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Mathematical modeling of the process of drilling mud filtrate penetration into the reservoir

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Abstract: The work is devoted to mathematical and numerical modeling of mud filtrate penetration processes when drilling wells in reservoirs containing oil. It is known that the distribution of saturation of pore space significantly affects the transformation of the field of the resistivity of the zone of penetration of the filtrate of the drilling mud. On the basis of two-dimensional self-similar solutions, the existence of a unique smooth solution close to the corresponding solution of the one-dimensional Stefan problem in self-similar variables is proved. The paper proposes an effective algorithm for the numerical solution of the problem under consideration. In modeling, the flow region is stellar relative to the center where the well is located, and the desired solution monotonically decreases along the radius of the well effect. Using special variables, the main problem is reduced to an equivalent boundary-value problem for a second-order nonlinear elliptic equation in a fixed domain. Further, with the help of a numerical solution, a comparative analysis was carried out.

Keywords: Mud filtration, electrical resistivity (ER), high-frequency induction isoparametric sounding method (HIISM), lateral logging sounding method (LLS).

I. INTRODUCTION

Throughout the following it is assumed that a liquid-saturated porous medium in near wellbore space is a substantially biphasic medium of one phase whose phases are the particles of the displaced liquid. Similar problems were considered in [1-7]. The first problem was investigated by I. Stephan [3] in 1889 and through the probability integral he managed to write out an explicit solution. In papers [4,7] A.M. Meirmanov and B.M. Anisyutin, various versions of self-similar solutions of the problems of filtration of a compressible fluid with free (unknown) boundaries were investigated. In the present paper, the above approach is developed to solve the problem of the propagation of mud filtrate in near-well space. In [5], A. Lazaridis numerically investigated the problem of melting a rectangular ingot and showed that for small values of time the corresponding solution is close to the self-similar solution. A different computational algorithm is proposed below and a comparative analysis with *HIISM*data is carried out.

II. SETTING OF THE PROBLEM

Assuming that the medium under consideration is hydrophilic in the vicinity of the injection well, it is assumed that the mathematical model of Rappoport-Lis in the case $\vec{u}_1 = -\vec{u}_2$ is assumed, i.e. water (displacement agent) is absorbed into the specimen, displacing the oil in the direction opposite to the motion of water, where $\vec{u}_i,\ \ i = 1,2$ is the velocity of water and oil. Then the corresponding equation with respect to saturation takes the form:

$$m \cdot \frac{\partial S}{\partial t} = div\left(\Phi(S) \cdot \nabla S\right), \qquad (1.1)$$

where*m* - porosity, $\Phi(S) = -\dfrac{f_1 \cdot f_2}{f_1 + \mu \cdot f_2} \cdot J'(S) \geq 0$ - a function that depends on the behavior of the functions f_i, $i=1,2$ and $J(s)$. The last inequality holds for a hydrophilic medium. A description of the process of countercurrent capillary impregnation is given in [1], and the corresponding mathematical problem in the one-dimensional case is investigated. Without loss of generality, it is assumed everywhere below that $m = 1$.Let (r, φ) be polar coordinates in the plane.A two-dimensional region G (t) is investigated, bounded by the well-known line $r=R_0$– the well radius and $r = R(\varphi,t), R_0 < R(\varphi,t)$ the sought line, and a nonnegative function

$\theta(r,\varphi,t)$. In addition, the equation (1.1) by the transformation $\theta = \int_{s_*}^{s} \Phi(\varsigma)d\varsigma$ in the polar coordinate system is given by the following form:

$$a(\theta)\cdot\frac{\partial\theta}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) + \frac{\partial}{\partial\varphi}\left(\frac{1}{r^2}\frac{\partial\theta}{\partial\varphi}\right) + f(r,\varphi,t) \quad \text{at } (r,\varphi)\in G(t), \tag{1.2}$$

on an unknown borderare fulfilled next conditions:

$$\theta = 0, \quad \frac{\partial\theta}{\partial t} = \left(\frac{\partial\theta}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\theta}{\partial\varphi}\right)^2 \quad \text{at } r = R(\varphi,t), \tag{1.3}$$

but on a well-known boundary (at the bottom of the well): $\theta = \theta_0$ at $r = R_0$ (1.4)

Coefficient $a(\theta) \ge a_0 \equiv const > 0$ and $f(r,\varphi,t)$ are known and quite smooth functions. In addition, at the initial moment of time, the desired functions satisfy the condition (1.3). The problem (1.2) - (1.4) is solved by transforming the original domain into a rectangular domain.

III. SOLUTION

Theorem 1.Let $a(\theta) \in C^2[0,\infty)$, $U_1(\varphi) \in C^{2+\alpha}[0,2\pi]$ and $U_0^2 \cdot a_0 > 2$, where U_0 =const> 0. Then, with sufficient small $\sigma,\ \theta = \theta_1(\varphi, \frac{r}{\sqrt{t}}),\ R = t^{-1/2}\cdot U_2(\varphi)$ with a period 2π periodic with respect to φ twice continuously differentiable in the domains $\Pi = \{\varphi:\ 0<\varphi<2\pi\}$ and $Q = \{(\varphi,\xi): 0<\varphi<2\pi,\ U_0 + \sigma\cdot U_1(\varphi) < \xi < U_2(\varphi)\}$ functions $U_2(\varphi)$ and $\theta(\varphi,\xi)$, whereconstant σ depends only from a_0, U_0, θ_0 and norm of function $U_1(\varphi)$ in space $C^{2+\alpha}[\overline{\Pi}]$.

Proceeding from the results of [4], to prove the theorem, we introduce the Mises variables: $\tau = t,\ \ x = \varphi,\ \ y = \theta(\varphi,r,t)$. ThenthedomainG(t) correspondstothedomain $\Omega = \{(x,y):\ 0<x<2\pi, 0<y<\theta_0\}$, and new unknown function $u(x,y,\tau) = r$ satisfies the following equation in**Ω**:

$$a(y)\cdot\frac{\partial u}{\partial t} - \frac{\partial}{\partial x}\left[\frac{u_x}{u^2}\right] + \frac{\partial}{\partial y}\left[\frac{1}{u^2}\cdot\left(1+\frac{u_x^2}{u^2}\right)\right] + \frac{1}{u} = 0 \tag{1.5}$$

and boundary conditions:

$$\frac{\partial u}{\partial\tau} + \frac{1}{u_y}\cdot\left(1+\frac{u_x^2}{u^2}\right) = 0 \quad \text{at } y = 0 \tag{1.6}$$

$$u = \tau^{1/2}\cdot\left[U_0 + \sigma\cdot U_1(\varphi)\right] \quad \text{at } y = \theta_0 \tag{1.7}$$

By the method of separation of variables with respect to time and spatial variables, we obtain the following problem:

$$\Lambda_1 U \equiv \frac{\partial}{\partial y}\left[\frac{1}{U_y}\left(1+\frac{U_x^2}{U^2}\right)\right] + \frac{\partial}{\partial x}\left[\frac{U_x}{U^2}\right] + \frac{1}{U} + \frac{a\cdot U}{2} = 0 \quad \text{at } (x,y)\in\Omega \tag{1.8}$$

$$\Lambda_2 U \equiv \frac{U}{2} + \frac{1}{U_y}\left(1+\frac{U_x^2}{U^2}\right) = 0 \quad \text{at } y = 0 \tag{1.9}$$

$$\Lambda_3 U \equiv U_0 + \sigma \cdot U_1(x) = U \text{ at } y = \theta_0. \qquad (1.10)$$

The solution of problem (1.8) - (1.10) is obtained with the help of the implicit function theorem for small perturbations of the one-dimensional Stefan problem in self-similar variables.

Theorem 2.Let $R_0(\varphi,t) = t^{1/2}\{U_0 + \delta U_1(\varphi)\}$, $U_0 = const > 0,\ U_1 \in C^{2+\infty}[0,2\pi]$ and $U_0^2 \cdot a_0 > 2$. Then for sufficiently small $\delta,\ |\delta| < \delta_*$, the problem (1.1) - (1.3) has a unique solution of the form $\theta = \theta_1\left(\varphi, r \cdot t^{-\frac{1}{2}}\right)$, $R = t^{-\frac{1}{2}} U_2(\varphi)$ with a periodic by φ with period 2πtwice continuously differentiable respectively in the domain $\Pi = \{\varphi:\ 0 < \varphi < 2\pi\}$ and $Q = \{(\varphi,\xi):\ 0 < \varphi < 2\pi, U_0 + \delta U_1(\varphi) < \xi < U_2(\varphi)\}$ with functions $U_2(\varphi)$ and $\theta_1(\varphi,\xi)$.

The constant δ_* depends only on a_0, U_0, θ_0 and the norm of the function $U_1(\varphi)$ in the space $C^{2+\infty}[\overline{\Pi}]$.

2. The formulation of an equivalent boundary value problem. Suppose that $\partial\theta/\partial r < 0$ and consider the new independent variables $\tau = t, x = \varphi, y = \theta(\varphi, r, t)$. In these variables, the $G(t)$ region corresponds to the $\Omega = \{(x,y): 0 < x < 2\pi,\ 0 < y < \theta_0\}$ domain, and the new sought function $u(x,y,\tau) = r$ of the **Ω** satisfies the equation

$$a(y)\frac{\partial u}{\partial \tau} - \frac{\partial}{\partial x}\left[\frac{u_x}{u^2}\right] + \frac{\partial}{\partial y}\left[\frac{1}{u_y}\left(1 + \frac{u^2{}_x}{u^2}\right)\right] + \frac{1}{u} = 0 \qquad (2.1)$$

and the boundary condition

$$\frac{\partial u}{\partial \tau} + \frac{1}{u_y}\left(1 + \frac{u^2{}_x}{u^2}\right) = 0 \text{ at } y = 0 \qquad (2.2)$$

$$u = \tau^{\frac{1}{2}}\left[U_0 + \delta U_1(\varphi)\right] \text{ at } y = \theta_0 \qquad (2.3)$$

Equation (2.1) and the boundary condition (2.2) are obtained from (1.1) and (1.2) by means of relations

$$\frac{\partial\theta}{\partial t} = -\frac{u_\tau}{u_y},\quad \frac{\partial\theta}{\partial\varphi} = -\frac{u_x}{u_y},\quad \frac{\partial\theta}{\partial r} = \frac{1}{u_y},\quad \frac{\partial}{\partial\varphi} = \frac{\partial}{\partial x} - \frac{u_x}{u_y}\cdot\frac{\partial}{\partial y},\quad \frac{\partial}{\partial r} = \frac{1}{u_y}\frac{\partial}{\partial y}.$$

It is easy to see that the boundary value problem (2.1) - (2.3) admits solutions of the form $u = \tau^{\frac{1}{2}} U\ (x,y)$:

$$L_1 U = \frac{\partial}{\partial y}\left[\frac{1}{U_y}\left(1 + \frac{U_x^2}{U^2}\right)\right]\frac{\partial}{\partial x}\left[\frac{U_x}{U^2}\right] + \frac{1}{U} + \frac{aU}{2} = 0,\quad (x,y)\in\Omega \qquad (2.4)$$

$$L_2 U = \frac{U}{2} + U_y^{-1}\left(1 + U_x^2 U^{-2}\right) = 0 \text{ at } y = 0 \qquad (2.5)$$

$$L_3 U = U_0 + \delta U_1(x) = U \text{ at } y = \theta_0 \qquad (2.6)$$

PROBLEM (A). It is required to define in the domain **Ω** a function $U(x,y)$ periodic in x with period 2π, belonging to the class $C^{2+\alpha}(\overline{\Omega})$ and satisfying the boundary-value problem (2.4) - (2.6).

3. Solvability of the problem (A). We obtain the solution of problem (A) with the help of the implicit function

theorem for small perturbations of the one-dimensional (in self-similar variables) Stefan problem:

$$\frac{d}{dy}\left[\left(\frac{d\Phi}{dy}\right)^{-1}\right]+\frac{a(y)}{2}\Phi+\Phi^{-1}=0, \quad 0<y<\theta_0, \tag{3.1}$$

$$\Phi(0)+2\left[\frac{d\Phi}{dy}(0)\right]^{-1}=0, \quad \Phi(\theta_0)=U_0. \tag{3.2}$$

LEMMA 1. For given positive U_0, θ_0 and twice continuously differentiable function $a(y)$, $a(y)\geq a_0>0$, such that $U_0^2a_0>2$, there exists a unique solution $\Phi(y)$ of the boundary value problem (3.1), (3.2) satisfying the condition

$$a(y)\Phi^2(y)>2 \text{ at } 0\leq y\leq\theta_0. \tag{3.3}$$

PROOF. We fix an arbitrary constant $b>U_0$ and consider the problem of determining the function $\upsilon(y)$, satisfying the equation (3.1) and the boundary conditions

$$\frac{d\upsilon}{dy}(0)=-\frac{2}{b}, \quad \upsilon(\theta_0)=U_0. \tag{3.4}$$

The solution of the latter is found from the integral equation

$$\upsilon(y)=U_0+\int_y^{\theta_0}\left[\frac{b}{2}+\int_0^s\left\{\frac{1}{\upsilon(\xi)}+\frac{a(\xi)}{2}\upsilon(\xi)\right\}d\xi\right]^{-1}ds\equiv F_0(\upsilon,b). \tag{3.5}$$

Assuming the solution of (3.5) is not negative, the estimate

$$U_0\leq\upsilon(y)\leq U_0+\frac{2}{b}\theta_0, \tag{3.6}$$

(More precisely, the $U_0\leq F_0(\upsilon,b)\leq U_0+\frac{2}{b}\theta_0$ for every positive function $\upsilon(y)$). Proceeding from this estimate, we consider the convex set S of all continuous functions $\upsilon(y)$ satisfying the inequalities (3.6). The right-hand side of the $F_0(\upsilon,b)$ of equation (3.5) is completely continuous (in the metric of $C[0,\theta_0]$) by an operator on the set S that takes this set into itself. By the Schauder theorem, there is at least one fixed point of the operator $F_0(\upsilon,b)$, which is a solution of the boundary value problem (3.1), (3.4). Assuming further in (3.5) of the $\upsilon(0)=b$, we obtain an equation for determining the constant b:

$$b=U_0+\int_0^{\theta_0}\left[\frac{b}{2}+\int_0^y\left\{\frac{1}{\upsilon(\xi)}+\frac{a(\xi)}{2}\upsilon(\xi)\right\}d\xi\right]^{-1}dy\equiv F_1(b)\cdot \tag{3.7}$$

The function $z=F_1(b)$ is bounded above by the function $z=U_0+\frac{2}{b}\theta_0$, whose graph intersects the line (U_0,b_*) at the point b_0. Since $F_1(\theta_0)>U_0$, there is at least one point b_0 of the intersection of the graph of $z=F_1(b)$ with the line $z=b$: $b_0=F_1(b_0)$ on (U_0,b_*). The constant b_0 determines the solution of the $\Phi(y)$ of the boundary value problem (3.1), (3.2). From the condition (1.4) and the inequality $\Phi(y)\geq U_0$ it follows that the condition (3.3) holds for $U_0^2a_0>2$.

We show that $\Phi(y)$ is the unique solution not only of the boundary value problem (3.1), (3.2), but also the unique solution of the problem (A) for the value of the parameter $\delta = 0$.

Let $U(x,y)$ be a solution of problem (A) satisfying the condition $U = U_0$ for $y = \theta_0$, condition (3.3) and $U_y < 0$, for $x, y \in \Omega$. For the difference $V = U - \Phi$ the equality

$$\int_{\Omega} \left\{ \frac{\Phi_y}{U_y U^2} V_x^2 + \frac{1}{\Phi_y U_y} V_y^2 + \left(\frac{a}{2} - \frac{1}{U\Phi} \right) V^2 \right\} dxdy = 0 \quad , \tag{3.8}$$

which is obtained if the equation for V(x,y) is multiplied by V(x,y) and integrated by parts over the domain Ω using the boundary conditions for $y = 0$, $y = \theta_0$, and the periodicity of the function V(x,y) with respect to the variable x. Since $\Phi_y \cdot U_y$ and $U \cdot \Phi \cdot a > 2$, (3.8) holds only for $V(x,y) \equiv 0$, which completes the proof of the lemma.

Let $C^{2+\alpha}(\overline{\Omega})$ be the space of all continuous functions U(x,y) having continuous second derivatives in **Ω** satisfying the Holder condition with exponent $\alpha,\ 0 < \alpha < 1$ and $|U|_{\Omega}^{(2+\alpha)}$ being the norm of the function U(x,y) in this space [4, p. 60]. We denote by H1 the space of all functions of class $C^{2+\alpha}(\overline{\Omega})$ periodic in x with period2π, and H0 is a Banach space whose elements are triples $f = (f_1, f_2, f_3)$ of $f_1 \in C^{\alpha}(\overline{\Omega})$, $f_2 \in C^{1+\alpha}(\overline{\Pi})$, $f_3 \in C^{2+\alpha}(\overline{\Pi})$ - periodic functions periodic in x with period 2π.

The problem (A) can be treated as a solution of the operator equation

$$LU \equiv (L_1U, L_2U, L_3U) = f \tag{3.9}$$

with the operator L acting from the Banach space H1 to the Banach space H0.

As was shown in Lemma 1, equation (3.9) has a unique solution $\Phi \in H_1$ for the right-hand side $f_0 = (0, 0, U_0)$. We show that the derivative of the operator L, computed on the element $\Phi \in H_1$, has a bounded inverse operator. To do this, we consider the linear boundary value problem

$$-\frac{\partial}{\partial y}\left[\frac{\upsilon_y}{\Phi_y^2}\right] - \frac{\partial}{\partial x}\left[\frac{\upsilon_x}{\Phi^2}\right] + \left(\frac{a}{2} - \frac{1}{\Phi^2}\right)\upsilon = g_1 \quad , x, y \in \Omega$$

$$\frac{\upsilon}{2} - \frac{\upsilon_y}{\Phi_y^2} = g_2(x) \quad \text{at } y = 0,$$

$$\upsilon = g_3(x) \quad \text{at } y = \theta_0$$

on the definition of the function $v \in H_1$ with respect to an arbitrary function $g \in H_0$. By virtue of condition (3.3), the maximum principle is valid for the formulated problem

$$|\upsilon|_{\Omega}^{(0)} \le \max\left(\lambda |g_1|_{\Omega}^{(0)}, 2|g_2|_{\Pi}^{(0)}, |g_3|_{\Pi}^{(0)}\right), \tag{3.10}$$

where $\lambda = 2\Phi^2(0)\left[U_0^2 a_0 - 2\right]^{-1}$. Indeed, the maximum of*v(x,y)*can not be attained at the boundaries of x=0 and x=2π, since in this case, by virtue of the periodicity of *v(x,y)*with respect to x, $\upsilon_x = 0$, which contradicts the Hopf-Giraud principle [2, p. 159]. If the maximum of *v(x,y)*is reached in **Ω** or at the boundaries of y=0, y=θ_0, then it is estimated in the usual way.

From [7], the estimate (3.10) and the periodicity of the function *v(x,y)*by the x, it follows that

$$|\upsilon|_{\Omega}^{(2+\alpha)} \le c\left(|g_1|_{\Omega}^{(\alpha)} + |g_2|_{\Pi}^{(1+\alpha)} + |g_3|_{\Pi}^{(2+\alpha)}\right),$$

With an absolute constant c. The last inequality is equivalent to the existence of the inverse operator to the derivative of the L operator, calculated on the element $\Phi \in H_1$. By the implicit function theorem, equation (3.9) has a unique solution $U \in H_1$ that depends continuously on δ in the norm of the space $C^{2+\alpha}(\overline{\Omega})$, if only $f = (0, 0, U_0 + \delta U_1)$ and $|\delta| < \delta_*$. Since $|\ln|\Phi_y||_{\Omega}^{(0)} < \infty$, the parameter δ_* can be chosen so small that

$$|\ln|U_y||_{\Omega}^{(0)} < \infty. \tag{3.11}$$

Letusformulatetheresult:

LEMMA 2. Under the hypotheses of Theorem 1, there exists a unique solution U(x,y) of problem (A) belonging to the space H1 that depends continuously on δ in the norm of this space and satisfies condition (3.11), provided only that $|\delta| < \delta_*$. The number δ_* depends on a_0, U_0, θ_0 and $|U_1|_{\Pi}^{(2+\alpha)}$.

It is easy to see that the function $\theta_1(\varphi, \xi)$ found from the identity $\xi = U[\varphi, \theta_1(\varphi, \xi)]$ (which is possible by virtue of condition (3.11)) and the function $R(\varphi, t) = t^{1/2} U_2(\varphi)$, $\theta(\varphi, r \cdot t) = \theta_1\left(\varphi, r \cdot t^{-\frac{1}{2}}\right)$ determines the solution f of the boundary value problem (1.1) - (1.3) and has the smoothness indicated in Theorem 1.

IV. CONCLUSION

We prove the existence of a unique smooth solution close to the corresponding solution of the one-dimensional Stefan problem in self-similar variables. The paper proposes an effective algorithm for the numerical solution of the problem under consideration. In modeling, the flow region is stellar relative to the center where the well is located, and the desired solution monotonically decreases along the radius of the well effect. Using special variables, the main problem is reduced to an equivalent boundary-value problem for a second-order nonlinear elliptic equation in a fixed domain. Further, with the help of a numerical solution, a comparative analysis was carried out.

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