Comparative Study of a Buck Converter Controlled by a PID Regulator in Continuous and Discrete Form

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Abstract: The control of a DC-DC converter is generally done through a PID controller operating in the continuous domain comprising logic and analog components. In the presented work, we have controlled a Buck converter in two different ways. The first way consists in using a PID controller operating in the continuous domain. The second way is based on a discrete-time PID controller which is generally realized in the form of a program and implemented on a programmable circuit. The DC-DC converter controlled in PWM mode is a Buck converter made under SIMSCAPE / Simulink. The simulation results obtained under MATLAB for the two types of controllers under different conditions are presented and compared.
Keywords: Buck Converter, PWM Mode, Continuous PID, Discrete PID, Simscape, Simulink/Matlab.

I. INTRODUCTION

Static converters are generally based on power electronics components used as switches. They have a control circuit which must act on the opening or closing of the switch according to a well-defined protocol and frequency. These control devices are implemented in the form of analog or digital circuits. At the present time, we are moving towards a programmed control which is managed by a programmable circuit.
The advantage of this method is its simplicity and versatility compared with a control implemented in the form of an electrical circuit .

The work presented aims to make a comparison between the control of a Buck converter, controlled by a continuous PID regulator which is generally realized whit a conventional analog electrical components and a discrete PID controller practically realized in the form of a computer program and implanted on a programmable circuit. In both cases, the signal from the PID controller will be used to modulate a control signal in PWM mode. The PWM signal will control the DC-DC converter in order to maintain the output voltage applied to a load at the same value as the reference voltage applied to the input of the converter.

We will simulate the response of the converter submitted to the two types of control using SIMULINK/ MATLAB. the objective being to make a comparison of the results obtained with both types of control.

II. BUCK CONVERTER AND CONTROL CHAIN

2.1 Buck converter and control system

The synoptic schema of the system which will be the subject of our study is represented in figure1.
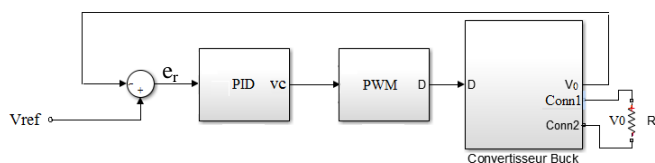

Fig. 1 Buck Converter and chain control

It consists of three main blocks: A Buck converter , a PWM generator and a PID controller.

The input of the regulator is the difference between a reference voltage V_{ref} and the voltage Vo obtained at the resistive load R_L which constitutes the load of the converter.

$$e_r = V_{ref} - V_O$$

e_r : represents the error between the voltage V_{ref} and the output voltage V_O across the load R_L. The objective being to minimize the error in order to adjust the output voltage V_O to the reference voltage V_{ref}. The control voltage Vc at the output of the PID regulator will modulate the sawtooth voltage delivered by a generator and supply the output with a PWM signal of fixed frequency (f_h) and variable duty cycle (α). This signal will act in

turn on the switch which is part of the converter in order to adjust the output voltage V_O to the reference voltage V_{ref}.

2.2 Buck Converter circuit
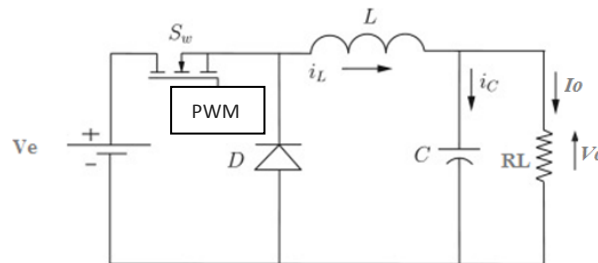The electrical circuit of the DC-DC converter used is shown in figure2:

Fig. 2 Buck Converter circuit

It is a Buck converter controlled by a PWM type impulse control device [1] [2].

The mode of operation adopted is that of Continuous Conduction Mode (CCM) [3]-[5].

The PWM regulator which will supply the pulses to the switch will act on the converter according to the result of comparison of the reference voltage (V_{ref}) and the output voltage (Vo) taken at the load R_L. It is the PID regulator which will send the control voltage Vc resulting from the comparison between Vref and Vo to the PWM circuit which will generate a rectangular signal with a fixed frequency (f_h) but with a variable duty cycle (α), image of the variation of the control signal Vc. The duty cycle, after a transient regime will stabilize at a fixed value when Vo across the load R_L reaches the voltage V_{ref}. It can be seen that the average output voltage Vo depends on the duty cycle α.

2.3 Converter model
When the switch (MOSFET transistor) is closed on the time interval [0, αT_h] with T_h the PWM signal period ($T_h = 1/f_h$). The diode is blocked. By neglecting the serial resistance of the coil and the resistance of leak of the capacitor, one can write, in the Laplace domain:

$$Ve(p) = Lp.i_L(p) + V_0(p) \quad (1) \qquad \text{and} \qquad i_L(p) = C.p.V_0(p) + \frac{V_0(p)}{R_L} \quad (2)$$

On the time interval [αT_h, T_h], the transistor is blocked and the diode is passing from where the new equations:

The transfer function in the Laplace domain can thus be expressed:

$$0 = Lp.i_L(p) + V_0(p) \quad (3) \qquad \text{and} \qquad i_L(p) = C.p.V_0(p) + \frac{V_0(p)}{R_L} \quad (4)$$

Average model: Assuming that the switching frequency is very high in front of the frequencies characteristics of the system and changes in the input α, the behavior of the system corresponds to the time average of the two states (open switch and closed switch). The response obtained is that obtained by averaging time (1) and (3) and then averaging (2) and (4). Which leads to the two new equations:

$$\alpha(p).Ve = Lp.i_L(p) + V_0(p) \quad (5) \qquad \text{and} \qquad i_L(p) = C.p.V_0(p) + \frac{V_0(p)}{R_L} \quad (6)$$

Which leads to the relationship :

$$\frac{V_0(p)}{\alpha(p)} = \frac{Ve}{LCp^2 + \frac{L}{R_L}p + 1} = \frac{Ve}{\frac{p^2}{\omega_0^2} + 2m\frac{p}{\omega_0} + 1} \quad (7)$$

This is the transfer function of a 2nd order low pass filter with the following characteristics:
$\omega_0 = \frac{1}{\sqrt{LC}}$, filter's natural frequency and $m = \frac{1}{2RL}\sqrt{\frac{L}{C}}$ the damping factor of the filter.

In order for the converter to operate in the continuous conduction mode (CCM), the inductor L must check the condition [6]:

$$L \geq \frac{0,125.Ve}{I_{smin} \cdot f_h} \qquad (8)$$

I_{smin}: the minimum intensity of the current that passed into the load.

By choosing an operating point with a nominal resistance $R_L = 10\Omega$, a current $I_{smin} = 0.2A$, a supply voltage of the converter Ve = 24V, a capacitance C = 10uF and a switching frequency f_h = 100kHz, it is possible to deduce the minimum value of L :

$$L \geq 150\mu H \qquad (9)$$

In our simulation, we will take L = 200µH

The converter circuit will be realized under Simscape/Simulink as shown in the figure3.
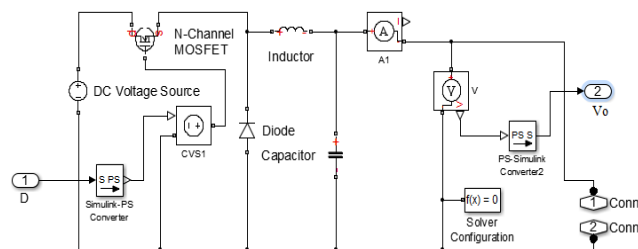

Fig. 3 Buck converter under Simscape

Input D represents the PWM signal that will drive the MOSFET.

2.4 PWM function
The circuit which will supply the PWM signal is represented in figure4.

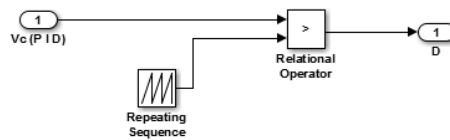Fig.4 PWM function

The sawtooth signal (frequency f_h = 100kHz) will be modulated by the voltage Vc from the PID controller as shown in the figure5.
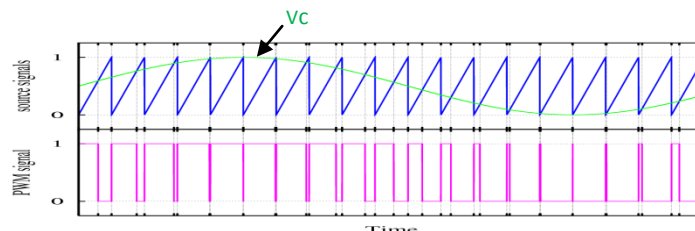

Fig.5 Evolution of PWM signals

Note that the duty cycle α varies in the interval [0,1] according to the control voltage Vc.

III. PID CONTROLLER

3.1 Continuous PID
The PID controller originally used is a classical parallel structure [7] provided by MATLAB's SIMULINK library.
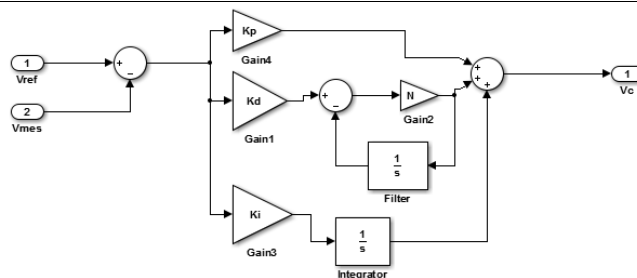
Fig.6 Continuous parallele PID under Simulink/Matlab

Its transfer function is given by the relation:

$$G(s) = \ Kp \ + Ki.\frac{1}{s} + \ Kd.\frac{N}{1+N.\frac{1}{s}} \qquad (10)$$

Kp, Ki, Kd respectively the coefficients of the terms proportional, Integral and derivative.
N: the cefficient of the derivative term of the low pass filter.

This continuous regulator is used in the case of an implementation of a conventional PID based on electrical components such as capacitance resistors and operational amplifier [8], [9].

3.2 Discrete PID
In the case of an implementation of the PID controller on a programmable circuit as in the case of a microcontroller, the use of a discrete-time PID controller is necessary [10].
So as in the continuous case, Matlab's SIMULINK library can provide a discrete-time PID with the same parallel structure as shown in the figure7.
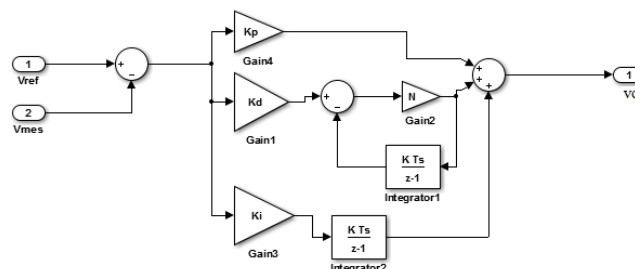

Fig.7 Discrete parallele PID controller under Simulink/Matlab

The transfer function in the discrete domain is given by the relation(11) for a Forward Euler method [11] [12]:

$$G(z) = \ Kp \ + Ki.\,T_s.\frac{1}{z-1} + \ Kd.\frac{N}{1+N.T_s.\frac{1}{z-1}} \qquad (11)$$

In our study we will make a comparison between the results obtained from a continuous PID and a discrete PID controller.

3.3 PID Tuning
If we consider the transfer function of the Buck converter given in relation(7), and using the values of L, C, f_h, and R_L previously chosen, we write the transfer function in the form:

$$H(s) = \ \frac{Ve}{LCs^2+\frac{L}{R_L}s+1} = \ \frac{24}{2.10^{-9}s^2+2.10^{-5}s+1} \qquad (12)$$

Matlab makes it possible to transform the transfer function H (s) into its equivalent function H (z) in the discrete domain by means of the integrated command c2d (sys, Ts). Ts being the chosen sampling period. Which give:

$$H(z) = \frac{0.1474\ z + 0.1449}{z^2 - 1.939\ z + 0.9512} \qquad (13)$$

Matlab's *PID Tuning tool* interface (figure8) is used to determine the coefficients Kp, Ki, Kd and N for each type of PID (PID (s) and PID (z)) corresponding to the transfer functions (12) and (13).
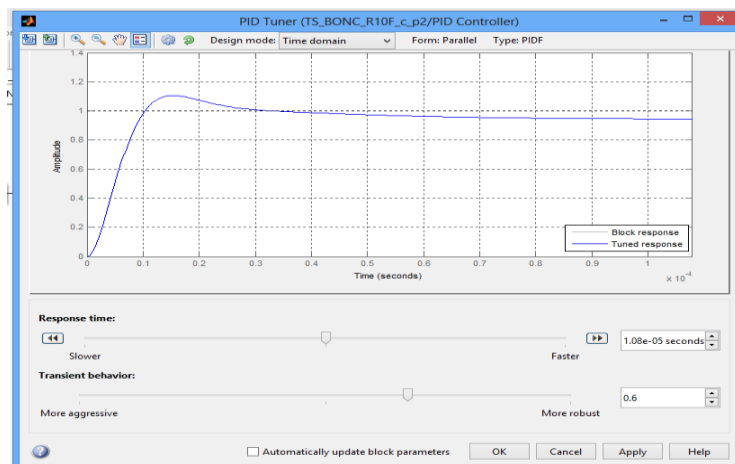

Fig.8 PID Tuning tool Interface

It is a very practical tool that allows you to adjust the PID coefficients according to the selected system response time , overshot and robustness (phase marging).

In both cases we will take the default values of these parameters.

The table-1 shows the values of these coefficients obtained after tunning.

Table-1: Coefficients of continuous and discrete PID

Continuous PID	Discrete PID (Ts = 5µs)
Kp = 0.4246	Kp = 0.2444
Ki = 2367,5	Ki = 1357,7
Kd = 1.5699	Kd = 8.3526
N = 393914.5	N = 298673.6
Overshot : 11,5%	*Overshot : 11,5%*
Phase marging:60° @ 8.47e+4 rad/s	*Phase marging:60° @ 5.72e+4 rad/s*

Ts = 5µs : sampling period chosen in our simulation.

IV.   SIMULATION RESULTS AND DISCUSION
4.1 Discrete PID setting

Figure9 (a) shows the response of the system to a scale varying from 0 to 8V with default coefficient settings. Note that the response obtained with the discrete PID is slower than that obtained with the continuous PID. The increase of the coefficient Ki (Ki =4357,7) of the discrete PID made it possible to reach the rise time of the continuous PID and to improve at the same time the response time of the system as indicated in the figure9 (b).
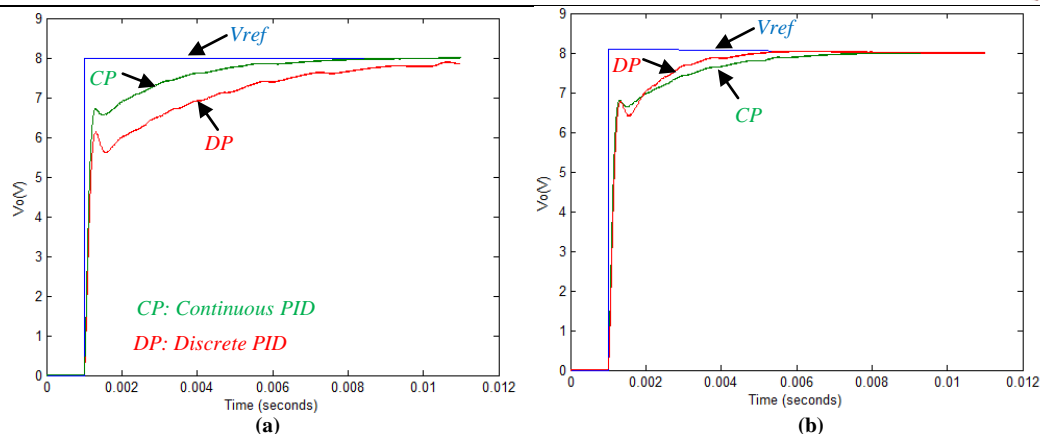
(a) **(b)**

Fig.9   Evolution of Vo before (a) and after (b) modification of the discrete PID coefficient Ki

4.2 System response for different load values:
Figure10 (a) ,(b), (c) and (d) shows the evolution of the system for different values of the load in response to a voltage step (V_{ref}).
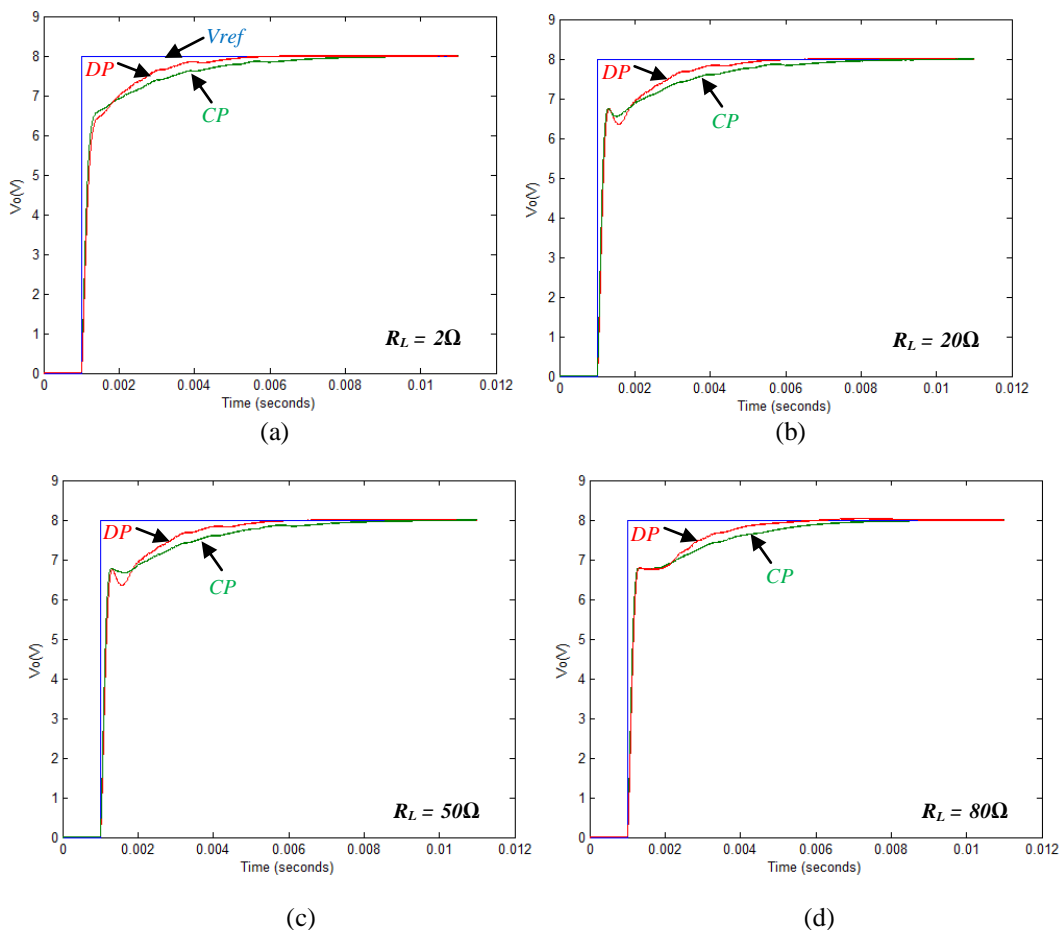

(a) (b)

(c) (d)

Fig.10   Evolution of the Vo for different values of the load in response to a voltage step.

Note that in all cases the response of the system is faster in the case of a regulation with the discrete PID. So for both the low load values and the high values, the response of the system regulated with the discrete PID is better than that obtained with the continuous PID.

4.3 Response of the system to a different values of step with a constant load

The response of the system to a voltage step of different values for a fixed load ($R_L = 10\Omega$) shows that the response of the system controlled by the discrete PID is better than that obtained with the continuous PID, especially for the high loads (low value of R_L) as shown in Figure 11(a).
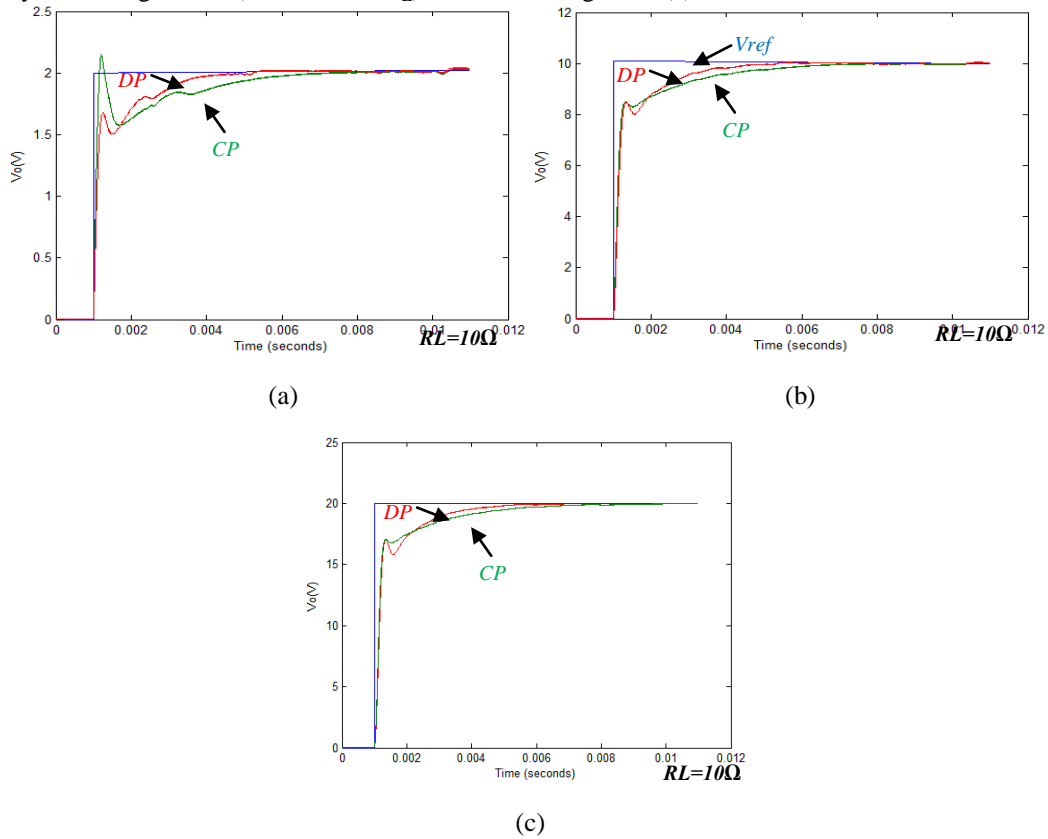

(a)

(b)

(c)

Fig. 11 Evolution of Vo with different values of step voltage reference

Note also that for the different values of the step amplitude, the response of the system is always faster in the case of the discrete PID.

4.4 Evolution of Vo with variable Vref amplitude

Finally the figure12 (a) and (b) shows the evolution of the voltage at the terminals of the load for a reference voltage V_{ref} varying with different levels in time. We see a similitude of the response of the system in both cases with a slight advantage of the PID regulator concerning the speed of the output signal to reach the reference voltage.
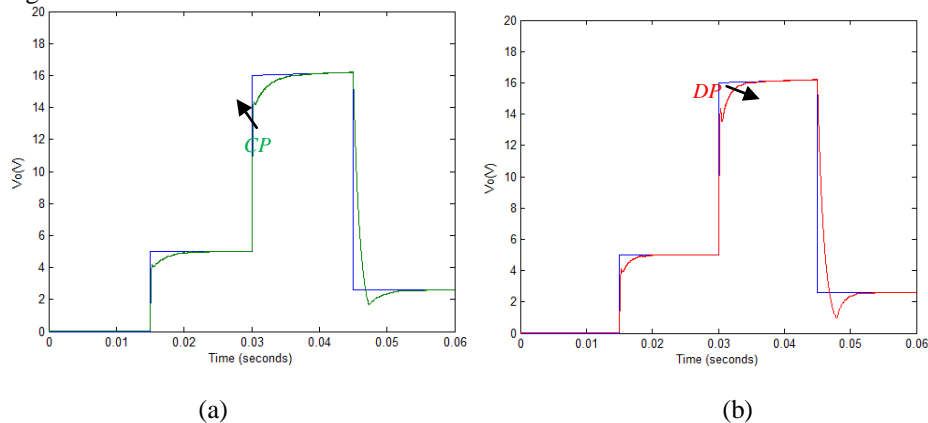

(a)

(b)

Fig. 12 Evolution of Vo in continuous PID (a) and discrete PID controller (b)

V. CONCLUSION

In this study, we compared the simulation results of the response of a Buck converter controlled by a PID controller in PWM mode operating in the continuous domain and in the discrete-time domain. At first, we used a PID controller operating in continuous domain. Then we replaced it with a discrete PID controller resulting from the transformation of the continuous PID transfer function by its equivalent in the discrete domain. The discrete PID controller becomes more and used because of the massive use of digital control instead of analog control. This type of control has the advantage of simplicity of realization and flexibility.

Thus the adjustment of the coefficients of the parameters can be done by programming and without intervention on the hard part.

The simulation results presented in this work showed that the modification of one of the coefficients of the discrete PID regulator allowed to improve its performances. This modification will be done programmatically in the case of an implementation on a programmable circuit without any intervention on the electrical part.

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