,3} + \Gamma^{1}_{10}\Gamma^{0}_{02} - \Gamma^{0}_{12}\Gamma^{0}_{00} - \Gamma^{1}_{12}\Gamma^{0}_{10} + \Gamma^{2}_{10}\Gamma^{0}_{22} - \Gamma^{2}_{12}\Gamma^{0}_{20} \\ & + \Gamma^{3}_{10}\Gamma^{0}_{30} - \Gamma^{3}_{12}\Gamma^{0}_{30} + \Gamma^{0}_{11}\Gamma^{1}_{02} + \Gamma^{2}_{11}\Gamma^{1}_{22} - \Gamma^{2}_{12}\Gamma^{1}_{21} + \Gamma^{3}_{11}\Gamma^{1}_{32} + \Gamma^{0}_{13}\Gamma^{3}_{02} - \Gamma^{0}_{12}\Gamma^{3}_{03} - \Gamma^{1}_{12}\Gamma^{3}_{13} \\ & + \Gamma^{2}_{13}\Gamma^{3}_{22} - \Gamma^{2}_{12}\Gamma^{3}_{23} + \Gamma^{3}_{13}\Gamma^{3}_{33} - \Gamma^{3}_{12}\Gamma^{3}_{33} \qquad (78) \end{aligned}$$

$$\begin{aligned} R_{13} = R_{31} = {} & \Gamma^{0}_{10,3} - \Gamma^{0}_{13,0} + \Gamma^{1}_{11,3} - \Gamma^{1}_{13,1} + \Gamma^{2}_{12,3} - \Gamma^{2}_{13,2} + \Gamma^{0}_{10}\Gamma^{0}_{03} - \Gamma^{0}_{13}\Gamma^{0}_{00} - \Gamma^{1}_{13}\Gamma^{0}_{10} + \Gamma^{2}_{10}\Gamma^{0}_{23} - \Gamma^{2}_{13}\Gamma^{0}_{20} \\ & + \Gamma^{3}_{10}\Gamma^{0}_{33} - \Gamma^{3}_{13}\Gamma^{0}_{30} + \Gamma^{0}_{11}\Gamma^{1}_{03} + \Gamma^{2}_{11}\Gamma^{1}_{23} + \Gamma^{3}_{11}\Gamma^{1}_{33} - \Gamma^{3}_{13}\Gamma^{1}_{31} + \Gamma^{0}_{12}\Gamma^{2}_{03} - \Gamma^{0}_{13}\Gamma^{0}_{02} - \Gamma^{1}_{13}\Gamma^{1}_{12} \\ & + \Gamma^{2}_{12}\Gamma^{3}_{23} - \Gamma^{2}_{13}\Gamma^{2}_{22} + \Gamma^{3}_{12}\Gamma^{2}_{33} - \Gamma^{3}_{13}\Gamma^{2}_{32} \qquad (79) \end{aligned}$$

$$\begin{aligned} R_{22} = {} & \Gamma^{0}_{20,1} - \Gamma^{0}_{22,0} + \Gamma^{1}_{21,2} - \Gamma^{1}_{22,1} + \Gamma^{3}_{23,2} - \Gamma^{3}_{22,3} + \Gamma^{0}_{20}\Gamma^{0}_{02} - \Gamma^{0}_{22}\Gamma^{0}_{00} + 2\Gamma^{1}_{20}\Gamma^{0}_{12} - \Gamma^{1}_{22}\Gamma^{0}_{10} + \Gamma^{2}_{20}\Gamma^{0}_{22} \\ & - \Gamma^{2}_{22}\Gamma^{0}_{20} + 2\Gamma^{3}_{20}\Gamma^{0}_{32} - \Gamma^{3}_{22}\Gamma^{0}_{30} - \Gamma^{0}_{22}\Gamma^{1}_{01} + \Gamma^{1}_{21}\Gamma^{1}_{12} - \Gamma^{1}_{22}\Gamma^{1}_{21} + \Gamma^{2}_{21}\Gamma^{1}_{22} - \Gamma^{2}_{22}\Gamma^{1}_{21} + 2\Gamma^{3}_{21}\Gamma^{1}_{32} \\ & - \Gamma^{3}_{22}\Gamma^{1}_{31} - \Gamma^{0}_{22}\Gamma^{3}_{03} - \Gamma^{1}_{22}\Gamma^{3}_{23} + \Gamma^{2}_{23}\Gamma^{3}_{22} - \Gamma^{2}_{22}\Gamma^{3}_{23} + \Gamma^{3}_{23}\Gamma^{3}_{32} \\ & - \Gamma^{3}_{22}\Gamma^{3}_{33} \qquad (80) \end{aligned}$$

$$\begin{aligned} R_{23} = R_{32} = {} & \Gamma^{0}_{20,3} - \Gamma^{0}_{23,0} + \Gamma^{1}_{21,3} - \Gamma^{1}_{23,1} + \Gamma^{2}_{22,3} - \Gamma^{2}_{23,2} + \Gamma^{0}_{20}\Gamma^{0}_{03} - \Gamma^{0}_{23}\Gamma^{0}_{00} + \Gamma^{1}_{20}\Gamma^{0}_{13} - \Gamma^{1}_{23}\Gamma^{0}_{10} - \Gamma^{2}_{23}\Gamma^{0}_{20} \\ & + \Gamma^{3}_{20}\Gamma^{0}_{33} - \Gamma^{3}_{23}\Gamma^{0}_{30} + \Gamma^{0}_{21}\Gamma^{1}_{03} - \Gamma^{0}_{23}\Gamma^{0}_{01} + \Gamma^{1}_{21}\Gamma^{1}_{13} - \Gamma^{1}_{23}\Gamma^{1}_{11} - \Gamma^{2}_{23}\Gamma^{1}_{21} + \Gamma^{3}_{21}\Gamma^{1}_{23} - \Gamma^{3}_{23}\Gamma^{1}_{31} \\ & + \Gamma^{0}_{22}\Gamma^{2}_{03} + \Gamma^{1}_{22}\Gamma^{2}_{13} + \Gamma^{3}_{22}\Gamma^{2}_{33} - \Gamma^{3}_{23}\Gamma^{2}_{32} \qquad (81) \end{aligned}$$

$$\begin{aligned} R_{33} = {} & \Gamma^{0}_{30,3} - \Gamma^{0}_{33,0} + \Gamma^{1}_{31,3} - \Gamma^{1}_{33,1} + \Gamma^{2}_{32,2} - \Gamma^{2}_{33,2} + \Gamma^{0}_{30}\Gamma^{0}_{03} - \Gamma^{0}_{33}\Gamma^{0}_{00} + 2\Gamma^{1}_{30}\Gamma^{0}_{13} - \Gamma^{1}_{33}\Gamma^{0}_{10} + 2\Gamma^{2}_{30}\Gamma^{0}_{23} \\ & - \Gamma^{2}_{33}\Gamma^{0}_{20} + \Gamma^{3}_{30}\Gamma^{0}_{33} - \Gamma^{3}_{33}\Gamma^{0}_{30} - \Gamma^{0}_{33}\Gamma^{1}_{01} + \Gamma^{1}_{31}\Gamma^{1}_{13} - \Gamma^{1}_{33}\Gamma^{1}_{11} + 2\Gamma^{2}_{31}\Gamma^{1}_{23} - \Gamma^{2}_{33}\Gamma^{1}_{21} + \Gamma^{3}_{31}\Gamma^{1}_{33} \\ & - \Gamma^{3}_{33}\Gamma^{1}_{31} - \Gamma^{0}_{33}\Gamma^{2}_{02} - \Gamma^{1}_{33}\Gamma^{1}_{12} + \Gamma^{2}_{32}\Gamma^{2}_{23} - \Gamma^{2}_{33}\Gamma^{2}_{22} + \Gamma^{3}_{32}\Gamma^{2}_{33} \\ & - \Gamma^{3}_{33}\Gamma^{2}_{32} \qquad (82) \end{aligned}$$

These expressions are henceforth called the Great Ricci Curvature Tensor formulae in Cartesian Coordinates.

2.3 The Great Ricci Curvature Vector in Cartesian Coordinates

In the Theory of Tensor Analysis, we can express the vector corresponding to the Great Ricci Curvature Tensor in Cartesian Coordinates, denoted by $\underline{R}$, is given in all gravitational field as:

$$(\underline{R})_{00} = (g_{00})^{-1}R_{00} \quad (83)$$
$$(\underline{R})_{01} = (g_{00})^{-\frac{1}{2}}(g_{11})^{-\frac{1}{2}}R_{01} \quad (84)$$
$$(\underline{R})_{10} = (g_{11})^{-\frac{1}{2}}(g_{00})^{-\frac{1}{2}}R_{10} \quad (85)$$
$$(\underline{R})_{02} = (g_{00})^{-\frac{1}{2}}(g_{22})^{-\frac{1}{2}}R_{02} \quad (86)$$
$$(\underline{R})_{20} = (g_{22})^{-\frac{1}{2}}(g_{00})^{-\frac{1}{2}}R_{20} \quad (87)$$
$$(\underline{R})_{03} = (g_{00})^{-\frac{1}{2}}(g_{33})^{-\frac{1}{2}}R_{03} \quad (88)$$
$$(\underline{R})_{30} = (g_{33})^{-\frac{1}{2}}(g_{00})^{-\frac{1}{2}}R_{30} \quad (89)$$
$$(\underline{R})_{11} = (g_{11})^{-1}R_{11} \quad (90)$$
$$(\underline{R})_{12} = (g_{11})^{-\frac{1}{2}}(g_{22})^{-\frac{1}{2}}R_{12} \quad (91)$$
$$(\underline{R})_{21} = (g_{22})^{-\frac{1}{2}}(g_{11})^{-\frac{1}{2}}R_{21} \quad (92)$$
$$(\underline{R})_{13} = (g_{11})^{-\frac{1}{2}}(g_{33})^{-\frac{1}{2}}R_{13} \quad (93)$$
$$(\underline{R})_{31} = (g_{33})^{-\frac{1}{2}}(g_{11})^{-\frac{1}{2}}R_{31} \quad (94)$$
$$(\underline{R})_{22} = (g_{22})^{-1}R_{22} \quad (95)$$
$$(\underline{R})_{23} = (g_{22})^{-\frac{1}{2}}(g_{33})^{-\frac{1}{2}}R_{23} \quad (96)$$
$$(\underline{R})_{32} = (g_{33})^{-\frac{1}{2}}(g_{22})^{-\frac{1}{2}}R_{32} \quad (97)$$
$$(\underline{R})_{33} = (g_{33})^{-1}R_{33} \quad (98)$$

This quantity is henceforth called the Generalized Great Ricci Curvature Vector formulae based upon great metric tensor in Cartesian Coordinates.

2.4 Great Riemann Curvature Scalar

Follow the definition of Riemann Curvature scalar from the theory of tensor analysis, we show how to express it in terms of the great metric tensor for all gravitational fields in nature in the Cartesian coordinate.

From the theory of tensor analysis, the Riemann curvature scalar in 4-dimensional space, R, is given in all gravitational fields in all orthogonal curvilinear coordinates x^{μ} by [3]

$$R = g^{\mu\nu}R_{\mu\nu} \quad (99)$$

where $g^{\mu\nu}$ is the metric tensor and $R_{\mu\nu}$ is the Ricci curvature tensor. This invariant quantity is known as the Riemann curvature scalar.

It therefore follows that the unique expression for the Riemann curvature scalar based upon the great metric tensor for all gravitational fields in nature in the Cartesian coordinate is given as

$$R = g^{00}R_{00} + g^{11}R_{11} + 2g^{12}R_{12} + 2g^{13}R_{13} + g^{22}R_{22} + 2g^{23}R_{23} + g^{33}R_{33} \quad (99)$$

3.0 Results and Discussion

In this paper we showed how to formulate the generalized great Riemannian curvature tensor formulae based upon the great metric tensor in Cartesian coordinates for all gravitational fields as (31) – (71). We then developed the generalization of the Great Ricci Curvature Tensor formulae as (73)- (82). Equation (83)- (98) are the generalized great Ricci curvature formulae in the Cartesian coordinates and equation (99) is called the great Riemann curvature scalar in Cartesian coordinates. These results so obtained in this paper are hitherto unknown and mathematically most sound and elegant and physically most natural and radial satisfactory. This is another exploitation of the application of the great metric tensor to the Riemannian geometry and its application in theoretical Physics

4.0 Conclusion

The door is henceforth opened for the expression of Riemann curvature tensor, Riemann Ricci curvature tensor, Riemann Ricci curvature vector and Riemann curvature scalar in all dimensions in all orthogonal curvilinear coordinates, based upon the great metric tensor for gravitational fields in nature.

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