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Economic Analysis of a System Having Duplicate Cold Standby Unit with Priority to Repair of Original Unit

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Abstract: Identical unit in the system play very important role to increase the reliability of the system. But in this situation, the cost of the system becomes very high to hold redundant unit. Idea that the system, having the minimum cost that duplicate unit has taken as cold standby with different failure rate as the failure rate of original unit. Also, the idea of priority to one discipline over to other discipline can improve the system performance. Keeping the idea of such system a model is developed having different failure rate with priority to repair of the original unit over repair of the duplicate unit. There is a single server, who play dual role to repair of original as well as duplicate unit whenever required. It is assume that the time to take repair activity follows negative exponential distribution whereas the distributions of unit are taken as arbitrary with different probability density functions. The expressions of various economic measures are analyzed in steady state using semi-Markov process and regenerative point technique. The graphs are sketched for arbitrary values of the parameters to delineate the behavior of some important performance measures to check the efficacy of the system model under such situations.

Key Words: Stochastic Analysis, Duplicate cold-standby, regenerative point, priority, steady state and semi-Markov process

Introduction

Redundancy is a common approach to enrich the reliability and availability of a system. In literature, the stochastic behavior of cold standby system has been widely discussed by many researchers, Osaki and Nakagawa [1971] discussed on a two-unit standby redundant system with standby failure. Nakagawa and Osaki [1975] analyzed stochastic behavior of a two-unit priority standby redundant system with repair. Subramanian *et.al.* [1976] highlighted reliability of a reparability system with standby failure. Gopalan and Nagarwalla [1985] analyzed cost-benefit of a one server two unit cold standby system with repair and age replacement. In a standby redundant system some additional paths are created for proper functioning of the system. Standby unit is the assistance to increase the reliability of the system. Singh and Sriniwas [1987] obtained stochastic analysis of a two unit cold standby system with preparation time for repair. In a cold standby redundant system as the operating unit fails, the standby unit takes its place and the failed unit goes under treatment. But it may be the possibility that the standby unit is already damaged owing to remain unused for a longer period of time or erosion etc. we may face a situation when the operating unit fails but the standby unit is already damaged. So far, the cold standby systems with the possibility of server failure have been discussed by the above researchers. Aggrawal *et.al.*[2010] highlighted the reliability characteristics of a cold-standby redundant system.

Recently, Malik and Sudesh K. Barak [2013] analyzed profit of a system reliability model under preventing maintenance and repair. Bhardwaj and Singh [2014] discussed semi-Markov approach for asymptotic performance analysis of a Standby system with server failure. Bhardwaj and Singh [2014] obtained steady state behavior of a cold-standby system with server failure and arbitrary repair, replacement and treatment. Malik, Bhardwaj and Komaldeep Kaur [2015] analyzed performance of a stochastic system with standby failure and maintenance. But the concept of standby failure needs more attention due to its significant contribution during study. So keeping this aspect in view, we developed a stochastic model of redundant system with standby failure. The model consists of two non-identical units; original unit is in operative mode and duplicate unit is in cold standby mode. The cold standby unit becomes operative after failure of the original unit. There is single server for repair activity whenever required. The time to take repair activity follows negative exponential distribution whereas the distributions of unit are taken as arbitrary with different probability density functions. The expression for various reliability measures such as transition probabilities, mean sojourn times, mean time to system failure, steady state availability are deduced by using semi-Markov process and regenerative point technique. The graphs are delineated for arbitrary values of the parameters to highlight the behavior of some important performance measures to check the efficacy of the system model under such situations. The suggested model is applicable in the hospital in which generator system working as duplicate unit and electric supply is treated as original unit. Also, this model is backbone of the industries.

Notations/Assumptions

E : Set of regenerative states $\{S_0, S_1, S_3, S_4\}$.

O / D_{cs} / D_o : The original unit is operative /duplicate unit is in cold standby mode /duplicate unit is operative.

λ/λ_1 : Constant failure rate of original unit /failure rate of duplicate unit.

O_{Fur}/D_{Fur} : The original failed unit /duplicate failed unit is under repair.

O_{Fwr}/D_{Fwr} : The original failed unit / duplicate failed unit is waiting for repair.

g(t) / G(t) : pdf /cdf of original unit repair time taken by the server.

$g_1(t)$ / $G_1(t)$: pdf /cdf of duplicate unit repair time taken by the server.

$q_{ij}(t)$ / $Q_{ij}(t)$: pdf / cdf of direct transition time from a regenerative state S_i to a regenerative state S_j without visiting any other regenerative state.

$q_{i,jk}(t)/Q_{i,jk}(t)$: pdf/cdf of first passage time from a regenerative state S_i to a regenerative state S_j or to a failed state S_j visiting state k once in (0,t].

$M_i(t)$: Probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state.

$W_i(t)$: Probability that the system is busy in state Si up at time t without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.

m_{ij} : Contribution to mean sojourn time (μ_i) in state S_i when system transit directly to state S_j so that $\mu_i = \sum_j m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*'}(0)$

Ⓢ/© : symbol for stieltjes convolution/ laplace convolution

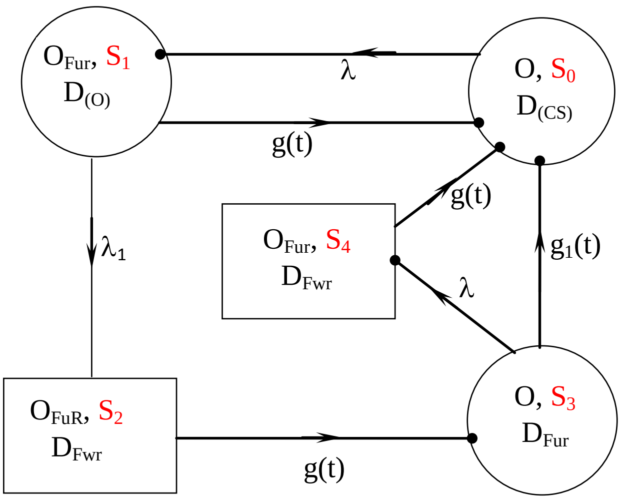

Fig.1: State Transition Diagram

Transition Probabilities

Simple probabilities considerations yield the following expressions for the non-zero elements p_{ij} by taking all distributions exponential assume as; $g_1(t) = \theta_1 e^{-\theta_1 t}$ and $g(t) = \alpha e^{-\alpha t}$

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \tag{1}$$

$$p_{01} = p_{41} = p_{23} = 1,\ p_{10} = \frac{\alpha}{\alpha+\lambda_1},\ p_{12} = \frac{\lambda_1}{\alpha+\lambda_1},\ p_{1,3;2} = \frac{\lambda_1}{\alpha+\lambda_1},\ p_{30} = \frac{\alpha}{\alpha+\lambda} \quad \text{and} \quad p_{34} = \frac{\lambda}{\alpha+\lambda} \tag{2}$$

Mean Sojourn Time

Let T denotes the time to system failure then the mean sojourn times (μ_i) in the state S_i are given by

$$\mu_i = E(t) = \int_0^\infty p(T > t)dt \tag{3}$$

Hence, $\mu_0 = \frac{1}{\lambda}$, $\mu_1 = \frac{1}{\lambda_1 + \alpha}$, $\mu_3 = \frac{1}{\lambda + \theta_1}$, $\mu_1' = \frac{1}{\lambda_1 + \alpha}$ and $\mu_3' = \frac{1}{\lambda + \alpha}$

(4)

Mean Time to System Failure

Let $\phi_i(t)$ be the c.d.f. of the first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$

$$\phi_0(t) = Q_{01}(t) Ⓢ \phi_1(t), \quad \phi_1(t) = Q_{10}(t) Ⓢ \phi_0(t) + Q_{12}(t)$$

(5)

Taking L.S.T. of relation (5) and solving for $\tilde{\phi}_0(s)$

$$\text{MTSF} = \lim_{s \to 0}\left(\frac{1 - \tilde{\phi}_0(s)}{s}\right) = \frac{\mu_0 + \mu_1}{p_{12}} = \frac{(\alpha + \lambda + \lambda_1)}{\lambda\lambda_1} \tag{6}$$

Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant t, given that the system entered the regenerative state S_i at t = 0.The recursive relations for $A_i(t)$ are as follows

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t), \quad A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + q_{1,3;2}(t) © A_3(t)$$

$$A_3(t) = M_3(t) + q_{30}(t) © A_0(t) + q_{3,1;4}(t) © A_1(t) \tag{7}$$

$M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ at time t without visiting to any other regenerative state where

$$M_0(t) = e^{-\lambda t}, M_1(t) = e^{-\lambda_1 t}\overline{G(t)} \text{ and } M_3(t) = e^{-\lambda t}\overline{G_1(t)} \tag{8}$$

Taking Laplace transform of equation (7) and (8), and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0 = \lim_{s \to 0} sA_0^*(s) = \frac{\mu_0(p_{10} + p_{1,3;2}\ p_{30}) + (\mu_1 + \mu_3 p_{1,3;2})}{\mu_0(p_{10} + p_{1,3;2}\ p_{30}) + (\mu_1' + \mu_3' p_{1,3;2})} = \frac{\alpha(\theta_1 + \lambda)(\alpha + \lambda + \lambda_1) + \lambda(\alpha + \lambda)(\theta_1 + \lambda + \lambda_1)}{(\theta_1 + \lambda)(\alpha + \lambda)(\alpha + \lambda + \lambda_1)}$$

(9)

Busy period analysis of the server

Let $B_i(t)$ be the probability that the server is busy at instant 't' given that the system entered the regenerative state S_i at t = 0. The recursive relations for $B_i(t)$ are as follows

$$B_0(t) = q_{01}(t) © B_1(t), \quad B_1(t) = W_1(t) + q_{01}(t) © B_0(t) + q_{1,3;2}(t) © B_3(t)$$

$$B_3(t) = W_3(t) + q_{30}(t) © B_0(t) + q_{3,1;4}(t) © B_1(t)$$

(10)

Where $W_i(t)$ is the probability that the server is busy in state S_i due to repairing of unit up to time t without making any transition to any other regenerative state or before returning to the same via one or more non-regenerative states; so

$$W_1(t) = e^{-\lambda_1 t}\overline{G(t)} + (\lambda_1 e^{-\lambda_1 t} © 1)\overline{G(t)} \text{ and } W_3(t) = e^{-\lambda t}\overline{G_1(t)} \tag{11}$$

Taking L.T. of relations (10) & (11) and solving for $B_0^*(s)$, the time for which server is busy is given as

$$B_0 = \lim_{s \to 0} sB_0^*(s) = \frac{(W_1 + W_3 p_{1,3;2})}{\mu_0(p_{10} + p_{1,3;2}\ p_{30}) + (\mu_1' + \mu_3' p_{1,3;2})} = \frac{\lambda(\alpha + \lambda)(\theta_1 + \lambda + \lambda_1)}{(\theta_1 + \lambda)(\alpha + \lambda)(\alpha + \lambda + \lambda_1)}$$

(12)

Expected Number of visits done by the server

Let $N_i(t)$ be the expected number of visits by the server in (0,t], given that the system entered the regenerative state S_i at t=0 .The recursive relations for $N_i(t)$ are as follows

$$N_0(t) = Q_{01}(t) \circledS [1 + N_1(t)] \; , \; N_1(t) = Q_{10}(t) \circledS N_0(t) + Q_{1,3;2}(t) \circledS N_3(t)$$

$$N_3(t) = Q_{30}(t) \circledS N_0(t) + Q_{3,1;4}(t) \circledS N_1(t)$$

(13)

Taking L.S.T. of the above relation and solving for $\tilde{N}_0(s)$, the expected number of visits by the server are given by

$$N_0 = \lim_{s \to 0} s\tilde{N}_0(s) = \frac{1 - p_{1,3;2}\ p_{3,1;4}}{\mu_0 (p_{10} + p_{1,3;2}\ p_{30}) + (\mu_1' + \mu_3' p_{1,3;2})} = \frac{\lambda^2}{(\alpha + \lambda)} \qquad (14)$$

Cost -Benefit Analysis

The profit occurred in the system model in steady state can be calculated as

$$P_0 = K_0A_0 - K_1B_0 - K_2N_0 \qquad (15)$$

Where K_0 = (5000), Revenue per unit up- time of the system, K_1 = (650), Cost per unit time for which server is busy and K_2 = (450), Cost per unit visits by the server.

Discussion

In this exploration, the effect of various parameters on system model is visualized. Graphs are penned down by assigning particular values to various parameters and costs shown in figure 2-4.

Figure 2: Shows the graphical behavior of Mean Time to System Failure with respect to rate λ by which the original unit fails. In this figure MTSF is rapidly declined when the rate 'λ' by which original unit fail increased for all possible changes in the other parameters. The curve L_1 and L_4 coincide to each other that indicated that the effect of repair rate of duplicate unit θ_1 is negligible.

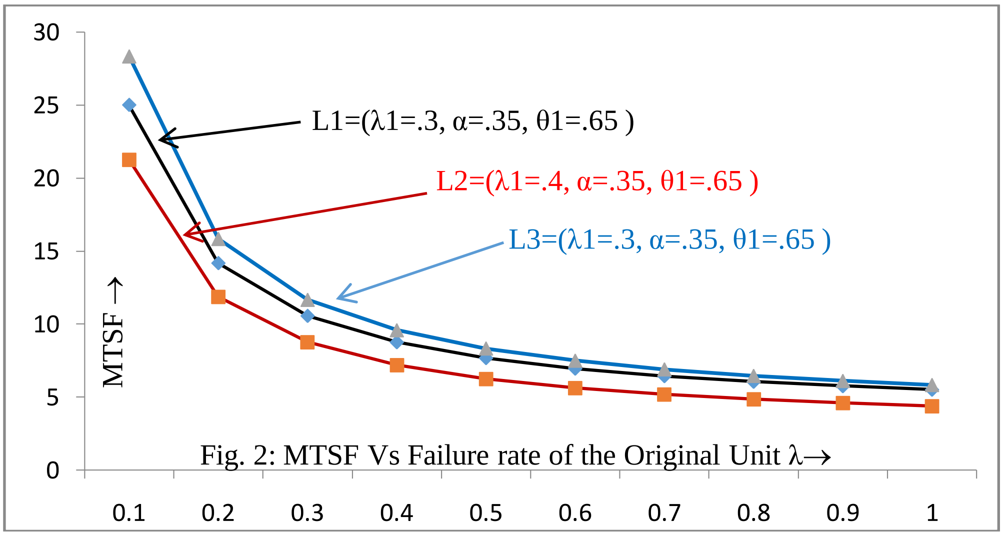

Fig. 2: MTSF Vs Failure rate of the Original Unit λ→

Figure 3: Shows the graphical behavior of Availability of the system with respect to rate λ by which the original unit fails. In this figure, availability is increasing in nature when the rate 'λ' by which original unit fail increased in the range (0.3 to 1.0) for all possible changes in the other parameters. The curve L_2 and L_4 coincide to each other that indicated that the effect of repair rate of duplicate unit θ_1 is negligible. But the availability of the system lies in its maximum range and increasing in nature. Hence, the idea of priority to repair of the original unit over repair of duplicate unit increases the availability of the system.

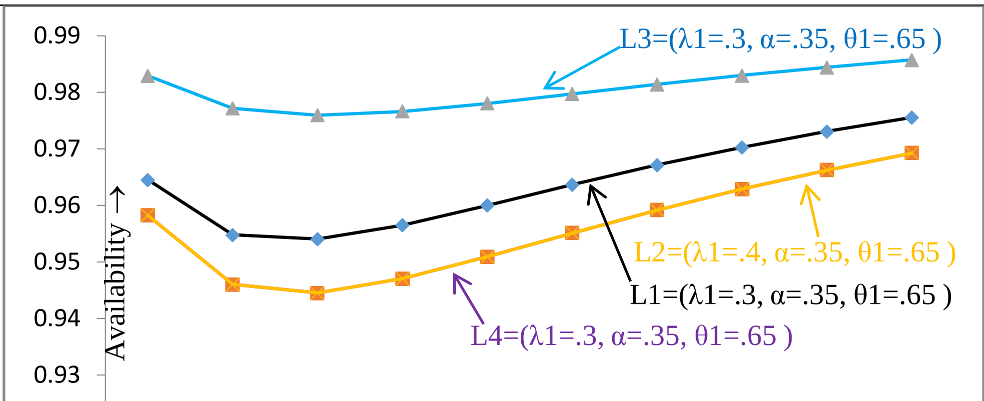

Fig. 3: Availability Vs Failure rate of the Original Unit λ→

Figure 4: Shows the graphical behavior of Profit of the system with respect to rate λ by which the original unit fails. In this figure, Profit also increased when the rate 'λ' by which original unit fail increased for all possible changes in the other parameters. This figure clearly shows that all the parameters having their valuable impact on the profit function. So priority to repair of original unit over the repair of duplicate unit increases the profit of the system.

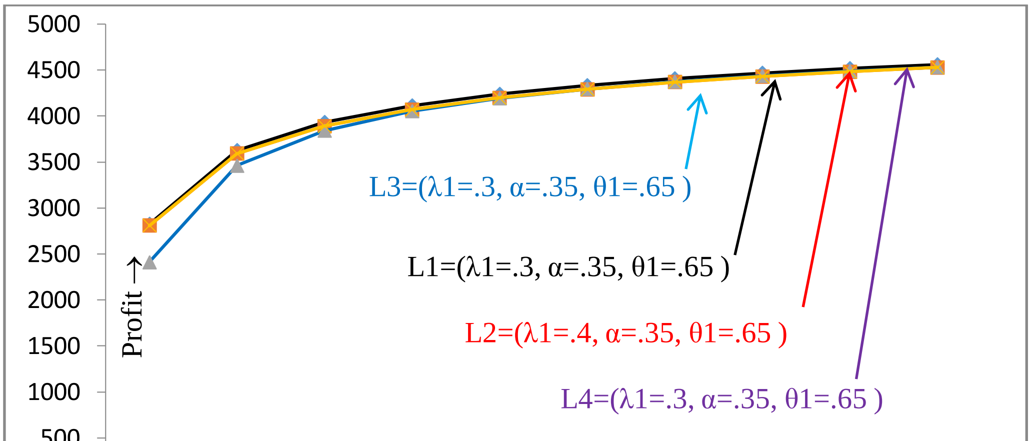

Fig. 4: Profit Vs Failure rate of the Original Unit λ→

Conclusion:

After the graphical discussion of the above model, the system can be more beneficial if priority is given to repair of the original unit over the repair of duplicate unit. Also, the failure rate of the original unit be controlled or better handing of the original unit can increase the performance of the system.

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