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New Interaction Solutions of the KP Equation

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Abstract: In this paper, three auxiliary equations method is presented. Analytical multiple function solutions including trigonometric function, exponential function, hyperbolic function and elliptic function can be easily obtained. New exact interaction solutions of the KP equation are obtained successfully by using the three auxiliary equations method. It is very significant to help physicists to analyze special phenomena in their relevant fields accurately.

Keywords: travelling wave, soliton, interaction solution, KP equation

AMS Subject Classification: 35Q51, 35Q53

INTRODUCTION

As we known, the complicated nature phenomena are often well described by nonlinear partial differential equations. The most representative nonlinear equation is the KP equation[1,2].

$$(\varepsilon^2 u_{xxx} + uu_x + u_t)_x + \lambda u_{yy} = 0. \tag{1}$$

The KP equation was first written in 1970 by Kadomtsew and Petviashvili.It came as a natural generalization of the KdV equation.The KP equation can be used to model water waves of long wavelength with weakly non-linear restoring forces and frequency dispersion. It can also be used to model waves in ferromagnetic media, as well as two-dimensional matter-wave pulses in Bose-Einstein condensates.

Soliton solutions of nonlinear partial differential equations have important applications in nonlinear optics, theoretical physics, plasma physics, fluid dynamics, semiconductors and other fields. It is meaningful to solve various exact solutions including trigonometric function, exponential function, hyperbolic function and other functions. The investigation of such analytical solutions helps us to understand the complicated physics phenomena well. In the past decades, many methods are proposed to obtain exact solutions of nonlinear partial differential equations: such as inverse scattering theory[3], Hirotas bilinear method[4], the truncated Painlev expansion[5], Darboux transformation[6] and so on. In recent years, a large number of powerful methods to solve nonlinear partial differential equations are considered. One of important methods is the auxiliary equation method[7,8].However, it is quite difficult to obtain the mixed solutions of KP equation, especially the mixed solutions formed by the nonlinear combination of elliptic function, hyperbolic function, trigonometric function and exponential function. In this paper, three auxiliary equations can be applied successfully to obtain multiple function solutions including trigonometric functions, exponential functions, hyperbolic functions and elliptic functions. This paper is organized as follow: Three auxiliary equations which we found out multiple function solutions in section 2. In section 3, we introduce the three auxiliary equations method. It can be applied to many nonlinear partial differential equations in different fields effectively. In section 4, this method is applied to the KP equation successfully. Many new exact interaction solutions are obtained. Some conclusions and discussions are given in section 5.

The new solutions of the novel auxiliary equation

For the novel auxiliary equation reads:

$$G^{''} = P + QG + RG^3 \quad , \tag{2}$$

where $G^{''} = G^{''}(\xi)$ .If P = R = 0, Q = 1, we obtain new multiple solutions of eq.(2) in the following:

$$G_1 = \frac{(\tanh(\eta)+\tan(\eta))e^{\eta}}{1+\tan(\eta)\coth(\eta)+\tanh(\eta)+\tan(\eta)}, \tag{3}$$

$$G_2 = \frac{(\tanh(\eta)+\coth(\eta))e^{\eta}}{1+\tanh(\eta)\tan(\eta)+\tan(\eta)+\coth(\eta)}, \tag{4}$$

$$G_3 = \frac{(\tanh(\eta)+\cot(\eta))e^{\eta}}{1+\cot(\eta)\coth(\eta)+\tanh(\eta)+\cot(\eta)}, \tag{5}$$

$$G_4 = \frac{(\cot(\eta)+\coth(\eta))e^{\eta}}{1+\tanh(\eta)\cot(\eta)+\cot(\eta)+\coth(\eta)}, \tag{6}$$

$$G_5 = \frac{(1+\tan(\eta)\coth(\eta))e^{\eta}}{1+\tanh(\eta)+\tan(\eta)+\tan(\eta)\coth(\eta)}, \tag{7}$$

$$G_6 = \frac{(1+\cot(\eta)\coth(\eta))e^{\eta}}{1+\tanh(\eta)+\cot(\eta)+\cot(\eta)\coth(\eta)}, \tag{8}$$

$$G_7 = \frac{(1+\tanh(\eta)\tan(\eta))e^{\eta}}{1+\tan(\eta)+\coth(\eta)+\tanh(\eta)\tan(\eta)}, \tag{9}$$

$$G_8 = \frac{(1+\tanh(\eta)\cot(\eta))e^{\eta}}{1+\cot(\eta)+\coth(\eta)+\tanh(\eta)\cot(\eta)}, \tag{10}$$

some imaginary solutions are omitted above, therefore real interaction solutions of nonlinear partial deferential equations are obtained when we apply this auxiliary equation into nonlinear partial deferential equations.

The three auxiliary equations method

Step1: For a given nonlinear partial differential equation with independent variables x,y,z,t,...:

$$P\,(t,x,y,z,u_t,u_x,u_{xt},u_{xx}\dots) = 0 \qquad (11)$$

We assume exact solutions of eq. (11) in the following form:

$$u(x,y,z,t)=\frac{a_1(x,y,z,t)G(x,y,z,t)}{F(x,y,z,t)\Phi(x,y,z,t)}+\frac{a_2(x,y,z,t)F(x,y,z,t)}{G(x,y,z,t)\Phi(x,y,z,t)}+\frac{a_3(x,y,z,t)\Phi(x,y,z,t)}{F(x,y,z,t)G(x,y,z,t)} \qquad \textbf{(12)}$$

where $G(x,y,z,t)$ satisfies eq. (2). $\Phi(x,y,z,t)$ and $F(x,y,z,t)$ satisfy the following auxiliary equations respectively.

$$\Phi' = \Phi \qquad (13)$$

$$F'^2 = A + BF^2 + CF^4 \qquad (14)$$

Substituting (12) into (11) along with (2),(13) and (14). We obtain a set of differential equations when we set the coefficients of $\Phi^{i_1}G^{i_2}F^{i_3}F'^{i_4}G'^{i_5}$ to zeroes. Therefore $a_i(x,y,z,t)$ will be determined by solving the set of differential equations. We will apply the method to the KP equation.

New Interaction Solutions of the KP Equation

The KP Equation is:

$$(\varepsilon^2 u_{xxx} + uu_x + u_t)_x + \lambda u_{yy} = 0. \qquad (15)$$

where ε, λ are constants.

We assume the solution of eq.(24) as the following,

$$u(x,y,t)=a_1(x)\frac{G(\eta)}{F(\theta)\Phi(\xi)}+a_2(x)\frac{F(\theta)}{G(\eta)\Phi(\xi)}+a_3(x)\frac{\Phi(\xi)}{F(\theta)G(\eta)},$$
$$\xi=k_1(x)+m_1(y)+l_1(t),\eta=k_2(x)+m_2(y)+l_2(t),\theta=k_3(x)+m_3(y)+l_3(t), \qquad (16)$$

where $a_1(x),a_2(x),a_3(x),k_1(x),m_1(y),l_1(t),k_2(x),m_2(y),l_2(t),k_3(x),m_3(y),l_3(t)$ are functions, they could be all determined in the later. Hence, substituting eq.(16) into eq.(15) along with (2),(13) and (14), a set of differential equations is obtained by equating the coefficients of $\Phi^{i_1}G^{i_2}F^{i_3}F'^{i_4}G'^{i_5}$ to zeroes. So that the unknown parameters $a_1(x),a_2(x),a_3(x),k_1(x),m_1(y),l_1(t),k_2(x),m_2(y),l_2(t),k_3(x),m_3(y),l_3(t)$ can be determined by solving the set of differential equations with Maple.

We get the following result:

$$\begin{aligned}
&a_1(x)=c_2a_2(x), a_2(x)=a_2(x), a_3(x)=0,\\
&k_1(x)=\ln(a_2(x))+c_4, m_1(y)=-\ln(c_7y-c_8), l_1(t)=l_1(t),\\
&k_2(x)=c_1, m_2(y)=c_5, l_2(t)=l_2(t),\\
&k_3(x)=c_3, m_3(y)=c_6, l_3(t)=l_3(t),\\
&A=A, B=B, C=C,
\end{aligned} \tag{17}$$

where c_i(i=1,2…8) are constants.

Substituting eq.(17) into eq.(16) and using the solutions of (2), (13) and (14), we obtain interaction solutions of eq.(15) in the following:

$$\begin{aligned}
u_1 &= c_2a_2(x)\frac{\tanh(\eta)+\tan(\eta)}{cn(\theta)[1+\tan(\eta)\coth(\eta)]e^{\xi}}\\
&+a_2(x)\frac{cn(\theta)[1+\tan(\eta)\coth(\eta)]}{[\tanh(\eta)+\tan(\eta)]e^{\xi}}+e^{\eta+\theta},
\end{aligned} \tag{18}$$

$$\begin{aligned}
u_2 &= c_2a_2(x)\frac{\tanh(\eta)+\coth(\eta)}{dn(\theta)[1+\tanh(\eta)\tan(\eta)]e^{\xi}}\\
&+a_2(x)\frac{dn(\theta)[1+\tanh(\eta)\tan(\eta)]}{[\tanh(\eta)+\coth(\eta)]e^{\xi}}+e^{\eta+\theta},
\end{aligned} \tag{19}$$

$$\begin{aligned}
u_3 &= c_2a_2(x)\frac{\tanh(\eta)+\cot(\eta)}{sn(\theta)[1+\cot(\eta)\coth(\eta)]e^{\xi}}\\
&+a_2(x)\frac{sn(\theta)[1+\cot(\eta)\coth(\eta)]}{[\tanh(\eta)+\cot(\eta)]e^{\xi}}+e^{\eta+\theta},
\end{aligned} \tag{20}$$

$$\begin{aligned}
u_4 &= c_2a_2(x)\frac{\cot(\eta)+\coth(\eta)}{sn(\theta)[1+\tanh(\eta)\cot(\eta)]e^{\xi}}\\
&+a_2(x)\frac{sn(\theta)[1+\tanh(\eta)\cot(\eta)]}{[\cot(\eta)+\coth(\eta)]e^{\xi}}+e^{\eta+\theta},
\end{aligned} \tag{21}$$

$$\begin{aligned}
u_5 &= c_2a_2(x)\frac{1+\tan(\eta)\coth(\eta)}{sn(\theta)[\tanh(\eta)+\tan(\eta)]e^{\xi}}\\
&+a_2(x)\frac{sn(\theta)[\tanh(\eta)+\tan(\eta)]}{[1+\tan(\eta)\coth(\eta)]e^{\xi}}+e^{\eta+\theta},
\end{aligned} \tag{22}$$

$$\begin{aligned}
u_6 &= c_2a_2(x)\frac{1+\cot(\eta)\coth(\eta)}{sn(\theta)[\tanh(\eta)+\cot(\eta)]e^{\xi}}\\
&+a_2(x)\frac{sn(\theta)[\tanh(\eta)+\cot(\eta)]}{[1+\cot(\eta)\coth(\eta)]e^{\xi}}+e^{\eta+\theta},
\end{aligned} \tag{23}$$

$$\begin{aligned}
u_7 &= c_2a_2(x)\frac{1+\tanh(\eta)\tan(\eta)}{sn(\theta)[\tan(\eta)+\coth(\eta)]e^{\xi}}\\
&+a_2(x)\frac{sn(\theta)[\tan(\eta)+\coth(\eta)]}{1+\tanh(\eta)\tan(\eta)e^{\xi}}+e^{\eta+\theta},
\end{aligned} \tag{24}$$

$$u_8 = c_2 a_2(x)\frac{1+\tanh(\eta)\cot(\eta)}{sn(\theta)[1+\cot(\eta)+\coth(\eta)]e^{\xi}}$$
$$+a_2(x)\frac{sn(\theta)[1+\cot(\eta)+\coth(\eta)]}{[1+\tanh(\eta)\cot(\eta)]e^{\xi}}+e^{\eta+\theta}, \qquad (25)$$

where $\xi = \ln(a_2(x)) + c_4 - \ln(c_7 y - c_8) + l_1(t), \eta = c_1 + c_5 + l_2(t), \theta = c_3 + c_6 + l_3(t).$

$a_2(x)$ is arbitrary functions of x. $l_1(t), l_2(t), l_3(t)$ are arbitrary functions of t. $c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8$ are arbitrary nonzero constants.

We get solutions (18)-(25) which contain trigonometric function, exponential function, hyperbolic functions and elliptic functions. They are all determined by solvable new arbitrary equation and they are novel interaction solutions of eq.(1) which are not obtained in ref[1, 2]. The phenomena of appearance of interaction waves are instantaneous and changeful. The interaction solutions are so complicated that the influences of the solutions are not easy to uncover. The effects are so significant, as nonlinear phenomena appear always everywhere in nature.

CONCLUSION AND DISCUSSION

In this paper, a new auxiliary equation is considered which we seek out multiple function solutions. Three auxiliary equations method is presented. Some new exact interaction solutions of the KP equation are obtained by using this method. This method can be easily and effectively applied to other partial differential equations. It draws great attention that solutions of the novel auxiliary equation themselves include trigonometric function, hyperbolic function, exponential function and other functions. It is not proposed in previous auxiliary equation methods. Complicate physical phenomena in nonlinear model systems will be described well when we analyze the typical interaction solutions we obtained in this paper.

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