International Journal of Latest Research in Engineering and Technology (IJLRET)
ISSN: 2454-5031
www.ijlret.com || Volume 03 - Issue 05 || May 2017 || PP. 72-78

Analysis of Step Response, Impulse and Ramp Response in the Continuous Stirred Tank Reactor System

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Abstract: The three main type reactor developed for batch, continuous-stirred tank, and plug-flow reactors are useful for modeling many complex chemical reactors. In this work, step response, impulse and ramp response is studied theoretician in a continuous stirred tank reactor. The aim of this study is to investigate the mathematical analysis on functions response in a continuous stirred tank reactor. The parameters assumed for analysis are the flow rate is constant for the whole system, $F = 0.085\frac{m^3}{\min}$, the volume of three thanks is the equal : $V_1= V_2= V_3= V_T= 1.05\ m^3$ and reaction rate $K= 0.04\ min^{-1}$. all three transfer function showed favorable results.

Keywords: Step Response, Ramp, Impulse Response, CSTR.

I. INTRODUCTION

There are several types of reactors used in chemical or biochemical industries. Continuous stirred tank reactors in the form of either single tank or (or more often) tanks in series, are used widely and these are particular suitable for liquid phase reactions (Danish et al, 2015).

The control of the operation of chemical reactors has attracted the attention of researchers for a long time (Adebekun,1996). The three main reactor types developed thus far batch, continuous-stirred tank, and plug-flow reactors are useful for modeling many complex chemical reactors.

Continuous stirred tank reactor system (CSTR) is a typical chemical reactor system . The CSTR consists of three tanks as showed in Figure 1. It is suppose that the overflow tanks are well-mixed , and the density is the equal in three tanks. So the assumptions for the overflow tanks, the volumes in the two tanks can be taken to be constant, and all flows are constant and same (Saad et al, 2013).

In this work, step response, impulse and ramp response is studied theoretician in a continuous stirred tank reactor. The aim of this study is to investigate the mathematical analysis on functions response in a continuous stirred tank reactor.

II. RESULT AND DISCUSSION

Modeling of the continuous stirred tank reactor system

Figure 1 shows the simple concentration process control. It is assumed that the overflow tanks are well-mixed isothermal reactors, and the density is the same in three tanks. Due to the assumptions for the overflow tanks, the volumes in the three tanks can be taken to be constant, and all flows are constant and equal. It is assumed that the inlet flow is constant. Figure 2 shows the block diagram of three tanks of chemical reactor.

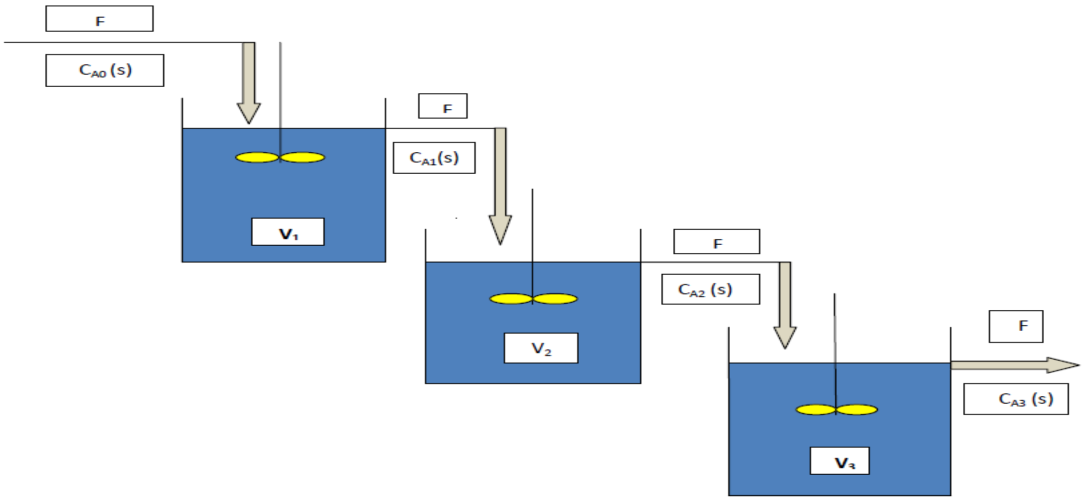

Fig1.The simple concentration process control

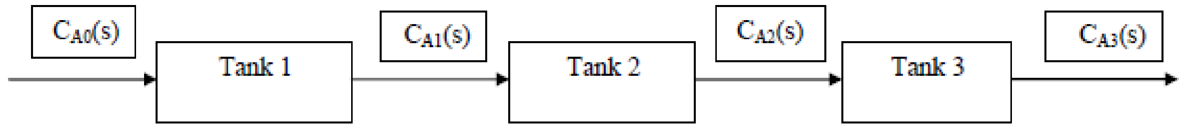

Fig. 2 The block diagram of the three tank system

The transfer function of the first tank can be expressed as follows:

$$FC_{A0} - FC_{A1} - V_1 K C_{A1} = V_1 \frac{dC_{A1}}{dt} \qquad (1)$$

Where F is the flow, $C_{A0}$is the inlet concentration of the first tank, $C_{A1}$is the outlet concentration of the first tank, C_{A2} is the outlet concentration of the Second tank and it too the inlet concentration of third tank, C_{A3} is the outlet concentration of third tank, V_1 is the volume of the first tank and K is the reaction rate. Equation 1 can be rearranged to be after simplify:

$$\frac{dC_{A1}}{dt} = \frac{1}{V_1}\left[FC_{A0} - FC_{A1} - V_1 K C_{A1}\right]$$

$$\frac{dC_{A1}}{dt} = \frac{F}{V_1} C_{A0} - C_{A1}\left[\frac{F + KV_1}{V_1}\right]$$

$$\frac{dC_{A1}}{dt} = \frac{F + KV_1}{V_1} C_{A1} = \frac{F}{V_1} C_{A0}$$

$$\frac{dCA1}{dt} + \frac{1}{\tau_1} C_{A1} = \frac{F}{V_1} C_{A0} \qquad (2)$$

By taking Laplace transform from equation (2):

$$SC_{A1}(s) + \frac{1}{\tau_1} C_{A1}(s) = \frac{F}{V_1} C_{A0}(s) \rightarrow$$

$$C_{A1}(s)\left[S + \frac{1}{\tau_1}\right] = \frac{F}{V_1} C_{A0}(s)$$

The transfer function of the first tank can be represented as (3):

$$\frac{C_{A1}(s)}{C_{A0}(s)} = \frac{\frac{F}{V_1}}{\left[S + \frac{1}{\tau_1}\right]} = \frac{\left[\frac{F}{V_1} \times \frac{V_1}{F + KV_1}\right]}{\tau_1 s + 1} = \frac{\frac{F}{F + KV_1}}{\tau_1 s + 1} = \frac{K_{p1}}{\tau_1 s + 1} \qquad (3)$$

Where K_{p1} is the gain of the transfer function of the first tank. The transfer function of the second tank can be expressed as follows:

$$FC_{A1} - FC_{A2} - V_2 K C_{A2} = V_2 \frac{dC_{A2}}{dt} \qquad (4)$$

Equation 4 can be rearranged to be (5):

$$\frac{dC_{A2}}{dt} + \frac{1}{\tau_2} C_{A2} = \frac{F}{V_2} C_{A1} \qquad (5)$$

By taking Laplace transform and rearranging equation 5, the transfer function of the second tank can be obtained:

$$\frac{C_{A2}(s)}{C_{A1}(s)} = \frac{K_{p2}}{\tau_2 s + 1} \qquad (6)$$

Where $K_{p2} = \frac{F}{F + KV_2}$ is the time constant for the second tank.

The transfer function of the third tank can be expressed as follows:

$$FC_{A2} - FC_{A3} - V_3 KC_{A3} = V_3 \frac{dC_{A3}}{dt} \qquad (7)$$

Equation 7 can be rearranged to be (8):

$$(8)\ \frac{dC_{A3}}{dt} + \frac{1}{\tau_3} C_{A3} = \frac{F}{V_3} C_{A2}$$

By taking Laplace transform and rearranging equation8, the transfer function of the third tank can be obtained:

$$\frac{C_{A3}(s)}{C_{A2}(s)} = \frac{K_{p3}}{\tau_3 s + 1} \qquad (9)$$

Where $K_{p3} = \frac{F}{F + KV_3}$

The transfer function of the combined three tanks with the assumed parameters can be presented equation (10):

$$G(s) = \frac{C_{A3}(s)}{C_{A0}(s)} = \frac{K_p{}^3}{(\tau_3 s + 1)^3} \qquad (10)$$

Figure 3 shows the block diagram of the open loop combined three tank system.

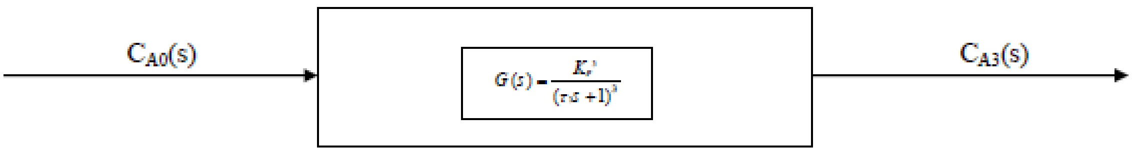

Fig3. The block diagram of the open loop system

The transfer function of the whole system can be obtained according to the following assumptions of parameters:

1- The flow rate is constant for the whole system $F = 0.085 \frac{m^3}{\min}$.
2- The volume of the three thanks is the equal: $V_1 = V_2 = V_3 = V_T = 1.05 m^3$
3- Reaction rate $K = 0.04 \min^{-1}$.

Due to time constants (τ) and the gains are same for three tank, they can be calculated as follow:

$$\tau = \frac{V}{F + KV} = \frac{1.05}{0.085 + (0.04 \times 1.05)} = 8.26$$

$$K_p = \frac{F}{F + KV} = 0.669$$

Transient response

The transfer function of a first- order system has been studied. Due to the study of response to different common forcing function: step, impulse, and ramp.

Step response

The transfer function, which is written by equation (11), is

$$G(s) = \frac{C_{A3}(s)}{C_{A0}(s)} = \frac{0.29942}{(8.26s+1)^3} \quad (11)$$

$$C_{A0}(t) = U(t) \Rightarrow C_{A0}(s) = \frac{1}{s} \quad (1.11)$$

Combining equation (11) and (1.11) gives:

$$C_{A3}(s) = \frac{0.29942}{s(8.26s+1)^3} \quad (2.11)$$

This can be expanded by partial fractions to give:

$$\frac{0.29942}{s(8.26s+1)^3} = \frac{A}{s} + \frac{B}{(8.26s+1)} + \frac{C}{(8.26s+1)^2} + \frac{D}{(8.26s+1)^3}$$

Multiply equation by the denominator:

$$\frac{0.29942s(8.26s+1)^3}{(8.26s+1)^3 s} = \frac{As(8.26s+1)^3}{s} + \frac{Bs(8.26s+1)^3}{(8.26s+1)} + \frac{Cs(8.26s+1)^3}{(8.26s+1)^2} + \frac{Ds(8.26s+1)^3}{(8.26s+1)^3}$$

Simplify:

$$0.29942 = A(8.26s+1)^3 + Bs(8.26s+1)^2 + Cs(8.26s+1) + Ds$$

Combining equation (11) and (1.11) gives:

$$C_{A3}(s) = \frac{0.29942}{s(8.26s+1)^3} \ (2.11)$$

This can be expanded by partial fractions to give:

$$\frac{0.29942}{s(8.26s+1)^3} = \frac{A}{s} + \frac{B}{(8.26s+1)} + \frac{C}{(8.26s+1)^2} + \frac{D}{(8.26s+1)^3}$$

Multiply equation by the denominator:

$$\frac{0.29942s(8.26s+1)^3}{(8.26s+1)^3 s} = \frac{As(8.26s+1)^3}{s} + \frac{Bs(8.26s+1)^3}{(8.26s+1)} + \frac{Cs(8.26s+1)^3}{(8.26s+1)^2} + \frac{Ds(8.26s+1)^3}{(8.26s+1)^3}$$

Simplify:

$$0.29942 = A(8.26s+1)^3 + Bs(8.26s+1)^2 + Cs(8.26s+1) + Ds$$

Solve the unknown parameters by plugging the real roots of the denominator:

$$s = 0, s = -\frac{1}{8.26}$$

For the denominator roots 0 and $s = -\frac{1}{8.26}$:

$$A = 0.29942, D = -2.47312$$

Plug in the solutions to the known parameters:

$$0.29942 = 0.29942(8.26s+1)^3 + Bs(8.26s+1)^2 + Cs(8.26s+1) + (-2.47312)s$$

Expand:

International Journal of Latest Research in Engineering and Technology (IJLRET)
ISSN: 2454-5031
www.ijlret.com || Volume 03 - Issue 05 || May 2017 || PP. 72-78

$$0.29942 = 68.2276Bs^2 + 168.74113s^3 + 16.52Bs^2 + 8.26Cs^2 + 61.28612s^2 + Cs + 4.94651s + Bs + 0.29942$$

Group elements according to power of s:

$$0.29942 = s^3(68.2276B + 168.74113) + s^2(8.26C + 16.52B + 61.28612) + s(4.94651 + B + C) + 0.29942$$

Equate the coefficients of similar terms on both sides to create a list of equation:

$$\begin{bmatrix} C + B + 4.94651 = 0 \\ 68.2276B + 8.26C + 61.28612 = 0 \\ 68.2276B + 168.4113 = 0 \end{bmatrix}$$

Solve system of equations:

$$B = -2.47321, C = -2.4733$$

Plug the solutions to the partial f partial fraction parameters to obtain the final results:

$$C_{A3}(s) = \frac{0.29942}{s} + \frac{(-2.47321)}{(8.26s+1)} + \frac{(-2.4733)}{(8.26s+1)^2} + \frac{(-2.47312)}{(8.26s+1)^3}$$

Inverse Laplace :Taking

$$C_{A3}(s) = \left[0.29942 - 0.29942e^{-0.121065t}(-0.0362507e^{-0.121065t} \times t)(-0.00219419e^{-0.121065t} \times t^2)\right]$$

Solve the unknown parameters by plugging the real roots of the denominator:

Ramp Response:

The transfer function, which is written by equation (12) is

$$G(s) = \frac{C_{A3}(s)}{C_{A0}(s)} = \frac{K_p^3}{(\tau_3 s+1)^3} \qquad (12)$$

$$C_{A0}(t) = t \Rightarrow C_{A0}(s) = \frac{1}{s^2}$$

Combining equation (12) and (1.12) gives:

$$C_{A3}(s) = \frac{K_p^3}{s^2(\tau_3 s+1)^3} = \frac{0.29942}{s^2(8.26s+1)^3}$$

This can be expanded by partial fractions to give:

$$\frac{0.29942}{s^2(8.26s+1)^3} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(8.26s+1)} + \frac{D}{(8.26s+1)^2} + \frac{E}{(8.26s+1)^3}$$

Multiply equation by the denominator:

$$\frac{0.29942s^2(8.26s+1)^3}{s^2(8.26s+1)^3} = \frac{As^2(8.26s+1)^3}{s} + \frac{Bs^2(8.26s+1)^3}{s^2} + \frac{Cs^2(8.26s+1)^3}{(8.26s+1)} + \frac{Ds^2(8.26s+1)^3}{(8.26s+1)^2} + \frac{Es^2(8.26s+1)^3}{(8.26s+1)^3}$$

Simplify:

$$0.29942 = As(8.26s+1)^3 + B(8.26s+1)^3 + Cs^2(8.26s+1)^3 + Ds^2(8.26s+1)^3 + Es^2$$

Solve the unknown parameters by plugging the real roots of the denominator:

$$s = 0, s = -\frac{1}{8.26}$$

For the denominator roots 0 and $s = -\frac{1}{8.26}$:

$$B = 0.29942$$
$$E = 20.42428$$

Plug in the solutions to the known parameters:

$$0.29942 = As(8.26s+1)^3 + 0.29942(8.26s+1)^3 + Cs^2(8.26s+1)^3 + Ds^2(8.26s+1)^3 + 20.42428s^2$$

Expand:

$$0.29942 = 68.2276Cs^4 + 563.5598As^4 + 204.6828As^3 + 8.26Ds^2$$
$$+168.74113s^3 + 16.52Cs^2 + Cs^2 + 81.7104s^2 + 24.78As^2 + Ds^2 + As + 7.41963s + 0.29942$$

Group elements according to power of s:

$$0.29942 = s^4\left[68.2276C + 563.5598A\right] + s^3\left[8.26D + 16.52C + 168.74113 + 204.6828A\right] \rightarrow$$
$$+s^2\left[24.78A + 81.7104 + C + D\right] + s\left[A + 7.41963\right] + 0.29942$$

Equate the coefficients of similar terms on both sides to create a list of equation:

$$\begin{bmatrix} A + 7.41963 = 0 \\ C + 81.7104 + 24.78A + D = 0 \\ 204.6828A + 8.26D + 168.74113 + 16.52C + 0 \\ 68.2276C + 563.55998A = 0 \end{bmatrix}$$

$$A = -7.41963, C = 61.2862, D = 40.8618$$

Plug the solutions to the partial fraction parameters to obtain the final results:

$$C_{A3}(s) = \frac{-7.41963}{s} + \frac{0.29942}{s^2} + \frac{61.2862}{8.26s + 1} + \frac{40.8618}{(8.26s + 1)^2} + \frac{20.42428}{(8.26s + 1)^3}$$

Inverse Laplace :Taking

$$C_{A3}(t) = \left[-7.41963 + 0.29942t + 7.41963e^{-\frac{1}{8.26}t} + 0.5989te^{-0.121065t} + 0.0181208e^{-0.121065} \times t^2\right]$$

Impulse response

The transfer function, which is written by equation (13) is

$$G(s) = \frac{Y(s)}{X(s)} = \frac{C_{A3}(s)}{C_{A0}(s)} = \frac{Kp^3}{(\tau_3 s + 1)^3}$$

$$C_{A0}(t) = \delta(t) \Rightarrow C_{A0}(s) = 1$$

Combining equation (13) and (1.13) gives:

$$C_{A3}(s) = \frac{Kp^3}{(\tau_3 s + 1)^3} = \frac{(0.669)^3}{(8.26s + 1)^3}$$

This can be expanded by partial fractions to give:

$$\frac{Kp^3}{(\tau_3 s + 1)^3} = \frac{A}{(\tau_3 s + 1)} + \frac{B}{(\tau_3 s + 1)^2} + \frac{C}{(\tau_3 s + 1)^3}$$

Multiply equation by the denominator:

$$\frac{0.29942(\tau_3 s + 1)^3}{(\tau_3 s + 1)^3} = \frac{A(\tau_3 s + 1)^3}{8.26s + 1} + \frac{B(\tau_3 s + 1)^3}{(\tau_3 s + 1)^2} + \frac{C(\tau_3 s + 1)^3}{(\tau_3 s + 1)^3} \rightarrow$$

Simplify:

$$0.29942 = A(\tau_3 s + 1)^2 + B(\tau_3 s + 1) + C$$

Solve the unknown parameters by plugging the real roots of the denominator:

$$s = -\frac{1}{8.26}$$

$$C = 0.29942$$

Plug in the solutions to the known parameters:

$$0.29942 = A(8.26s+1)^2 + B(8.26s+1) = 0.29942$$

Expand:

$$0.29942 = 68.2276As^2 + 16.5As + A + 8.26Bs + B + 0.29942$$

Group elements according to power of s:

$$0.29942 = 68.2276As^2 + s\left[16.5A + 8.26B\right] + \left[0.29942 + B + A\right]$$

Equate the coefficients of similar terms on both sides to create a list of equation:

$$\begin{bmatrix} 16.5A + 8.26B = 0 \\ 0.29942 + B + A = 0.29942 \\ 68.2276A = 0 \end{bmatrix}$$

$$A = 0, B = 0, C_{A3}(s) = \frac{0.29942}{(8.26s+1)^3}$$

Plug the solutions to the partial fraction parameters to obtain the final results:

$$\left[\frac{0}{8.26s+1} + \frac{0}{(8.26s+1)^2} + \frac{0.29942}{(8.26s+1)^3}\right] = C_{A3}(s)$$

$$C_{A3}(s) = \frac{0.29942}{(8.26s+1)^3}$$

Inverse Laplace:

$$C_{A3}(t) = 0.000265651e^{-0.121065t} \times t^2$$

III. CONCLUSION

The mathematical analysis on transform of the response is investigate in a continuous stirred tank reactor. All three transfer function showed favorable results.

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