International Journal of Latest Research in Engineering and Technology (IJLRET)
ISSN: 2454-5031
www.ijlret.com || Volume 03 - Issue 12 || December 2017 || PP. 55-64

On Study of MHD flow of a Viscous Fluid on Nonlinear porous shrinking sheet

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Abstract: The present article aims to investigate the analytical and numerical solution of a magnetohydrodynamic (MHD) flow of a viscous fluid towards a nonlinear porous shrinking sheet. The analytical solution of this problem is computed by The Adomian decomposition method (ADM). The ADM is provided an analytical solution in the form of an infinite power series. The governing equation of the MHD flow is simplified to boundary value problem (BVP) by similarity transformations and solved numerically using the Runge-Kutta method with the shooting technique. The Pertinent parameters appearing in the problem are discussed graphically and presented in tables. Convergent [m,m] Pad´e approximants are obtained and compared with the numerical results. It is found that the Pad´e approximants agree well with the numerical results. Comparison with homotopy analysis method (HAM) reveals that excellent agreement.
Keywords: MHD flow, ADM, HAM, shrinking sheet.

1 Introduction

In the beginning of the 1980s, a new method for exactly solving nonlinear functional equations has been proposed by George Adomian, the so called Adomian decomposition method [1-3]. Over these years, this method has been applied to solve a wide range of problems arising from physics, biology, engineering. This method uses a decomposition of the nonlinear operator as a series function [4,5]. A considerable amount of research work has been invested recently in applying this method to a wide class of linear and nonlinear ordinary differential equations, partial differential equations and integral equations. In fluid mechanics, the problems are usually governed by systems of partial differential equations. If somehow, a system can be reduced to an ordinary differential equations, this constitutes a considerable mathematical simplification of the problem. If the number of independent variables can be reduced, then partial differential equations can be replaced by ordinary differential equation. In the modeling of boundary layer, this is sometimes possible and in some cases, the system of partial differential equations reduces to a system involving a third order differential equation[6].

The phenomena of velocities on the boundary towards a fixed point are known as shrinking phenomena, which often occur in the situations such as rising shrinking balloon. The flow over a shrinking surface is an important problem in many engineering processes with applications in industries such as the hot rolling, wire drawing and glass wire production. In nature, the presence of pure air or water is impossible. Some foreign mass may be present either naturally or mixed with the air or water. The present trend in the field of magnetic strength analysis is to give a mathematical model for the system to predict the reactor performance [7,8]. Only limited attention has been focused on the study of shrinking phenomena[7-18]. However, in certain situations, the shrinking sheet solutions do not exist since the vorticity cannot be confined in a boundary layer. These solutions may exist if either the magnetic field or the stagnation flow is taken into account. It is worth mentioning that the flow of an electrically conducting fluid past a porous plate under the effect of a magnetic field has attracted the attention of many investigators in view of its applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors, oil exploration, geothermal energy extractions, and the boundary layer control in the field of aerodynamics. A number of MHD studies discuss the effects of magnetic fields on the hydrodynamic flow in various configurations[8,19-26]. The purpose of the present paper is to discuss the analytical and numerical solution of the two-dimensional electrically conducting viscous fluid past a porous nonlinear porous shrinking sheet. The governing equations are simplified by a suitable similarity transformation, and the reduced nonlinear boundary value problem is then solved analytically by the ADM and numerically by Runge-Kutta method with the shooting technique. Convergent [*m*,*m*] Pad´e approximants are obtained and compared with the numerical results. It is found that the Pad´e approximants agree well with the numerical results. By comparing with HAM reveals that there exist an excellent agreement.

2 Mathematical formulation

Let us consider the steady two-dimensional MHD flow of an incompressible viscous fluid over a nonlinear porous shrinking sheet at $y = 0$. The fluid is electrically conducting under the influence of an applied magnetic field $B(x)$ normal to the shrinking sheet. The induced magnetic field is neglected. The governing equations for the flow problem are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u \tag{2}$$

where u and v are the velocity components in the x and y directions, respectively, ν is the kinematic viscosity, ρ is the density, and σ is the electrical conductivity of the fluid. In Eq. (2), the external electric field and polarization effects are negligible[19]. Thus,

$$B(x)=B_0\, x^{\frac{n-1}{2}} \tag{3}$$

The boundary conditions corresponding to the nonlinear porous shrinking sheet are given as

$$u(x,0)=-c\,x^n\,, v(x,0)=-V_0\, x^{\frac{n-1}{2}}\,,\quad u(x,y)\to 0\; as\; y\to\infty \tag{4}$$

where V_0 is the porosity of the plate. We introduce the similarity variables and non-dimensional variables as following

$$\eta=\sqrt{\frac{U_w}{\nu x}}\,y\;,\quad u=c\,x^n f'(\eta), U_w=\frac{c\,(n+1)}{2}x^n \tag{5}$$

$$v=-\sqrt{\frac{U_w\nu}{x}}\left[f(\eta)+\frac{n-1}{n+1}\eta\, f'(\eta)\right] \tag{6}$$

Making use of Eqs. (5) and (6), the continuity equation is automatically satisfied and the momentum equation takes the form

$$f'''+ff''-\beta f'^2-Mf'=0 \tag{7}$$

The boundary conditions for the problem under consideration are

$$f(0)=K,\quad f'(0)=-1,\quad f'(\infty)=0, \tag{8}$$

where K is the wall mass transfer parameter, M is the magnetic parameter, and β is the non-dimensional parameter. They are given as follows:

$$\beta=\frac{2n}{n+1},\quad M=\frac{2\sigma B_0^2}{\rho c(1+n)},\quad K=\frac{V_0}{\sqrt{\frac{c\upsilon(n+1)}{2}}} \tag{9}$$

3 Method of solution

Adomian decomposition method is used for solving operate equations of the form

$$u=v+Nu \tag{10}$$

Where $N: X\to X$ is a nonlinear mapping from a Banach space X into itself and $v\in X$ is known. Since the method does not resort to linearization or assumptions of week nonlinearity, the nonlinearities which can be handled are quite general and the solutions generated may be more realistic than those achieved by simplifying the model of the physical problem to achieve conditions required for other techniques [27]. Adomian method assumes that the solution u can be expanded as an infinite series:

$$u = \sum_{n=0}^{\infty} u_n \tag{11}$$

With $u_n \in X$, the image Nu has an expansion

$$Nu = \sum_{n=0}^{\infty} A_n \tag{12}$$

With $A_n \in X, \forall n$ Substituting and into (10) gives

$$\sum_{n=0}^{\infty} u_n = v + \sum_{n=0}^{\infty} A_n \tag{13}$$

Which is satisfied formally if we set $u_0 = v,\ u_{n+1} = A_n$

Thus, we need to determine the so-called Adomian polynomials $A_n; n = 1,2,3,...$ To determine the $A_n's$, a scalar parameter λ is introduced to set

$$u(\lambda) = \sum_{n=0}^{\infty} \lambda^n u_n \tag{14}$$

And

$$Nu(\lambda) = \sum_{n=0}^{\infty} \lambda^n A_n \tag{15}$$

Where

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} Nu(\lambda) \bigg|_{\lambda=0} \tag{16}$$

By applying the Adomian method to the flow problem (7) with boundary conditions (8). Let

$$Lf = -ff'' + \beta f'^2 + Mf' \tag{17}$$

Where the differential operator L is given by $L = \frac{d^3}{d\eta^3}$

The inverse operator L^{-1} is therefor considered an 3- fold integral operator defined by

$$L^{-1} = \int_0^{\eta}\int_0^{\mu}\int_0^{\eta} (.)\, d\eta\, d\eta d\eta$$

Operating with L^{-1} on (7), we have

$$f(\eta) = K - \eta + \frac{1}{2}\alpha\eta^2 + L^{-1}(-ff'' + \beta f'^2 + Mf') \tag{18}$$

where $\alpha = f''(0)$ to be determined based on the boundary condition at infinity. The Adomian decomposition method introduces the following expression:

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta) \tag{19}$$

for the solution $f(\eta)$ of (7), where the components $f_n(\eta)$ will be determined recurrently according to a recursive relation. Moreover, the method defines the nonlinear function $F(f(\eta))$ by an infinite series of polynomials

$$F(f(\eta)) = \sum_{n=0}^{\infty} A_n \tag{20}$$

where A_n are the so-called Adomian polynomials that represent the nonlinear term $F(f(\eta))$ and can be calculated for various classes of nonlinear operators according to specific algorithms presented previously. Subsituting (19) and (20) into (18) yields

$$\sum_{n=0}^{\infty} f_n = K - \eta + \frac{1}{2}\alpha\eta^2 + L^{-1}\left(\sum_{n=0}^{\infty} A_n\right) \tag{21}$$

Where

$$f_0 = K - \eta + \frac{1}{2}\alpha\eta^2, \quad f_{k+1} = L^{-1}(A_k); \quad k \ge 0 \tag{22}$$

By using (16), we obtain the following few terms of the Adomian polynomials A_n:

$$A_0 = -f_0 f_0'' + \beta f_0'^2 + M f_0',$$
$$A_1 = -f_1 f_0'' - f_0 f_1'' + 2\beta f_0' f_1' + M f_1',$$
$$A_2 = -f_2 f_0'' - f_0 f_2'' - f_1 f_1'' + 2\beta f_0' f_2' + M f_1' + \beta f_1'^2,$$

and so on . Frome (22) we determine f_n:

$$f_0 = K - \eta + \frac{1}{2}\alpha\eta^2$$
$$f_1 = c_1\eta^3 + c_2\eta^4 + c_3\eta^5$$
$$f_2 = c_4\eta^4 + c_5\eta^5 + c_6\eta^6 + c_7\eta^7 + c_8\eta^8$$
$$f_3 = c_9\eta^5 + c_{10}\eta^6 + c_{11}\eta^7 + c_{12}\eta^8 + c_{13}\eta^9 + c_{14}\eta^{10} + c_{15}\eta^{11}$$

and so on, where constants $c_i, i = 1,\ldots,15$ are defined as following

$$c_1 = \frac{-1}{3!}(K\alpha + \beta + M)$$

$$c_2 = \frac{\alpha}{4!}(1 - 2\beta + M)$$

$$c_3 = \frac{\alpha^2}{5!}(-1 + 2\beta),$$

$$c_4 = -\frac{1}{4}K c_1$$

$$c_5 = \frac{1}{20}(-4Kc_2 + 2c_1 - 2\beta c_1 + c_1 M)$$

$$c_6 = \frac{1}{5!}(-\alpha c_1 - 20Kc_3 + 12c_3 - 3\alpha c_1 - 8\beta c_2 + 6\alpha\beta c_1 + 4Mc_2)$$

$$c_7 = \frac{1}{210}(-\alpha c_2 + 20c_3 - 6\alpha c_2 + 5Mc_3 - 10\beta c_3 + 8\beta\alpha c_2),$$

$$c_8 = \frac{\alpha}{336}(-11c_3 + 10\beta c_3)$$

$$c_9 = \frac{1}{20}(-4Kc_4 + c_1M)$$

$$c_{10} = \frac{1}{30!}(Mc_2 - 2c_4\beta - 5Kc_5 + 3c_4)$$

$$c_{11} = \frac{1}{210}(-\alpha c_4 - 6c_1^2 - 6\alpha c_4 + 10c_5 - 30Kc_6 + 9\beta c_1^2 + 8\alpha\beta c_4 - 10\beta c_5 + 5Mc_3)\ ,$$

$$c_{12} = \frac{1}{336}(-\alpha c_5 - 18c_1c_2 - 10\alpha c_5 + 30c_6 - 42Kc_7 + 24\beta c_1c_2 + 10\alpha\beta c_5 - 12\beta c_6)$$

$$c_{13} = \frac{1}{504}(-\alpha c_6 - 26c_1c_3 - 12c_2^2 - 15\alpha c_6 + 42c_7 - 56Kc_8 + 30\beta c_1c_3 + 16\beta c_2^2 + 12\alpha\beta c_6 - 14\beta c_7)$$

$$c_{14} = \frac{1}{720}(-\alpha c_7 - 32c_2c_3 - 21\alpha c_7 + 56c_8 + 40\beta c_2c_3 + 14\alpha\beta c_7 - 16\beta c_8)$$

$$c_{15} = \frac{1}{990}(-\alpha c_8 - 20c_3^2 - 28\alpha c_8 + 25\beta c_3^2 + 16\alpha\beta c_8)$$

Due to lengthy calculations, the analytical results of higher order have been shown graphically. The series solution of equation (7) with boundary conditions (8) can be written in the form of the following infinite series (20). According to the series solutions (20) the accuracy of Adomian method solution increases by increasing the number of solutions terms which it can be by helping of the MATHEMATICA computer software.

4 Convergence and accuracy of the ADM solution

We aim to study the mathematical behavior of the solution $f(\eta)$ that can be achieved by forming Pad´e approximants which have the advantage of manipulating the polynomial approximation into a rational function to gain more information about $f(\eta)$. To show its advantages, we compare the $[m,m]$ Pad´e approximants and numerical results of $f''(0)$ for the flow with the differential equation (7) with boundary conditions (8) at $K=1, \beta=1$ and different values of M in Table 1. It can be seen the $[m,m]$ Pad´e approximants converge and agree well with the numerical solution.

5 Numerical Solution

The nonlinear ordinary differential equation (7) with boundary conditions (8) is solved numerically using a shooting technique with fourth order Runge-Kutta method by choosing suitable guess values for $f''(0)$ with prescribed parameters M, β and K The computation was carried out by using the MATLAB system to satisfy the convergence criteria of 10^{-6} with different step sizes. In this method, equation (7) with boundary conditions (8) is converted into the following system of linear equation of first order

$$f = z_1\ ,\ f' = z_2\ ,\ f''' = z_3$$
$$z_3' = -z_1z_3 + \beta z_2^2 + M z_2 \tag{23}$$

With the initial conditions

$$f(0) = K,\quad f'(0) = -1,\quad f'(\infty) = 0, \tag{24}$$

6 Results and discussion

The computations have been carried out by assuming various values of the parameters involved in the problem and results are represented through Tables 1,2 and graphs (1-6). The results of approximations for the ADM (present method) and HAM [7] solutions with different of $M, K, and\ \beta$ are present in Table 2. Is observed that $f''(0)$ increases with the mass suction parameter K and magnetic parameter M but an opposite behavior is noted in case of non- dimensional parameter β which displays excellent consent between the current method and HAM [7].

Table 1 Comparison of The [*m*,*m*] Pad´e approximants and numerical solution of $f''(0)$ for $K=1,\ \beta=1$ and different values of M

[m,m]	M=2.0	M=2.3	M=2.6	M=2.9
[15,15]	1.61809	1.74503	1.86017	1.96632
[16,16]	1.61808	1.74503	1.86017	1.96632
[17,17]	1.61808	1.74502	1.86016	1.96631
[18,18]	1.61807	1.74502	1.86016	1.96631
[19,19]	1.61807	1.74501	1.86016	1.96630
[20,20]	1.61807	1.74501	1.86015	1.96629
Numerical	1.61807	1.74501	1.86015	1.96629

Table 2 Comparison of the ADM and HAM solution of $f''(0)$ with different values of $M, K, and\ \beta$

$K=1,\ \beta=1$			$K=1, M=1$			$M=2,\ \beta=0.1$		
M	ADM	HAM	β	ADM	HAM	K	ADM	HAM
2.0	1.61807	1.61806	0.0	1.86263	1.86205	0.0	1.28268	1.27098
2.1	1.66193	1.66192	0.1	1.83945	1.83943	0.1	1.31880	1.31880
2.2	1.70419	1.70418	0.2	1.81651	1.81650	0.2	1.36922	1.36864
2.3	1.74501	1.74500	0.3	1.79322	1.79323	0.3	1.41985	1.42044
2.4	1.78454	1.78453	0.4	1.76952	1.76951	0.4	1.47434	1.47434
2.5	1.82290	1.82289	0.5	1.74539	1.74541	0.5	1.53100	1.53028
2.6	1.86015	1.86015	0.6	1.72091	1.72089	0.6	1.58822	1.58822
2.7	1.89643	1.89643	0.7	1.69591	1.69593	0.7	1.64816	1.64816
2.8	1.93178	1.93178	0.8	1.67046	1.67049	0.8	1.71004	1.71004
2.9	1.96629	1.96629	0.9	1.64452	1.64454	0.9	1.77383	1.77381
3.0	2.00000	2.00000	1.0	1.61809	1.61806	1.0	1.83945	1.83943

Figure.1 depicts the variation with η of the functions $f,\ f'\ and\ f''$ with $K=0,\ \beta=1\ and\ M=4$. The comparison of the ADM and numerical results of velocity profiles f' and f at $K=0,\ \beta=1\ and\ M=4$ are shown in In Fig.2 and 3. In Fig. 4, depicts the influence of magnetic parameter M on velocity profiles $f'(\eta)$. It is observed that an increasing in the values of M, the velocity profile increases. In Fig. 5, the effects of the mass suction parameter K on $f'(\eta)$ are presented. An increasing in the velocity is observed with an increasing in the mass suction parameter. In Fig. 6, the effects of non- dimensional parameter β on $f'(\eta)$ are presented. A decreasing in the velocity is observed with an increasing in the non-dimensional parameter. It is concluded that the results obtained are compatible with those in [7] .

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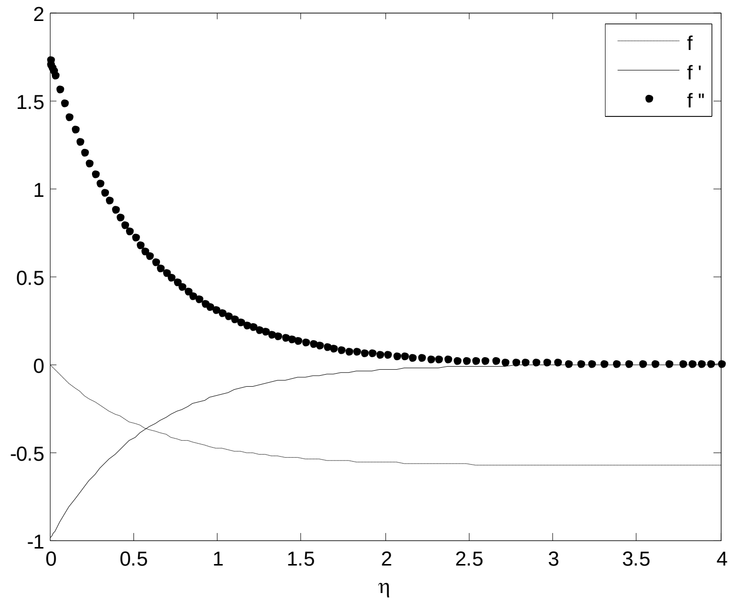

Fig. 1. Variation with η of the functions f, f' *and* f'' with $K=0$, $\beta=1$ *and* $M=4$.

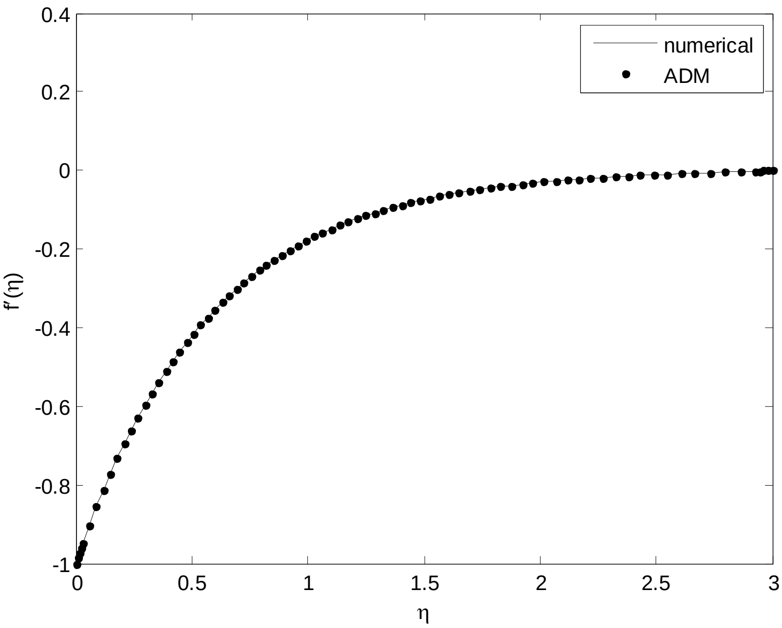

Fig. 2. Comparison of the ADM and numerical results of velocity profile f' at $K=0$, $\beta=1$ *and* $M=4$

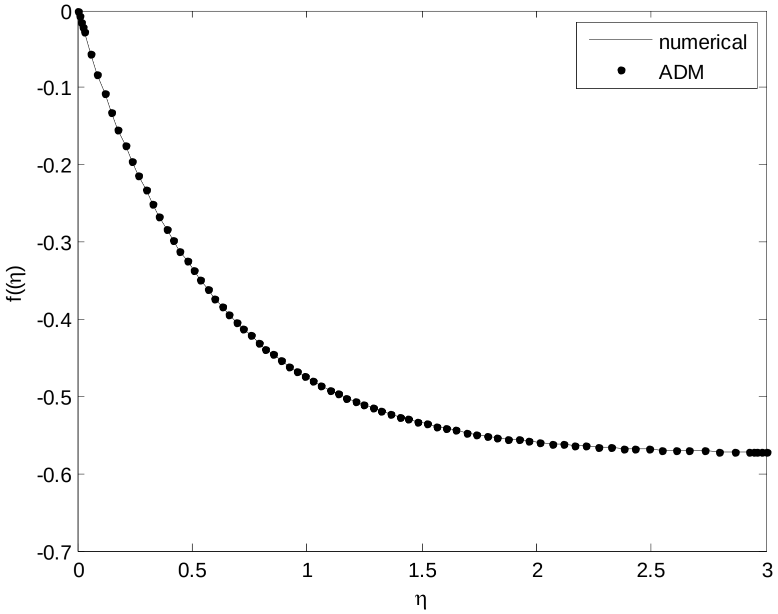

Fig. 3. Comparison of the ADM and numerical results of velocity profile f at $K=0,\ \beta=1\ and\ M=4$

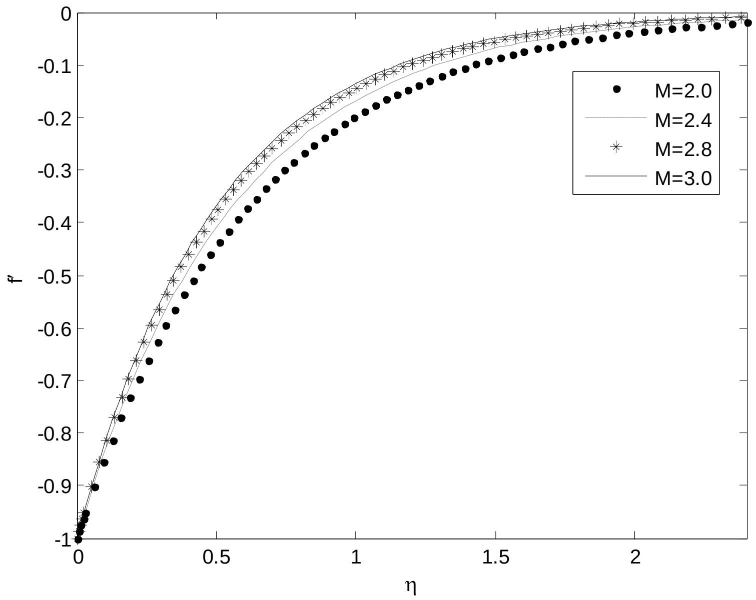

Fig. 4. Magnetic effects on velocity profile with $K=1 and\ \beta=1$

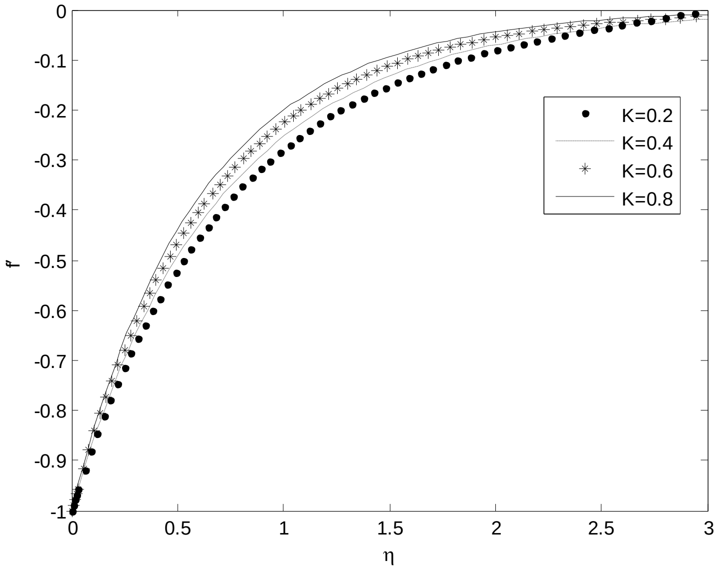

Fig. 5. Suction effects on velocity profile with $M = 2\, and\;\; \beta = 0.1$

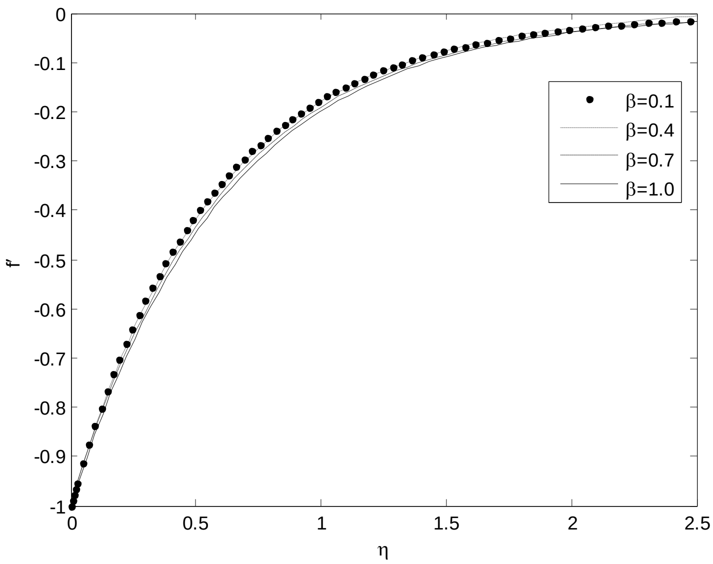

Fig.6. Velocity profile under different values of β with K=1 and M=2.

7 Conclusion

The magnatichydrodynamic flow of a viscous fluid towards a nonlinear porous shrinking sheet is studed. The governing equations of the MHD flow are solved analytically and numerically the results are used to investigate the effects of various parameters on velocity, the main conclusions drawn from the study are as follows:

-The mass suction parameter K and the magnetic parameter M have increasing effects on velocity profiles.

- A decreasing in the velocity is noticed with increasing in the values of the parameter β .

- From Tables and graphs, it is concluded that the results obtained are compatible with those in [7] .

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